Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2025**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

[Turn over

1 **(a)** Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $3x - 1 < |2x - 3|$. [2]

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2 It is given that $2\ln p + \ln(p-1) - \frac{1}{2}\ln(q+1) = 3$.

Find q in terms of p. [3]

3 Find the complex numbers z for which $\frac{z+5i}{z-5}$ is real and $|z| = \sqrt{17}$. Give your answers in the form $z = x + iy$, where x and y are real. [6]

4 The parametric equations of a curve are

$$x = e^{\tan t}, \quad y = 3\tan^2 t.$$

Find the equation of the tangent to the curve at the point (e, 3). Give your answer in the form $y = mx + c$, where m and c are exact. [6]

5 The polynomial $3x^3 + pax^2 + 7a^2x + qa^3$ is denoted by $f(x)$, where p, q and a are constants and $a \neq 0$.

When $f(x)$ is divided by $(x+2a)$ the remainder is $-22a^3$. When $f(x)$ is divided by $(3x-a)$ the remainder is $-a^3$.

Find the values of p and q. [5]

6 It is given that $z_1 = 3e^{\frac{1}{4}\pi i}$, $z_2 = \frac{3}{2}e^{\frac{1}{6}\pi i}$ and $\omega = 2e^{\frac{1}{2}\pi i}$.

(a) State the values of ωz_1 and ωz_2. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [2]

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(b) On a sketch of an Argand diagram with origin O, show the points A, B, C and D representing the complex numbers z_1, z_2, ωz_1 and ωz_2 respectively. [2]

(c) State the geometric effects of multiplying z_1 and z_2 by ω. [2]

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7 **(a)** Express $5\sin\left(x+\frac{1}{6}\pi\right)-4\cos x$ in the form $R\sin(x-\alpha)$, where $R>0$ and $0<\alpha<\frac{1}{2}\pi$. State the exact value of R and give the value of α correct to 3 decimal places. [4]

(b) Hence solve the equation $5\sin\left(2\theta+\frac{1}{6}\pi\right)-4\cos 2\theta=\sqrt{7}$ for $0\leqslant\theta\leqslant\pi$. Give your answers correct to 2 decimal places. [4]

8 With respect to the origin O, the points A and B have position vectors $2\mathbf{i}+4\mathbf{k}$ and $5\mathbf{i}+\mathbf{j}+6\mathbf{k}$ respectively. The line l_1 passes through the points A and B.

(a) Find a vector equation for the line l_1. [2]

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The line l_2 has equation $\mathbf{r} = 2\mathbf{i}+\mathbf{j}+5\mathbf{k}+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$.

(b) Show that l_1 and l_2 do **not** intersect. [4]

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(c) Find the acute angle between the directions of l_1 and l_2. [3]

9 The constant a is such that $\int_1^a 6x \ln x \, dx = 4$.

(a) Show that $a = \exp\left(\frac{1}{6}\left(\frac{5}{a^2} + 3\right)\right)$, where $\exp(x)$ denotes e^x. [5]

(b) Verify by calculation that a lies between 2 and 2.1. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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10 **(a)** Find the quotient and remainder when $x^3 + 5x^2 - 2x - 15$ is divided by $x^2 - 3$. [3]

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(b) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{x^3 + 5x^2 - 2x - 15}{6y(x^2 - 3)}.$$

It is given that $y = 2$ when $x = 2$.

Solve the differential equation to obtain an expression for y^2 in terms of x. [5]

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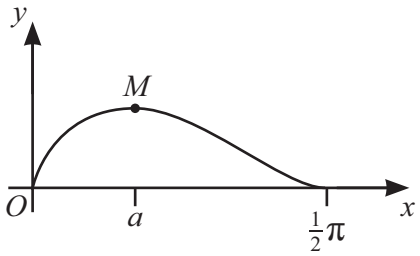

The diagram shows the curve $y = \cos x\sqrt{\sin 2x}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve has a maximum point at M, where $x = a$.

(a) Find the exact value of a. [6]

(b) The region enclosed between the x-axis and the curve is rotated through 2π radians about the x-axis.

Find the exact volume of the solid generated. [5]