



CANDIDATE  
NAME

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CENTRE  
NUMBER

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CANDIDATE  
NUMBER

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## 0606/21

May/June 2024

**2 hours**

No additional materials are needed.

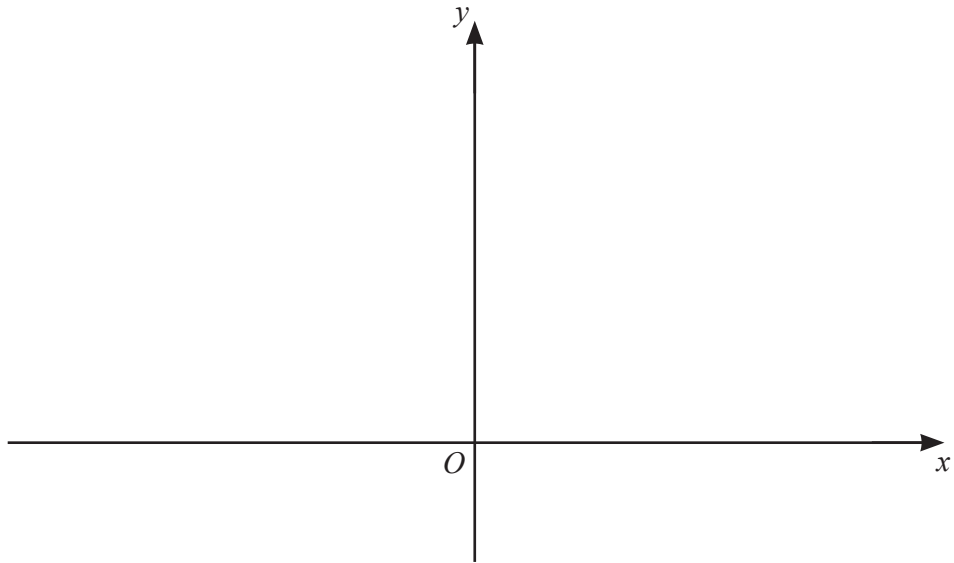
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

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**[Turn over**

- 1 (a) On the axes, sketch the graph of  $y = |4x - 6|$ , showing the points where the graph meets the axes. [2]



- (b) Solve the equation  $|4x - 6| = |2x|$ . [3]

- 2 (a) Write  $3 + 4x - 2x^2$  in the form  $a + b(x + c)^2$ , where  $a$ ,  $b$  and  $c$  are integers. [3]



- (b) Hence write down the range of the function  $f(x) = 3 + 4x - 2x^2$ , where  $x \in \mathbb{R}$ . [1]

- 3 Use algebra to show that the equation  $5x(x - 3) = 5x - 26$  has no real solutions. [3]



**4 DO NOT USE A CALCULATOR IN THIS QUESTION.**

- (a) Find the exact distance between the two points where the curve  $9(x-1)^2 + 4(y-3)^2 = 36$  cuts the  $y$ -axis. [4]

- (b) Find the coordinates of the points where the curve with equation  $2x^2 + 83xy = x^3y - 20x$  intersects the curve with equation  $y = \frac{1}{x}$ . Give each of your answers in the form  $a + b\sqrt{c}$ , where  $a$  and  $b$  are rational and  $c$  is the smallest integer possible. [6]

5 There are 3 women, 2 men and 4 children in a choir.



(a) The choir stands in a single straight line.

(i) Find the number of possible arrangements if the first person and last person are both women. [2]

(ii) Find the number of possible arrangements if all the children stand next to each other. [2]

(b) Four of the choir are selected to sing in a group.

(i) Find the number of different selections if no man is chosen. [2]


(ii) Find the number of different selections if at least 2 women are chosen. [2]

- 6 Variables  $x$  and  $y$  are such that  $y = \cos x \sin^2 x$ . Use differentiation to find the approximate change in  $y$  as  $x$  increases from 3 to  $3 + h$ , where  $h$  is small. [5]



- 7 It is given that  $y = mx^2 + \frac{x}{2} + n$ , where  $m$  and  $n$  are non-zero constants. It is also given that  $3\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^2 - y$  for all values of  $x$ . Find the values of  $m$  and  $n$ . [4]



- 8 (a) In an arithmetic progression, the sum of the first 30 terms is  $-1065$ .  
 The sum of the **next** 20 terms is  $-2210$ .  
Find the first term and the common difference.

[5]

- (b) A geometric progression is such that the first term is 4 and the sum of the first three terms is 7. Find the two possible values of the common ratio and find the sum to infinity for the convergent progression. [5]



9 The functions  $f$  and  $g$  are defined by



$$f(x) = \frac{3x^2}{4x-1} \quad \text{for } x < 0$$

$$g(x) = \frac{1}{x^2} \quad \text{for } x < 0.$$

(a) Explain why the function  $fg$  does **not** exist. [1]

(b) Given that the function  $gf$  does exist, find and simplify an expression for  $gf(x)$ . [2]

(c) Show that  $f^{-1}(x)$  can be written as  $\frac{px - \sqrt{x(qx+r)}}{3}$  where  $p$ ,  $q$  and  $r$  are integers. [4]

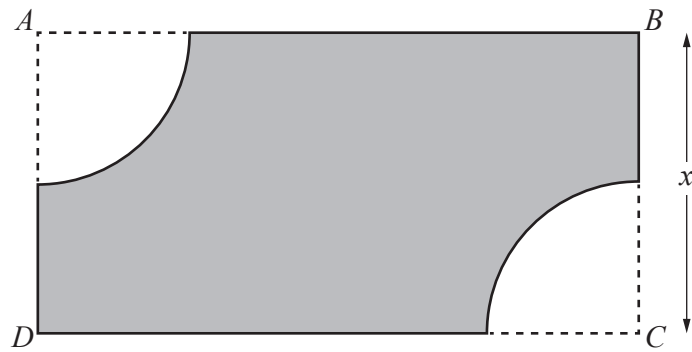
- 10 (a) Show that  $(\tan x + \sec x)^2$  can be written as  $\frac{1 + \sin x}{1 - \sin x}$ . [4]



- (b) Hence solve the equation  $(\tan 3\theta + \sec 3\theta)^2 = 6$  for  $0^\circ \leq \theta \leq 180^\circ$ . [4]

11 In this question all lengths are in centimetres.

7

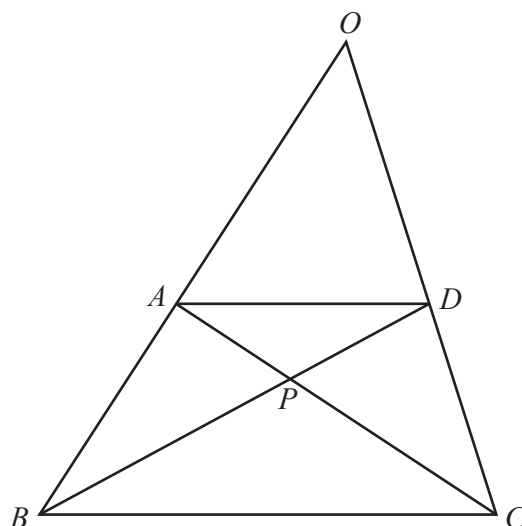


The diagram shows a rectangle  $ABCD$  with  $BC = x$ .  
The area of the rectangle is  $400 \text{ cm}^2$ .

Two identical quarter-circles of radius  $\frac{x}{2}$ , with centres  $A$  and  $C$ , are removed from the rectangle to make the shaded shape.

Given that  $x$  can vary, find the value of  $x$  that gives the minimum value of the perimeter of the shaded shape and hence find this minimum value. [7]

Continuation of working space for Question 11.



The diagram shows a triangle  $OBC$ .

$OA : OB = 4 : 7$  and  $OD : OC = 4 : 7$ .

$$\overrightarrow{OB} = \mathbf{b} \text{ and } \overrightarrow{OC} = \mathbf{c}$$

The point  $P$  is the point of intersection of  $AC$  and  $BD$  such that  $\overrightarrow{AP} = \lambda \overrightarrow{AC}$  and  $\overrightarrow{BP} = \mu \overrightarrow{BD}$  where  $\lambda$  and  $\mu$  are scalars.

- (a) Find two expressions for  $\overrightarrow{OP}$ , each in terms of  $\mathbf{b}$ ,  $\mathbf{c}$  and a scalar, and hence show that  $P$  divides both  $AC$  and  $DB$  in the ratio  $4 : 7$ . [7]

- (b) The point  $Q$  is such that  $\overrightarrow{OQ} = \frac{2}{7}\mathbf{b} + \frac{2}{7}\mathbf{c}$ .

Use a vector method to show that  $O$ ,  $Q$  and  $P$  are collinear. Justify your answer.

[2]