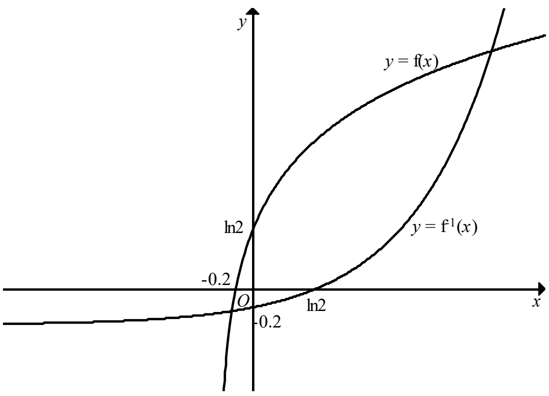
Question	Answer	Marks	Guidance
1	$9kx+1=kx^2+3(2k+1)x+4$, leading to $kx^2+x(3-3k)+3\ \ [=0]$	**M1**	For equating the two equations and attempt to obtain a 3 term quadratic equation equated to zero.
	$(3-3k)^2-(4\times 3k)$ oe	**M1**	**Dep** on previous M mark for attempt to use the discriminant in any form
	$3k^2-10k+3$ oe	**M1**	**Dep** on previous M mark for simplification to a 3 term quadratic expression in terms of k
	Critical values 3 and $\frac{1}{3}$	**A1**	For both
	$\frac{1}{3}<k<3$	**A1**	Mark the final answer
2	$x=\dfrac{-(2\sqrt{3}+5)\pm\sqrt{(2\sqrt{3}+5)^2-4(3-5\sqrt{3})(-1)}}{2(3-5\sqrt{3})}$	**M1**	For the use of the quadratic formula
	$x=\dfrac{-(2\sqrt{3}+5)\pm\sqrt{12+20\sqrt{3}+25+12-20\sqrt{3}}}{2(3-5\sqrt{3})}$	**M1**	For expansion of the square root, must see at least 4 terms
	$x=\dfrac{-12-2\sqrt{3}}{2(3-5\sqrt{3})}$ oe, $x=\dfrac{2-2\sqrt{3}}{2(3-5\sqrt{3})}$ oe	**A1**	For both
	$x=\dfrac{-12-2\sqrt{3}}{2(3-5\sqrt{3})}\times\dfrac{3+5\sqrt{3}}{3+5\sqrt{3}}$ oe or $x=\dfrac{2-2\sqrt{3}}{2(3-5\sqrt{3})}\times\dfrac{3+5\sqrt{3}}{3+5\sqrt{3}}$ oe with an attempt to simplify	**M1**	For attempt to rationalise at least one of *their* solutions (must be similar structure) Sufficient detail must be seen, at least 3 terms in the numerator
	$\frac{1}{2}+\frac{\sqrt{3}}{2}$	**A1**	Must have sufficient detail shown
	$\frac{2}{11}-\frac{\sqrt{3}}{33}$	**A1**	Must have sufficient detail shown

Question	Answer	Marks	Guidance
3(a)	$b = \frac{1}{8}$	**B1**	
	$11 = a\sin\frac{4\pi}{8} + c$ $5 = a\sin\left(\frac{-4\pi}{3\times 8}\right) + c$	**M1**	For attempt to form two simultaneous equations using given points, together with an attempt to obtain at least one unknown. Allow use of *their b*.
	$a = 4$	**A1**	
	$c = 7$	**A1**	
3(b)	Using symmetry	**M1**	For e.g. period is 16π, symmetrical about the line $x = 8\pi$
	For obtaining max at 4π and min at 12π	**M1**	
	$x = 12\pi$	**A1**	
	$y = 3$	**A1**	
	Alternative method 1		
	Minimum value when $y = 3$	**(B2)**	**FT** on *their* $-a + c$
	When $y = 3$, $x = 12\pi$.	**(2)**	**M1** for attempt to solve *their* $3 = a\sin bx + c$ using *their* values of *a*, *b* and *c* to get *x* = ...
	Alternative method 2		
	Min occurs $\frac{3}{4}$ through sine cycle so $x = 12\pi$	**(B2)**	
	When $x = 12\pi, y = 3$	**(2)**	**M1** for attempt to solve $y = a\sin b(12\pi) + c$ using *their* values of *a*, *b* and *c*
	Alternative method 3		
	$\frac{dy}{dx} = ab\cos bx$ $(ab)\cos bx = 0$	**(M1)**	
	$x = 4\pi,\ 12\pi$	**(M1)**	**Dep** for attempt to solve to obtain *x* =
	$x = 12\pi$	**(A1)**	
	$y = 3$	**(A1)**	cao

Question	Answer	Marks	Guidance
4(a)	$\frac{(2x-1)+4}{(2x-1)^2}=\frac{2x+3}{(2x-1)^2}$	B1	
	Alternative method		
	$\frac{(2x-1)^2+4(2x-1)}{(2x-1)^3}=\frac{4x^2+4x-3}{(2x-1)^3}$ $=\frac{(2x-1)(2x+3)}{(2x-1)^3}=\frac{2x+3}{(2x-1)^2}$	(B1)	
4(b)	Use of $\int \frac{1}{2x-1}+\frac{4}{(2x-1)^2}\,\mathrm{d}x$ to obtain $\frac{1}{2}\ln(2x-1)-\frac{2}{(2x-1)}$	2	**B1** for $\frac{1}{2}\ln(2x-1)$ or equivalent **B1** for $-\frac{2}{(2x-1)}$, allow unsimplified
	$\left[\frac{1}{2}\ln(2x-1)-\frac{2}{(2x-1)}\right]_2^5$ $\left(\frac{1}{2}\ln 9-\frac{2}{9}\right)-\left(\frac{1}{2}\ln 3-\frac{2}{3}\right)$	M1	For application of limits, must be in the form $a\ln(2x-1)+\frac{b}{(2x-1)}$
	$=\frac{4}{9}+\ln\sqrt{3}$	2	**A1** for $\ln\sqrt{3}$ **A1** for $\frac{4}{9}$
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(3x\times\frac{4x}{(2x^2-3)}\right)-3\ln(2x^2-3)}{9x^2}$ oe	3	**B1** for $\frac{4x}{(2x^2-3)}$ **M1** for attempt to differentiate a quotient or product **A1** for all terms other than $\frac{4x}{(2x^2-3)}$ correct.
5(b)	When $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0.133$	M1	For substitution of $x=2$ into *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ and use of h
	$0.133h$	A1	
5(c)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{4}{0.133}$	M1	For $\frac{4}{\textit{their}\text{ value of }\frac{\mathrm{d}y}{\mathrm{d}x}\text{ from (b)}}$
	30.2	A1	

Question	Answer	Marks	Guidance
6	$\frac{dy}{dx} = 3\sec^2 3x$	2	**M1** for $a\sec^2 3x$
	When $x = \frac{\pi}{12}$, $y = 2$	B1	
	When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 6$	M1	
	Gradient of perpendicular is $-\frac{1}{6}$	M1	For $-\frac{1}{\textit{their}\ \frac{dy}{dx}}$, must be numeric
	Equation of normal $y - 2 = -\frac{1}{6}\left(x - \frac{\pi}{12}\right)$	M1	For attempt at a normal equation using *their* $-\frac{1}{6}$ and 2
	Area of triangle = 12	2	**M1 dep** for attempt at correct area using *their* 2 and *their* $12 + \frac{\pi}{12}$
7	$-\frac{1}{2}(2-3x)^{\frac{2}{3}}$	2	**M1** for $a(2-3x)^{\frac{2}{3}}$, $a \neq -\frac{1}{2}$ Allow unsimplified
	When $x = -2$, $\frac{dy}{dx} = -6$ leading to $c = -4$	2	**M1 Dep** for correct attempt to find the value of the arbitrary constant
	$\frac{1}{10}(2-3x)^{\frac{5}{3}}$ nfww	2	**M1** for $b(2-3x)^{\frac{5}{3}}$, $b \neq \frac{1}{10}$ Allow unsimplified
	When $x = -2$, $y = 10.2$ leading to $d = -1$	M1	**Dep** on previous M mark for attempt to find the value of a second arbitrary constant
	$y = \frac{1}{10}(2-3x)^{\frac{5}{3}} - 4x - 1$	A1	
8(a)	$\begin{pmatrix} -40 \\ 42 \end{pmatrix}$	B1	Allow $2\begin{pmatrix} -20 \\ 21 \end{pmatrix}$

Question	Answer	Marks	Guidance
8(b)	$\begin{pmatrix}5\\-3\end{pmatrix}+\begin{pmatrix}-40\\42\end{pmatrix}t$	**B1**	**FT** on *their* answer to (a), must be numeric but not $\begin{pmatrix}-20\\21\end{pmatrix}$
8(c)	$\begin{pmatrix}-35t+4\\44t-2\end{pmatrix}-\begin{pmatrix}5-40t\\-3+42t\end{pmatrix}$	**M1**	Allow if in the incorrect order, **FT** on *their* (b), must have correct structure
	$\begin{pmatrix}5t-1\\2t+1\end{pmatrix}$	**A1**	
8(d)	$AB=\sqrt{(5t-1)^2+(2t+1)^2}$	**M1**	For attempt at modulus and square root using *their* answer to (c)
	$\sqrt{29t^2-6t+2}$	**A1**	
8(e)	$29t^2-6t-4=0$	**M1**	For attempt to solve the square of *their* answer to (d) $-6=0$
	0.49 only	**A1**	
9(a)(i)	–0.4	**B1**	
9(a)(ii)	$\mathrm{f}(x)\in\mathbb{R}$ oe	**B1**	
9(a)(iii)	$x=\ln(5y+2)$ oe $\mathrm{e}^x=5y+2$ oe	**M1**	For a correct attempt to find the inverse
	$\mathrm{f}^{-1}(x)=\dfrac{\mathrm{e}^x-2}{5}$	**A1**	Must be in the correct form
	$x\in\mathbb{R}$	**B1**	
9(a)(iv)		**4**	**B1** for two correctly shaped graphs in the correct quadrants **B1** for a correct graph for $y=\mathrm{f}(x)$ with correct intercepts **B1** for a correct graph for $y=\mathrm{f}^{-1}(x)$ with correct intercepts **B1** all correct with symmetry implied, exact intercepts and two points of intersection

Question	Answer	Marks	Guidance
9(b)	$g^2(x) = \left(\left(x^{\frac{1}{2}} - 4\right)^{\frac{1}{2}} - 4\right)$	**M1**	For a correct order of operations
	$\left(\left(x^{\frac{1}{2}} - 4\right)^{\frac{1}{2}} - 4\right) = -2$ leading to $x^{\frac{1}{2}} = 8$,	**M1**	**Dep** on previous M mark for a correct attempt at a solution. Must deal with $x^{\frac{1}{2}}$ correctly to obtain the final solution
	$x = 64$	**A1**	
10(a)	Common difference = $4\sin 3x$ soi	**B1**	
	$390 = \frac{20}{2}\left(2\sin 3x + 19\left(4\sin 3x\right)\right)$	**M1**	**M1** for attempt at sum to 20 terms using *their* common difference, equating to 390 and attempt to solve to obtain $\sin 3x = ...$
	$\sin 3x = 0.5$	**A1**	
	$x = \frac{\pi}{18}, \frac{5\pi}{18}$	**3**	**M1** for a correct attempt to solve, may be implied by one correct solution, allow if not exact **A1** for 1 correct solution **A1** for a second correct solution and no others in the range
10(b)(i)	Common ratio = $0.5\cos y$	**B1**	
	$-0.5 \leqslant 0.5\cos y \leqslant 0.5$	**B1**	Correct use of $\lvert\text{common ratio}\rvert < 1$
10(b)(ii)	$9 = \frac{20\cos y}{1 - 0.5\cos y}$	**B1**	For attempt to use sum to infinity equation correctly and solve
	$\cos y = \frac{18}{49}$ or 0.367...	**2**	**M1** for solution of *their* equation, must have r as a multiple of $\cos y$, to obtain $\cos y = ...$
	1.19	**A1**	