Question	Answer	Marks	Guidance
1(a)	$y = -\frac{1}{2}x + 25$ isw	3	**M1** for $m = \frac{29-23}{-8-4}$ oe or $-\frac{1}{2}$ and **M1 FT** for $\frac{y-23}{x-4} = their\left(-\frac{1}{2}\right)$ oe or $y = their\left(-\frac{1}{2}\right)x + c$ and $23 = -\frac{1}{2} \times 4 + c$ oe **OR** **M1** for solving $23 = 4m + c$ $29 = -8m + c$ for $m = -\frac{1}{2}$ or $c = 25$ and **M1 FT** for correctly using *their m* or *their c* to find *c* or *m*
	Solves *their* linear equation simultaneously with $y = 2x + 5$ to find *x* or *y*	M1	**FT** *their* $y = -\frac{1}{2}x + 25$ oe
	(8, 21)	A1	
1(b)	$\sqrt{8^2 + 21^2}$ oe	M1	**FT** *their* (8, 21)
	$\sqrt{505}$ isw or 22.5 or 22.47[22…] rot to 2 or more dp	A1	
2	$x^2 + 2kx = -2x - 6k - 1$	M1	
	$x^2 + (2k+2)x + 6k + 1 [= 0]$	A1	
	Correctly uses $b^2 - 4ac$ [*0] for *their* equation $(2k + 2)^2 - 4(6k + 1)$ [*0]	M1	where * is any inequality sign or =; **FT** *their* 3-term quadratic in *x* and *k*
	$4k^2 - 16k$ [*0] nfww	A1	
	$k = 4$	A1	**dep** on all previous marks awarded

Question	Answer	Marks	Guidance
2	**Alternative method**		
	$2x + 2k$	**(M1)**	
	$k = -x - 1$ or $x = -k - 1$ oe	**(A1)**	
	$-2(-k-1) - 6k - 1 = (-k-1)^2 + 2k(-k-1)$ oe or $-2x - 6(-1-x) - 1 = x(x + 2(-1-x))$ oe	**(M1)**	**FT** *their* k of the form $ax + b$ where a and b are non-zero constants or *their* x of the form $ck + d$ where c and d are non-zero constants
	$k^2 - 4k$ [= 0] or $x^2 + 6x + 5$ [= 0] and $x = -5$ [$x = -1$] nfww	**(A1)**	
	$k = 4$	**(A1)**	**dep** on all previous marks awarded
3	$\frac{16+9\sqrt{3}}{(2+\sqrt{3})^2}$	**B1**	
	$\frac{\left(16+9\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)}$ or $\frac{16+9\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$	**M1**	**FT** $\frac{c\left(16+9\sqrt{3}\right)}{a+b\sqrt{3}}$ where a, b and c are non-zero constants
	$112 - 64\sqrt{3} + 63\sqrt{3} - 108$ or $\frac{-112+64\sqrt{3}-63\sqrt{3}+108}{-1}$	**A1**	
	$4-\sqrt{3}$ or $-\sqrt{3}+4$ cao, nfww	**A1**	
	Alternative method		
	$\frac{16+9\sqrt{3}}{(2+\sqrt{3})^2}$	**(B1)**	
	$\frac{\left(16+9\sqrt{3}\right)\left(2-\sqrt{3}\right)^2}{\left(2+\sqrt{3}\right)^2\left(2-\sqrt{3}\right)^2}$ or $\frac{16+9\sqrt{3}}{\left(2+\sqrt{3}\right)^2} \times \frac{\left(2-\sqrt{3}\right)^2}{\left(2-\sqrt{3}\right)^2}$	**(M1)**	
	$112 - 64\sqrt{3} + 63\sqrt{3} - 108$ or $64 - 32\sqrt{3} - 32\sqrt{3} + 48 +$ $36\sqrt{3} - 54 - 54 + 27\sqrt{3}$	**(A1)**	
	$4-\sqrt{3}$ or $-\sqrt{3}+4$ cao, nfww	**(A1)**	

Question	Answer	Marks	Guidance
4(a)	$\dfrac{e^{2x+2}}{e^{\frac{x}{2}}}=10$ oe, soi	**B1**	
	$e^{1.5x+2}=10$ oe	**M1**	**FT** $\dfrac{e^{2x+k}}{e^{\frac{x}{2}}}=10$ oe or $\dfrac{e^{kx+2}}{e^{\frac{x}{2}}}=10$ oe where k is an integer and $k>0$ or $\dfrac{e^{2x+2}}{e^{\frac{x}{n}}}=10$ oe where n is an integer and $n>1$ or $n=-2$
	$1.5x+2=\ln 10$ oe	**M1**	**FT** an expression of, or equivalent to, the form $e^{ax+b}=10$ oe where a and b are non-zero constants
	$x=\dfrac{2}{3}(\ln 10-2)$ oe, isw or 0.202 or 0.2017[23…] rot to 4 or more dp isw	**A1**	
4(b)	$\dfrac{y^2}{4y-9}=9^{\frac{1}{2}}$ nfww or $\log_9\dfrac{y^2}{4y-9}=\log_9 9^{\frac{1}{2}}$ oe	**M2**	**M1** for at least one correct log law used in a correct equation e.g. $\log_9 y^2-\log_9(4y-9)=\dfrac{1}{2}$ or $\log_9\dfrac{y^2}{4y-9}=\dfrac{1}{2}$ or $2\log_9 y-\log_9(4y-9)=\dfrac{1}{2}\log_9 9$
	$y^2-12y+27\,[=0]$ nfww	**A1**	
	$(y-3)(y-9)=0$	**DM1**	**dep** on at least **M1** previously awarded
	$y=3$, $y=9$ nfww	**A1**	
5(a)	Correct first derivative: $3x^2-14x+12$	**M2**	**M1** for two terms of $x^3-7x^2+12x-5$ differentiated correctly
	[At $x=1$] gradient of tangent: 1	**A1**	
	$y-1=$ *their*$(-1)(x-1)$ oe or $y=-x+c$ **and** $1=-1+c$ soi	**M1**	**FT** $\dfrac{-1}{\textit{their}\left.\dfrac{dy}{dx}\right\vert_{x=1}}$
	$y-1=-1(x-1)$ or $y=-x+2$ oe, isw	**A1**	

Question	Answer	Marks	Guidance
5(b)	$x^3 - 7x^2 + 12x - 5 = their(-x+2)$	**B1**	**FT** *their* $y = ax + b$ where a is a non-zero constant
	Uses the correct linear factor $x - 1$ and the correct cubic $x^3 - 7x^2 + 13x - 7[=0]$ to find a quadratic factor with at least two terms correct	**M1**	
	$x^2 - 6x + 7$	**A1**	
	Correct use of formula or completing the square on *their* 3-term quadratic, e.g., $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4[1](7)}}{2[1]}$ or $x = \frac{6 \pm \sqrt{36 - 4[1](7)}}{2}$	**M1**	**FT** *their* 3-term quadratic providing it is from an attempt at finding a quadratic factor and the discriminant is not negative
	$x = 3 \pm \sqrt{2}$	**A1**	
6	$\left[\frac{(x+2)^2}{x} =\right]$ $x + 4 + \frac{4}{x}$ soi	**B2**	**B1** for two terms correct or for $\frac{x^2 + 4x + 4}{x}$
	$\left[\frac{x^2}{2} + 4x + 4\ln x\right]_2^3$	**B2**	**B1** for $\left[\frac{x^2}{2} + ... + 4\ln x\right]_2^3$ or $[... + 4x + 4\ln x]_2^3$ or $\left[\frac{x^2}{2} + 4x + k\ln x\right]_2^3$ with $k \neq 0$
	$\left[\frac{9}{2} + 12 + 4\ln 3\right] - \left[\frac{4}{2} + 8 + 4\ln 2\right]$	**M1**	**dep** on at least previous **B1** for integration
	$6.5 + 4\ln\left(\frac{3}{2}\right)$ or exact equivalent	**A1**	
7(a)	Velocity: $3e^{2t} - 4e^{-2t} - 1$ isw	**B2**	**B1** for $3e^{2t}$ or $-4e^{-2t}$
	Acceleration: $6e^{2t} + 8e^{-2t}$ isw	**B1**	**FT** $me^{2t} + ne^{-2t} + k$ where m, n and k are constants

Question	Answer	Marks	Guidance
7(b)	$3e^{4t} - e^{2t} - 4 = 0$ or $3\left(e^{2t}\right)^2 - e^{2t} - 4 = 0$	**B1**	
	$\left(3e^{2t} - 4\right)\left(e^{2t} + 1\right) = 0$	**M1**	**FT** *their* 3-term quadratic in e^{2t} oe
	$e^{2t} = \frac{4}{3}$ nfww	**A1**	
	$\frac{1}{2}\ln\frac{4}{3}$ oe, isw or 0.144 or 0.1438[41…] rot to 4 or more dp and no other solutions	**A1**	
7(c)	$6 \times e^{2\left(\frac{1}{2}\ln\frac{4}{3}\right)} + 8 \times e^{-2\left(\frac{1}{2}\ln\frac{4}{3}\right)}$	**M1**	**FT** $pe^{2t} + qe^{-2t}$ where p and q are non-zero constants and *their* **positive** $\frac{1}{2}\ln\frac{4}{3}$ from part (b)
	14 nfww	**A1**	
8(a)	Derivative of sin2x: $2\cos 2x$ soi	**B1**	
	Product rule: $x \times 2\cos 2x + [1]\sin 2x$ isw	**B1**	**FT** *their* 2cos2x
8(b)	$y = \frac{\pi}{4}$ soi, isw	**B1**	
	gradient of tangent: 1 soi	**B1**	**dep** on correct derivative
	$y = x$ or $y - x = 0$ or $x - y = 0$	**B1**	**dep** on correct derivative
8(c)	$\left[x\sin 2x + \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$ nfww	**M3**	**M2** for $x\sin 2x + k\cos 2x$ where $k > 0$ or $k = -\frac{1}{2}$; nfww or **M1** for $\int 2x\cos 2x\,dx = x\sin 2x - \int \sin 2x\,dx$ or $\frac{-\cos 2x}{2} + \int 2x\cos 2x\,dx = x\sin 2x$
	$\frac{\pi}{6}\sin\frac{\pi}{3} + \frac{1}{2}\cos\frac{\pi}{3} - \frac{1}{2}\cos 0$	**A1**	
	$\frac{\pi\sqrt{3}}{12} - \frac{1}{4}$ or $\frac{\pi\sqrt{3} - 3}{12}$	**A1**	

Question	Answer	Marks	Guidance
9(a)	Correct pair of simplified linear equations in a and d with terms collected, e.g., $3a+3d=-36$ isw or $a+d=-12$ isw $3a+30d=72$ isw or $a+10d=24$ isw	B3	**B2** for one correct simplified equation or **B1** for $a+a+d+a+2d=-36$ or $\frac{3}{2}\{2a+(3-1)d\}=-36$ or $a+9d+a+10d+a+11d=72$ or $\frac{12}{2}\{2a+(12-1)d\}-\frac{9}{2}\{2a+(9-1)d\}=72$ or $12a+66d-9a-36d=72$ or $\frac{3}{2}\{2(a+9d)+(3-1)d\}=72$
	Solves two linear equations for d or a e.g. $27d=108 \rightarrow d=\ldots$ or $9d=36 \rightarrow d=\ldots$ or $a+10(-12-a)=24 \rightarrow a=\ldots$ $27a=-432 \rightarrow a=\ldots$	M1	**FT** *their* linear equations in a and d providing at least **B1** earned and the equations have a solution
	$d=4$ and $a=-16$ nfww	A1	
9(b)	$1.2^n * 101$	B3	where * is any inequality sign or =; **B2** for $\frac{[1](1.2^n-1)}{(1.2-1)}*500$ or **B1** for $r=1.2$ soi
	$n\log 1.2 * \log 101$ or $\log_{1.2}101$ soi	M1	**FT** $1.2^n *$ *their* 101 providing **B2** has been awarded and (*their* 101) >0
	$n=26$	A1	**dep** on all previous marks awarded

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Question	Answer	Marks	Guidance
10(a)	Writes cot$x$ and tan$x$ in terms of sin$x$ and cos$x$: $\dfrac{\sin x}{1-\dfrac{\cos x}{\sin x}}+\dfrac{\cos x}{1-\dfrac{\sin x}{\cos x}}$	**M1**	OR $\dfrac{\sin x\left(1-\dfrac{\sin x}{\cos x}\right)+\cos x\left(1-\dfrac{\cos x}{\sin x}\right)}{\left(1-\dfrac{\cos x}{\sin x}\right)\left(1-\dfrac{\sin x}{\cos x}\right)}$
	Simplifies denominator: $\dfrac{\sin x}{\dfrac{\sin x-\cos x}{\sin x}}+\dfrac{\cos x}{\dfrac{\cos x-\sin x}{\cos x}}$	**A1**	OR $\dfrac{\sin x\left(\dfrac{\cos x-\sin x}{\cos x}\right)+\cos x\left(\dfrac{\sin x-\cos x}{\sin x}\right)}{\left(\dfrac{\sin x-\cos x}{\sin x}\right)\left(\dfrac{\cos x-\sin x}{\cos x}\right)}$
	Writes as two simple algebraic fractions: $\dfrac{\sin^2 x}{\sin x-\cos x}+\dfrac{\cos^2 x}{\cos x-\sin x}$	**A1**	OR writes as a single simple algebraic fraction: $\dfrac{\sin^2 x(\cos x-\sin x)+\cos^2 x(\sin x-\cos x)}{(\sin x-\cos x)(\cos x-\sin x)}$
	Writes as a difference with a common denominator: $\dfrac{\sin^2 x}{\sin x-\cos x}-\dfrac{\cos^2 x}{\sin x-\cos x}$	**A1**	OR $\dfrac{\sin^2 x(\cos x-\sin x)-\cos^2 x(\cos x-\sin x)}{(\sin x-\cos x)(\cos x-\sin x)}$
	Correct simplification to given answer, e.g., $\dfrac{(\sin x-\cos x)(\sin x+\cos x)}{(\sin x-\cos x)}=\sin x+\cos x$ or $\dfrac{\cancel{(\sin x-\cos x)}(\sin x+\cos x)}{\cancel{(\sin x-\cos x)}}[=\sin x+\cos x]$	**A1**	All steps correct and final step fully justified by factorising

Question	Answer	Marks	Guidance
10(b)	$10\cos^2 x + 3\cos x - 1 [= 0]$ or $\sec^2 x - 3\sec x - 10 [= 0]$	B2	**B1** for $\frac{9\cos x}{\sin x} + \frac{3}{\sin x} = \frac{\sin x}{\cos x}$ or better or $9 + \frac{3\tan x}{\sin x} = \tan^2 x$ or better OR **M1** for one sign error in $10\cos^2 x + 3\cos x - 1 [= 0]$ or $\sec^2 x - 3\sec x - 10 [= 0]$
	$(5\cos x - 1)(2\cos x + 1)[= 0]$ or $(\sec x - 5)(\sec x + 2)[= 0]$	M1	**FT** *their* 3-term quadratic in cos*x* or sec*x*
	[cos*x* = $\frac{1}{5}$ **and** cos*x* = $-\frac{1}{2}$ OR sec*x* = 5 **and** sec*x* = –2 leading to] 78.5 or 78.46[30…] rot to 2 or more dp 281.5 or 281.53[69…] rot to 2 or more dp 120 240 and no extras in range $0 < x < 360$	A2	**A1** for any two correct angles [found using cos*x* = $\frac{1}{5}$ **and** cos*x* = $-\frac{1}{2}$ OR sec*x* = 5 **and** sec*x* = –2]; ignore extras