

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method marks, awarded for a valid method applied to the problem.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B** Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

- awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfwf not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1(a)		3	B1 for correct shape with max and min in the correct quadrant. Ignore labelling of their maximum point if incorrect coordinates. B1 for -5 , -2 , $\frac{1}{2}$ marked on x -axis. Must have a cubic shape B1 for 2 marked on the y -axis. Must have a cubic shape
1(b)	$x \leq -5, -2 \leq x \leq \frac{1}{2}$	2	B1 for each If B0 then SC1 for $x < -5, -2 < x < \frac{1}{2}$

Question	Answer	Marks	Guidance
2(a)	$q(x) = 3x^2 - 10x - 8$ $r = 5$	3	M1 for a valid attempt to obtain the quotient by algebraic long division, synthetic division, factorising $(2x - 5)$ or by forming an identity A1 for each
2(b)	$[p(x) - 5] = (3x^2 - 10x - 8)(2x - 5)$	M1	For a correct attempt to factorise
	$(3x + 2)(x - 4)(2x - 5)$	A1	Must see $p(x) - 5$ as a product of linear factors
2(c)	$x = -\frac{2}{3}, 4, \frac{5}{2}$	B1	FT on <i>their</i> 3 terms quadratic $q(x)$
3(a)	$1 = \lg 10$ soi	B1	
	$\lg \frac{10(x^2 - 1)}{(x - 1)^2}$ oe	M1	For correct use of logs power rule or multiplication or division rule. i.e.: Award M1 for example: $2 \log(x - 1) = \log(x - 1)^2$ $\log(x^2 - 1) = \log(x - 1) + \log(x + 1)$ $\log 10 + \log(x^2 - 1) = \log 10(x^2 - 1)$ $\log(x^2 - 1) - \log(x - 1)^2 = \log \frac{(x^2 - 1)}{(x - 1)^2}$
	$\lg \frac{10(x - 1)(x + 1)}{(x - 1)^2}$	DM1	Dep on previous M1 for a correct attempt to factorise and an attempt to simplify
	$\lg \frac{10(x + 1)}{x - 1}$	A1	
3(b)	$[4 \log_5(x + 1)] = \frac{9}{\log_5(x + 1)}$ or $\frac{4}{\log_{(x+1)} 5} [= 9 \log_{(x+1)} 5]$	B1	For change of base
	$(\log_5(x + 1))^2 = \frac{9}{4}$ or $(\log_{(x+1)} 5)^2 = \frac{4}{9}$	2	M1 for a correct method in forming a quadratic equation Dep on previous B1
	$x + 1 = 5^{(\pm)\frac{3}{2}}$	M1	Dep on dealing with logarithms correctly Allow if \pm is missing
	$x = -1 + 5\sqrt{5}, -1 + \frac{1}{25}\sqrt{5}$	A1	

Question	Answer	Marks	Guidance
4(a)	$n = 5$	B1	
	$5 \times 3^4 p = 810$	M1	For considering the second term. Allow for use of <i>their n</i>
	$p = 2$	A1	
	$10 \times 3^3 \times p^2 = q$	M1	For considering the third term. Allow for use of <i>their n</i> and p^2
	$q = 1080$	A1	
4(b)	${}^6C_2 (2y)^4 \left((-)\frac{1}{3y^2} \right)^2$	M1	For identifying the correct term. Condone errors with brackets and coefficients. Could be implied by a correct answer
	$\frac{80}{3}$ oe	A1	Must be exact. Allow $\frac{240}{9}$ or $26\frac{2}{3}$
5(a)	$a = 5, b = \frac{3}{4}$ (oe), $c = -4$	3	B1 for each
5(b)	$[p =]1, [p =]5$	2	B1 for each Do not allow if written as inequalities
6	$\ln(2x - 3)$	B1	Allow unsimplified
	$\frac{1}{3x - 5}$ oe	B1	
	$\ln 5 + \frac{1}{7} - 1$	M1	For correct application of limits in <i>their</i> integral, Must be in the correct form $a \ln(2x - 3) \pm \frac{b}{3x - 5}$
	$\ln 5 - \frac{6}{7}$ cao	A1	

Question	Answer	Marks	Guidance
7	Use of $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$	B1	$4 + \cot^2 \theta = 9x^2$ gets B0 M0 A0 unless recovered
	$(3x-2)^2 + 1 = \left(\frac{1}{y}\right)$ oe	M1	
	$y = \frac{1}{(3x-2)^2 + 1}$ oe	A1	
	Alternative Method: Use of $\cot \theta = \frac{\cos \theta}{\sin \theta}$ leading to $y = \frac{\cos^2 \theta}{(3x-2)^2}$ oe	M1	For use of the identity to substitute $\cot \theta$ to get $2 + \frac{\cos \theta}{\sqrt{y}} = 3x$ and rearranging to get $y = \frac{\cos^2 \theta}{(3x-2)^2}$
	Use $\sin^2 \theta + \cos^2 \theta = 1$ to get $y = \frac{1-y}{(3x-2)^2}$ oe	M1	For use of the identity to substitute $\cos^2 \theta$ to get $y = \frac{1-\sin^2 \theta}{(3x-2)^2}$ and use of $\sin^2 \theta = y$
$y = \frac{1}{(3x-2)^2 + 1}$ oe	A1		
8	$\sin\left(2\alpha - \frac{\pi}{3}\right) = (\pm)\frac{1}{2}$	B1	Condone if \pm is missing for this mark
	$\alpha = -\frac{5\pi}{12}, -\frac{\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}$ oe in terms of π	4	M1 for correct attempt to obtain one solution using correct order of operations A3 for 4 correct solutions and no extras in the range, or A2 for 3 correct solutions or A1 for 2 correct solutions
9(a)	For use of $e^0 = 1$ or $\ln 1 = 0$	B1	
	$4x - y (= 0)$	B1	For simplifying powers of e leading to a linear equation in x and y
	$4x^3 = 256$ or $\frac{y^3}{16} = 256$	M1	For attempt to obtain a cubic equation in one variable using <i>their</i> linear $4x - y = 0$ with an attempt to solve
	$x = 4, y = 16$	2	A1 for each A0 for $y = \pm 16$

Question	Answer	Marks	Guidance
9(b)	$e^{1-2x} = \frac{1}{e^{2x-1}}$	B1	Correct use of the reciprocal
	$10(e^{2x-1})^2 - 11e^{2x-1} - 6 = 0$ or $6(e^{1-2x})^2 + 11e^{1-2x} - 10 = 0$	M1	For a correct attempt to form a quadratic equation in e^{2x-1} or in e^{1-2x}
	$e^{2x-1} = \frac{3}{2} \left(e^{2x-1} = -\frac{2}{5} \right)$ or $e^{1-2x} = \frac{2}{3} \left(e^{1-2x} = -\frac{5}{2} \right)$	M1	Dep on attempt to solve <i>their</i> quadratic equation
	$x = \frac{1}{2} + \frac{1}{2} \ln \frac{3}{2}$ or exact equivalent only	A1	Allow $x = \frac{1 + \ln \frac{3}{2}}{2}$ and $x = \frac{1}{2} \ln \left(\frac{3}{2} e \right)$ for A1 A0 if negative root not discounted
	Alternative Method: $\frac{10e^{2x}}{e} - 11 = \frac{6e}{e^{2x}}$	B1	Correct use of the reciprocal
	$10(e^{2x})^2 - 11e(e^{2x}) - 6e^2 = 0$	M1	For correct attempt to form a quadratic equation
	$e^{2x} = \frac{11e \pm 19e}{20}$	M1	Dep on attempt to solve <i>their</i> quadratic equation
9(b)	$x = \frac{1}{2} \ln \left(\frac{3}{2} e \right) \left[= \frac{1}{2} \ln \frac{3}{2} + \frac{1}{2} \right]$	A1	Allow $x = \frac{1 + \ln \frac{3}{2}}{2}$ and $x = \frac{1}{2} \ln \left(\frac{3}{2} e \right)$ for A1 A0 if negative root not discounted
10(a)	$t = \frac{\pi}{2}$	2	M1 for attempt at solution of $3 \sin 2t = 0$ implied by 90 or $\frac{\pi}{2}$ A0 for $t = 90$
10(b)	$-\frac{3}{2} \cos 2t$	2	M1 for $k \cos 2t$, $k \neq 6$
	When $t = 0$, $s = 0$ so $c = \frac{3}{2}$	M1	Dep on attempt to find c using <i>their</i> s
	$s = \frac{3}{2} - \frac{3}{2} \cos 2t$	A1	Must be an expression for s

Question	Answer	Marks	Guidance
10(c)	Distance travelled = $2 \times \left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_0^{\frac{\pi}{2}}$ using symmetry or $2 \times \left(\frac{3}{2} - \frac{3}{2} \cos \left(2 \frac{\pi}{2} \right) \right)$ using symmetry $\left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_0^{\frac{\pi}{2}} + \left \left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_{\frac{\pi}{2}}^{\pi} \right $ oe	2	M1 dep on <i>their s</i> from (b) unless restarted. Must be in the form of $k \cos 2t$ (+c) Condone the use of 0 to π as limits. Limits must be the correct way round and subtracted. M1dep for correct substitution of limits at least once and correct use of symmetry
	6	A1	
11	When $x = 3, y = 2$	B1	
	$\frac{dy}{dx} = (3x - 1)^{-\frac{2}{3}}$	M1	Allow for $k(3x - 1)^{-\frac{2}{3}}$
	When $x = 3, \frac{dy}{dx} = \frac{1}{4}$	A1	
	Tangent equation: $y - 2 = \frac{1}{4}(x - 3)$	M1	Allow using <i>their y</i> and <i>their</i> $\frac{dy}{dx}$
	Intercepts on the axes: $x = -5, y = \frac{5}{4}$	A1	For both
	Midpoint of $AB: \left(-\frac{5}{2}, \frac{5}{8} \right)$	M1	Dep on M mark for equation of tangent FT on <i>their</i> intercepts
	Grad of perp bisector = -4	M1	FT on <i>their</i> $\frac{dy}{dx}$ or from <i>their x</i> and <i>y</i> intercept
	Perp bisector equation: $y - \frac{5}{8} = -4 \left(x + \frac{5}{2} \right)$	M1	Allow using <i>their</i> -4 and <i>their</i> midpoint
	$a - \frac{5}{8} = -4 \left(a + \frac{5}{2} \right)$	M1	Dep on previous M mark for use of (a, a) in <i>their</i> perp bisector equation
	$a = -\frac{15}{8}, -1.875$	A1	

Question	Answer	Marks	Guidance
12(a)	$\frac{dy}{dx} = \frac{x^2 \left(\frac{1}{x} \right) - 2x \ln 3x}{x^4} \text{ oe}$ $\frac{dy}{dx} = x^{-2} \times \frac{3}{3x} + \frac{-2}{x^3} \times \ln 3x$	3	B1 for $\frac{1}{x}$ or $\frac{3}{3x}$ M1 for a correct attempt at a quotient or a product A1 for all terms apart from log derivative correct.
	$\frac{1 - 2 \ln 3x}{x^3}$		
12(b)	$\frac{\ln 3x}{x^2} = \int \frac{1 - 2 \ln 3x}{x^3} dx$ $2 \int \frac{\ln 3x}{x^3} dx = \int \frac{1}{x^3} dx - \frac{\ln 3x}{x^2} \text{ oe}$	M1	For using integration as a reverse of differentiation (reverse of part (a)) Allow using <i>their A</i> and <i>B</i> but do not allow any extra terms added or subtracted from <i>their part (a)</i>
	$\left(\int \frac{1}{x^3} dx = \right) - \frac{1}{2x^2} \text{ oe nfw}$	B1	FT on their <i>A</i>
	$-\frac{1}{4x^2} - \frac{\ln 3x}{2x^2} + c \text{ oe}$	2	M1 dep for rearranging simplification and an attempt to integrate ax^{-3} . Allow if +c is missing A1 must include +c