MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial Marks
1(a)	8 – 4x = 10 oe soi and 8 – 4x = −10 oe soi OR $16x^2 - 64x - 36 [= 0]$ oe	**M1**	
	$x = -\frac{1}{2}$, $x = \frac{9}{2}$	**A2**	mark final answer **A1** for $x = -\frac{1}{2}$ or $x = \frac{9}{2}$
1(b)	$-30x^2 + 105x - 75\ [*0]$ oe where * is any inequality sign or =	**M1**	condone one sign or arithmetic error
	Critical values 2.5 and 1	**2**	**M1** for factorises or solves a 3-term quadratic to find critical values
	$x < 1$ $x > 2.5$	**A1**	mark final answer

Question	Answer	Marks	Partial Marks
2	$\dfrac{a+b\sqrt{5}}{1+7\sqrt{5}}=\dfrac{20}{4+2\sqrt{5}}$ or $\dfrac{20(1+7\sqrt{5})}{4+2\sqrt{5}}$ oe, soi	**B1**	
	$[20\times]\dfrac{1+7\sqrt{5}}{4+2\sqrt{5}}\times\dfrac{4-2\sqrt{5}}{4-2\sqrt{5}}$ or $[10\times]\dfrac{1+7\sqrt{5}}{2+\sqrt{5}}\times\dfrac{2-\sqrt{5}}{2-\sqrt{5}}$ oe	**M1**	condone one slip providing it is not in the rationalisation factor
	$\dfrac{20(28\sqrt{5}-70+4-2\sqrt{5})}{16-20}$ or $\dfrac{10(14\sqrt{5}-35+2-\sqrt{5})}{4-5}$ oe	**A1**	
	a = 330 and b = –130 oe, nfww	**A1**	
	Alternative method		
	$\dfrac{a+b\sqrt{5}}{1+7\sqrt{5}}=\dfrac{20}{4+2\sqrt{5}}$ oe, soi	**(B1)**	
	Cross multiplies and multiplies out: $20+140\sqrt{5}=4a+4b\sqrt{5}+2a\sqrt{5}+10b$	**(M1)**	condone one sign or arithmetic error
	Correct pair of simultaneous equations $4a + 10b = 20$ oe $2a + 4b = 140$ oe and solves for a = 330 or b = –130	**(A1)**	
	a = 330 and b = –130 oe, nfww	**(A1)**	
3(a)	$\dfrac{3}{4}\mathbf{a}+\dfrac{1}{4}\mathbf{b}$ or equivalent simplified expression	**B2**	**B1** for $\mathbf{a}+\dfrac{1}{4}(\mathbf{b}-\mathbf{a})$ or $\mathbf{b}+\dfrac{3}{4}(\mathbf{a}-\mathbf{b})$ oe or for $3(\overrightarrow{OP}-\mathbf{a})=\mathbf{b}-\overrightarrow{OP}$ oe

Question	Answer	Marks	Partial Marks
3(b)	$\mathbf{q} = \begin{pmatrix} 24 \\ -12 \end{pmatrix}$ oe	2	**M1** for $12\sqrt{5} \times \frac{1}{\sqrt{6^2 + (-3)^2}} \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ oe, soi
	$\mathbf{r} = \begin{pmatrix} -15 \\ 15 \end{pmatrix}$ oe	2	**M1** for $15\sqrt{2} \times \frac{1}{\sqrt{(-5)^2 + 5^2}} \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ oe, soi If M0 M0, then **SC1** for the unit direction vectors $\frac{1}{\sqrt{45}} \begin{pmatrix} 6 \\ -3 \end{pmatrix}$ or better and $\frac{1}{\sqrt{50}} \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ or better
	$\lvert\mathbf{q} + \mathbf{r}\rvert = \left\lvert \begin{pmatrix} 9 \\ 3 \end{pmatrix} \right\rvert = \sqrt{9^2 + 3^2}$	M1	**FT** *their* (**q** + **r)** providing at least M1 previously awarded
	[unit vector in direction **q** + **r** =] $\frac{1}{\sqrt{90}} \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ oe, isw	A1	
4(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6\sin x\cos x - \sin x$ oe, isw	B2	**B1** for an attempt to differentiate both terms with one term correct
	$3\sin^2 x + \cos x + \frac{\cos x}{\sin x}(6\sin x\cos x - \sin x)$	M1	**FT** *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the form $k\sin x\cos x \pm \sin x$
	Correct simplified step e.g. $3\sin^2 x + \cos x + 6\cos^2 x - \cos x$ or $3\sin^2 x + 6\cos^2 x$ or $3 + 3\cos^2 x$ leading to $3(1 + \cos^2 x)$ nfww	A1	
4(a)(ii)	$\cos^2 x = \frac{1}{3}$	M1	**FT** *their* *k* providing $0 < k \le 4$
	$\cos x = [\pm]\sqrt{\frac{1}{3}}$ oe	M1	**dep** on previous **M1; FT** *their k*
	±0.955 or ±0.9553[1…] rot to 4 or more sf ±2.19 or ±2.186[2…] rot to 4 or more sf	A2	and no other angles in range **A1** for any two correct angles, ignoring extras

Question	Answer	Marks	Partial Marks
4(b)(i)	$\left(1-\frac{1}{2}x^{-\frac{1}{2}}\right)\sec^2\left(x-\sqrt{x}\right)$ oe, isw	2	**M1** for $\mathrm{f}(x)\sec^2\left(x-\sqrt{x}\right)$
4(b)(ii)	Correctly writes $\left(1-\frac{1}{2}x^{-\frac{1}{2}}\right)\sec^2\left(x-\sqrt{x}\right)=$ $\frac{2\sqrt{x}-1}{2\sqrt{x}}\sec^2(x-\sqrt{x})$ or $\frac{2\sqrt{x}-1}{2\sqrt{x}\cos^2(x-\sqrt{x})}$ **and** states an answer $k\tan\left(x-\sqrt{x}\right)$ or states $\frac{1}{2}\int\frac{2\sqrt{x}-1}{\sqrt{x}\cos^2(x-\sqrt{x})}\,\mathrm{d}x=\tan(x-\sqrt{x})$	M1	where k is a non-zero constant; dependent on **part (b)(i)**
	$2\tan\left(x-\sqrt{x}\right)+c$ nfww	A1	
5	Correct quotient rule: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\ln 3x)[1]-x\left(\frac{1}{3x}\times 3\right)}{(\ln 3x)^2}$ oe OR correct product rule using $y = x(\ln 3x)^{-1}$: $\frac{\mathrm{d}y}{\mathrm{d}x}=x\left(-(\ln 3x)^{-2}\times\frac{3}{3x}\right)+[1](\ln 3x)^{-1}$	2	**M1** for $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\ln 3x)[1]-x\times \mathit{their}\left(\frac{1}{3x}\times 3\right)}{(\ln 3x)^2}$ OR for $\frac{\mathrm{d}y}{\mathrm{d}x}=x\times \mathit{their}\left(-(\ln 3x)^{-2}\times\frac{3}{3x}\right)$ $+[1](\ln 3x)^{-1}$
	$\frac{\delta y}{h}=\frac{\ln 3-1}{(\ln 3)^2}$ oe, soi	M1	**FT** *their* $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=1}$ providing quotient rule or appropriate product rule attempted
	$\delta y=\frac{\ln 3-1}{(\ln 3)^2}h$ or $\delta y=0.0817h$ nfww	A1	must have evidence of correct derivative
6	$\left[-\frac{1}{4}\mathrm{e}^{2-4x}\right]_{-0.25}^{0.5}$ oe	B2	**B1** for $k\mathrm{e}^{2-4x}$, $k\neq -4$
	Correct use of correct limits: $-\frac{1}{4}\mathrm{e}^0-\left(-\frac{1}{4}\mathrm{e}^3\right)$ oe	M1	**FT** *their* $-\frac{1}{4}\mathrm{e}^{2-4x}$ providing **B1** awarded
	$-\frac{1}{4}+\frac{1}{4}\mathrm{e}^3$ or exact equivalent, isw	A1	

Question	Answer	Marks	Partial Marks
7(a)	Correctly eliminates x or y e.g. $4x^2-3\left(\frac{2}{x}\right)^2+x\left(\frac{2}{x}\right)=24$ oe or $4\left(\frac{2}{y}\right)^2-3y^2+y\left(\frac{2}{y}\right)=24$ oe	**M1**	
	Rearranges to a 3-term quadratic in x^2 or y^2 soi e.g. $4x^4-22x^2-12[=0]$ or $2x^4-11x^2-6[=0]$ or $3y^4+22y^2-16[=0]$ oe	**A1**	
	Factorises or solves *their* 3-term quadratic in x^2 or y^2 soi e.g. $(2x^2+1)(x^2-6)$ or $(3y^2-2)(y^2+8)$	**M1**	
	$x^2=6$ oe, nfww or $y^2=\frac{2}{3}$ nfww	**A1**	
	$\left(\pm\sqrt{6},\pm\frac{2}{\sqrt{6}}\right)$ or $\left(\pm\sqrt{6},\pm\sqrt{\frac{2}{3}}\right)$ oe, nfww	**A1**	and no other values; **dep** on at least the first M1 A1
7(b)	$\sqrt{(x_P-x_Q)^2+(y_P-y_Q)^2}$ oe, soi	**M1**	**FT** providing *their* x_P, x_Q and *their* y_P, y_Q are non-zero
	$\frac{4}{3}\sqrt{15}$	**A1**	
8(a)	Points plotted at x: 1, 3, 5, 10, 12 $\lg y$: 1.6, 2.2, 2.8, 4.3, 4.9 soi and ruled, single straight line of best fit	**B2**	**B1** for at least 4 correctly plotted points
8(b)	$\lg y=\lg A+x\lg b$ soi	**B1**	
	$\lg A=$ *their* 1.3 soi	**M1**	**dep** on using linear points
	$\lg b=$ *their* $\frac{4.9-1.6}{12-1}$ oe or $\lg b=0.3$ oe soi	**M1**	**dep** on using linear points
	$A=10^{1.3}$ isw and $b=10^{\frac{3}{10}}$ isw	**A2**	**A1** for $A=10^{1.3}$ isw or $b=10^{\frac{3}{10}}$ isw
	$A=20$ and $b=2$ nfww	**A1**	If zero scored, award **SC1** for $A=20$ and **SC1** for $b=2$ found without using the graph in any way

Question	Answer	Marks	Partial Marks
8(c)	lg1500 = 3.2 or 3.17[60…]	**M1**	
	OR $x = \log_{their\,b}\left(\frac{1500}{theirA}\right)$		**FT** *their A* and *b*
	OR $x = \frac{\lg 1500 - their \lg A}{their \lg b}$		**FT** *their* lg*A* and lg*b*
	awrt 6.2 to awrt 6.4 isw	**A1**	
9(a)	$[A=]\frac{1}{2}x^2 \times 0.5 + \frac{1}{2}(x+2)^2 \times 2 + \frac{1}{2}y^2[\times 1]$ soi	**B1**	
	$[P =]$ $x + 0.5x + 2 + 2(x+2) + (x+2-y) + y + y$	**M1**	Attempts to form an expression in *x* and *y* for the perimeter using arc lengths and lengths of lines
	Equates *P* to 24 and rearranges: $y = 16 - \frac{9}{2}x$	**A1**	
	$A = \frac{5}{4}x^2 + 4x + 4 + 128 - 72x + \frac{81}{8}x^2$ oe leading to given answer $A = \frac{91}{8}x^2 - 68x + 132$	**A1**	
9(b)	$\frac{dA}{dx} = \frac{91}{4}x - 68$	**M1**	
	Solves $\frac{dA}{dx} = 0$ for x	**M1**	**FT** *their* $\frac{dA}{dx}$ providing at least one term is correct
	$x = \frac{272}{91}$ or $2\frac{90}{91}$ or 2.99 or 2.989[01…] rot to 4 or more sf	**A1**	
	$A = \frac{91}{8}\left(\frac{272}{91}\right)^2 - 68\left(\frac{272}{91}\right) + 132$	**M1**	**FT** *their x*
	$A = \frac{2764}{91}$ or $30\frac{34}{91}$ or 30.4 or 30.37[36…] rot to 4 or more sf	**A1**	

Question	Answer	Marks	Partial Marks
10(a)	$a^n = b^4$ and $na^{n-1}\left(\frac{1}{a}\right) = 48b^3$ oe	**M1**	
	Eliminates b from one equation using the other equation e.g. $\frac{a^{n-2}}{a^{\frac{3}{4}n}} = \frac{48}{n}$	**M1**	**dep** previous **M1**
	Simplifies a terms e.g. $a^{\frac{n}{4}-2} = \frac{48}{n}$ or $a^{\frac{3n}{4}-6} = \left(\frac{48}{n}\right)^3$	**A1**	
	Uses an appropriate power and completes to the given form e.g. $\left(a^{\frac{n}{4}-2}\right)^2 = \left(\frac{48}{n}\right)^2$ or $\left(a^{\frac{3n}{4}-6}\right)^{\frac{2}{3}} = \left(\frac{48}{n}\right)^{3\times\frac{2}{3}}$ → $a^{\frac{n}{2}-4} = \left(\frac{48}{n}\right)^2$	**A1**	
10(b)	Correct equation in a, b, n $\frac{n(n-1)}{2}\times a^{n-2}\times\frac{1}{a^2} = 1056b^2$ oe, soi	**M1**	
	Correct equation in a, n $\frac{n(n-1)}{2}\times a^{n-2}\times\frac{1}{a^2} = 1056a^{\frac{n}{2}}$ oe	**A1**	
	$\left[\frac{n(n-1)}{2}\times a^{\frac{n}{2}-4} = 1056 \text{ oe} \rightarrow\right]$ Correct equation in n only $\frac{n(n-1)}{2}\times\left(\frac{48}{n}\right)^2 = 1056$ oe	**A1**	
	$n^2 - 12n = 0$ or $n - 12 = 0$ oe	**A1**	
	$n = 12$ only	**A1**	
	$a = 4$ only and $b = 64$ only	**A1**	

Question	Answer	Marks	Partial Marks
11	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$ or $\frac{dS}{dr} = 6$ soi	**B1**	
	$S = 2\pi r(4r)$ or $8\pi r^2$	**B1**	
	$16\pi r = 6$	**M1**	**FT** *their* $S = k\pi r^2$ with k a positive integer to give $2k\pi r = 6$
	$r = \frac{6}{16\pi}$ oe, isw	**A1**	
	$S = \frac{9}{8\pi}$ oe, isw	**A1**	