MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

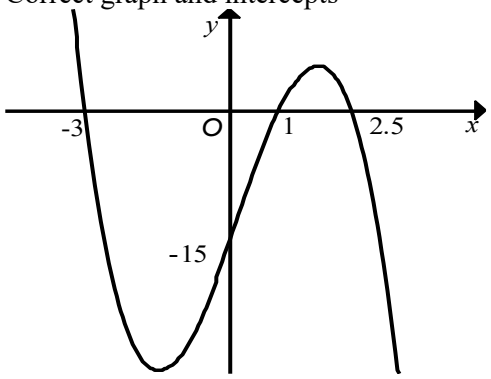
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

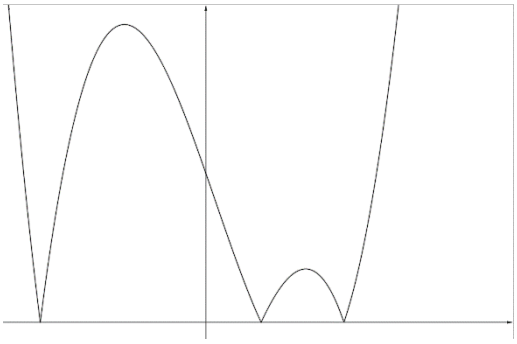
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial Marks
1(a)	Correct graph and intercepts 	**B3**	**B1** for correct shape; the ends must extend above and below the x-axis **B1** for correct roots indicated; must have attempted a cubic shape **B1** for correct y-intercept indicated; must have attempted a cubic shape

Question	Answer	Marks	Partial Marks
1(b)(i)	$-3 \leqslant x \leqslant 1$, $x \geqslant 2.5$ mark final answer	**B2**	**FT** *their* **(a)** providing it is an equivalent cubic shape and has 3 stated or indicated roots for **B2, B1** or **SC1** **B1** for one correct inequality out of two If 0 scored then **SC1** for $-3 < x < 1$, $x > 2.5$ or $-3 < x \leqslant 1$, $x > 2.5$ or $-3 \leqslant x < 1$, $x > 2.5$
1(b)(ii)	Graph of correct shape, with cusps, positive y-intercept and x-intercepts which match **(a)** 	**B1**	**FT** *their* **(a)** providing it is an equivalent cubic shape
2(a)	$\left[4\sin\frac{x}{4}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	**B2**	**B1** for $k\sin\frac{x}{4}$ where $k > 0$ or $k = -4$
	$4\sin\frac{\pi}{8} - 4\sin\frac{\pi}{12}$	**M1**	**FT** provided at least **B1** awarded
	0.495 or 0.4954[57…] rot to 4 or more sf	**A1**	**dep** on all previous marks awarded
2(b)	$\frac{1}{4}\ln(4x-3) - \frac{x^{-2}}{2}\ (+c)$ oe, isw or $\frac{1}{4}\ln(x-0.75) - \frac{x^{-2}}{2}\ (+c)$ oe, isw	**B3**	**B2** for $\frac{1}{4}\ln(4x-3)$ or $\frac{1}{4}\ln(x-0.75)$ or **B1** for $\frac{1}{4}\ln 4x - 3$ or $\frac{1}{4}\ln x - 0.75$ or $k\ln(4x-3)$ or $k\ln(x-0.75)$ where $k \neq \frac{1}{4}$ and **B1** for $\frac{x^{-2}}{-2}$ oe

Question	Answer	Marks	Partial Marks
3(a)	$7x^2 + 9x + 5[= 0]$ or $-7x^2 - 9x - 5[= 0]$	**B2**	**B1** for two terms correct in $7x^2 + 9x + 5 = 0$ or at most one term incorrect in $12x^2 + 11x + 2 = 5x^2 + 2x - 3$ oe
	$9^2 - 4(7)(5)$ or $(-9)^2 - 4(-7)(-5)$ oe	**M1**	**FT** *their* 3-term quadratic
	–59 and no real roots or 81 – 140 < 0 and no real roots oe	**A1**	
3(b)	$\left(\sqrt[3]{x}\right)^2 + 4\sqrt[3]{x} - 12 = 0$ oe soi or $y = \sqrt[3]{x}$ and $y^2 + 4y - 12 = 0$ oe soi	**B1**	
	Factorises or solves *their* 3-term quadratic in $\sqrt[3]{x}$	**M1**	**FT** *their* 3-term quadratic in $\sqrt[3]{x}$ or a stated substituted unknown
	$x = 8 \quad x = -216$	**A2**	**A1** for $\sqrt[3]{x} = 2 \quad \sqrt[3]{x} = -6$
4(a)	33	**B1**	
4(b)(i)	$6\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 12\left(\frac{1}{2}\right) + 5 = 0$ oe or $6\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{12}{2} + 5 = 0$ oe or $\frac{3}{4} + \frac{1}{4} - 6 + 5 = 0$ oe	**B1**	
4(b)(ii)	Finds the quadratic factor $3x^2 + 2x - 5$	**M2**	**M1** for any two terms correct in $3x^2 + 2x - 5$
	$(2x - 1)(3x + 5)(x - 1)$ oe	**A1**	If 0 scored then **SC2** for justifying $x - 1$ as a factor and writing down $(2x - 1)(3x + 5)(x - 1)$ without any incorrect work seen
4(b)(iii)	$[\sin\theta = 0.5]$ $\theta = 30$ nfww $[\sin\theta = 1]$ $\theta = 90$ nfww and no value of θ from $3\sin\theta + 5 = 0$	**B2**	**B1** for $\sin\theta = 0.5$ or $\theta = 30$ or **B1** for $\sin\theta = 1$ nfww or $\theta = 90$ nfww

Question	Answer	Marks	Partial Marks
5	$\frac{dy}{dx} = 10e^{2x-1}$ [+0] oe	**M2**	**M1** for $\frac{dy}{dx} = ke^{2x-1}$, $k \neq 10$ or **SCM1** for $\frac{dy}{dx} = 10e^{2x-1} + c$ where c is algebraic or numerical
	$\left.\frac{dy}{dx}\right\|_{x=1} = 10e$ and $y = 6e$	**A1**	**FT** *their* $\frac{dy}{dx}$ provided **M1** or **SCM1** awarded and a value is found
	$y - 6e = 10e(x-1)$ oe or $y = 10ex + c$ and $6e = 10e + c$	**M1**	**FT** *their* $\left.\frac{dy}{dx}\right\|_{x=1}$ and *their y*
	$y = 10ex - 4e$ isw	**A1**	
	[x-coordinate of P =] 0.4 oe, isw	**A1**	**dep** on correct equation of tangent with exact values
6(a)	Convincing correct statement from which the answer can be easily determined e.g. $\sin^3 x\left(\frac{1}{\sin x} \times \frac{\sin x}{\cos x}\right)$ oe or $\sin^2 x\left(\sin x \times \frac{1}{\sin x} \times \frac{1}{\cot x}\right)$ oe or $\sin^3 x\left(\frac{1}{\sin x} \div \frac{\cos x}{\sin x}\right) = \sin^3 x\left(\frac{1}{\cos x}\right)$ oe or $\sin^3 x\left(\frac{1}{\sin x} \div \frac{1}{\tan x}\right) = \sin^3 x\left(\frac{1}{\sin x} \times \tan x\right)$ oe	**2**	**M1** for either cosec$x$ correctly written as $\frac{1}{\sin x}$ oe seen in a correct expression or for cot$x$ correctly written as $\frac{\cos x}{\sin x}$ or $\frac{1}{\tan x}$ oe seen in a correct expression e.g. $\sin^3 x(\text{cosec}\, x \times \tan x)$ or $\sin^3 x\left(\text{cosec}\, x \div \frac{\cos x}{\sin x}\right)$ or $\sin^3 x\left(\frac{1}{\sin x} \div \frac{\cos x}{\sin x}\right)$ or $\sin^3 x\left(\frac{1}{\sin x} \div \frac{1}{\tan x}\right)$
	Correct completion to given answer: $\sin^2 x \tan x$	**A1**	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Partial Marks</th></tr>
<tr><td rowspan="4">6(b)</td><td>Factorises:
$\tan x\left(\cos^2 x-\frac{1}{2}\right)=0$ oe or
$\tan x\left(\frac{1}{2}-\sin^2 x\right)=0$ oe or
correctly rewrites and then factorises:
$\sin x\left(\cos^2 x-\frac{1}{2}\right)=0$ oe or
$\sin x\left(\frac{1}{2}-\sin^2 x\right)=0$ oe or
$\tan x\left(1-\tan^2 x\right)=0$ oe</td><td>M1</td><td>Note: division by tanx is M0

Note: division by sinx is M0</td></tr>
<tr><td>[tanx = 0 or sinx = 0] [x =] 0</td><td>A1</td><td></td></tr>
<tr><td>cosx = [±]$\sqrt{\frac{1}{2}}$ oe or
sinx = [±]$\sqrt{\frac{1}{2}}$ oe or
tan x = [±]1</td><td>M1</td><td>nfww</td></tr>
<tr><td>[x =]
$\pm\frac{\pi}{4}$ or ± 0.785 or ± 0.7853 to ± 0.7854,

$\pm\frac{3\pi}{4}$ or ± 2.36 or ± 2.356 to ± 2.3562</td><td>A2</td><td>with no extras in range; nfww

A1 for any two out of four correct, ignoring extras</td></tr>
<tr><td>7(a)</td><td>282 240</td><td>2</td><td>M1 for 9! – 2! × 8! oe</td></tr>
<tr><td>7(b)</td><td>120</td><td>2</td><td>M1 for 5! or $^5P_5$ oe</td></tr>
<tr><td>8(a)</td><td>Points plotted at
<table><tr><td>x</td><td>15</td><td>30</td><td>45</td><td>60</td><td>75</td></tr><tr><td>lny</td><td>2.3</td><td>2.5 or 2.6</td><td>3.1</td><td>3.5 or 3.6</td><td>3.9</td></tr></table>soi
and
single, ruled, straight line of best fit drawn</td><td>B2</td><td>B1 for at least 4 correctly plotted points</td></tr>
</table>

Question	Answer	Marks	Partial Marks
8(b)	$\ln y = 0.03x + 1.8$	**2**	**M1** for m = awrt 0.02 to awrt 0.03 or c = awrt 1.7 to awrt 2.0 or for the straight line form in terms of $\ln A$ and k: $\ln y = \ln A + kx$[lne]
	$A = 6$ or $A = 7$ and $k = 0.03$ or $k = 0.02$	**B3**	Must have been found using linear points **or** linear equation **B2** for $A = 6$ or $A = 7$ or A in range: awrt 6 or awrt 7 or **B1 FT** for $\ln A =$ *their* 1.8 or $A = e^{\textit{their}1.8}$ and **B1 FT** for $k = 0.03$ or 0.02 or k in range: awrt 0.02 or awrt 0.03 **Maximum of 2 marks if one or both values not rounded to 1 sf** If **B0** scored, award **SC1** for $A = 6$ or $A = 7$ and **SC1** for $k = 0.03$ or $k = 0.02$ found not using transformed data

Question	Answer	Marks	Partial Marks
8(c)	A value of x in range $33 \leqslant x \leqslant 37.5$ nfww, isw	**2**	**M1** for $\ln y = 2.8$ or 2.83[32...] OR A value of x in range $29.5 \leqslant x < 33$ or $37.5 < x \leqslant 45$ nfww OR **M1 STRICT FT** for $x = \dfrac{\ln 17 - \textit{their} \ln A}{\textit{their } k}$ **STRICT FT** *their* stated ln$A$ and *their* stated k or *their* stated linear equation in **(b)** OR $x = \dfrac{1}{\textit{their } k} \ln\left(\dfrac{17}{\textit{their} A}\right)$ **STRICT FT** *their* stated A and *their* stated k or *their* stated exponential equation in **(b)**

Question	Answer	Marks	Partial Marks
9	$A(2, 0)$ or $x = 2$ [$x = 6$]	**2**	**M1** for factorising or solving $32x - 4x^2 - 48 = 0$
	Finds equation CD/ x-coordinate of C or D or maximum point : $x = 4$	**2**	**M1** for $\frac{2+6}{2}$ or $32 - 8x = 0$
	$D(4, 8)$ or [equation AB is $y =$] $4x - 8$	**2**	**M1** for $y = \frac{2}{3}\times 12$ or $m_{AB} = \frac{12-0}{5-2}$ soi
	$\left[\frac{32}{2}x^2 - \frac{4}{3}x^3 - 48x\right]_2^4$ or $\left[\frac{28}{2}x^2 - \frac{4}{3}x^3 - 40x\right]_2^4$	**B1**	**must be seen**
	Correct plan **including** correct substitution of upper and lower limits at some point e.g. $\left[16x^2 - \frac{4}{3}x^3 - 48x\right]_2^{their4} - \frac{1}{2}\times(their4 - 2)\times their8$ or $\left[16x^2 - \frac{4}{3}x^3 - 48x\right]_2^{their4} - [2x^2 - 8x]_2^{their4}$ or $\left[14x^2 - \frac{4}{3}x^3 - 40x\right]_2^{their4}$	**M1**	**dep** on attempt to integrate **FT** *their* 4 and *their* 8 if needed or **FT** *their* 4 and *their* $4x - 8$ of the form $mx + c$ if needed or **FT** *their* 4 and $(32 - their4)x - 4x^2 + (-48 - their(-8))$ providing clear evidence of the derivation of this has been seen
	$\frac{40}{3}$ isw or 13.3[33….] **nfww**	**A1**	**dep on all previous marks awarded**
10(a)	Valid explanation using f: f is one-one oe	**B1**	
10(b)	Complete method to find inverse function: Swaps the variables and rearranges or rearranges and swaps the variables	**M1**	Condone one sign or arithmetic error but must have the correct order of operations
	$\left[f^{-1}(x) =\right] -\sqrt{\ln x - 3}$ isw or $\left[f^{-1}(x) =\right] -\sqrt{\ln\frac{x}{e^3}}$ oe isw	**A2**	**A1** for $\left[f^{-1}(x) =\right]$ $[\pm]\sqrt{\ln x - 3}$ or $\left[f^{-1}(x) =\right]$ $[\pm]\sqrt{\ln\frac{x}{e^3}}$ oe
	Domain f^{-1}: $x > e^3$	**B1**	
	Range f^{-1}: $f^{-1} < 0$	**B1**	

Question	Answer	Marks	Partial Marks
10(c)	$g(x) = f^{-1}(e^{2x})$ soi or $g(x) = -\sqrt{\ln e^{2x} - 3}$	**M1**	**FT** *their* expression for f^{-1}
	$-\sqrt{2x-3}$	**A1**	If 0 scored, allow **SCB1** for $-\sqrt{2x-3}$ found from solving $(g(x))^2 + 3 = 2x$ and using existence of composite functions to deduce that the square root must be negative
11	$2^n + n \times 2^{n-1} \times \frac{x}{2} + \frac{n(n-1)}{2[!]} \times 2^{n-2} \times \left(\frac{x}{2}\right)^2$ soi	**B1**	implied by three correct equations or e.g. $2^n + {}^nC_1 \times 2^{n-1} \times \frac{x}{2} + {}^nC_2 \times 2^{n-2} \times \left(\frac{x}{2}\right)^2$ **and** sight of ${}^nC_1 = n$ **and** ${}^nC_2 = \frac{n(n-1)}{2[!]}$ clearly in the working
	Forms three correct equations e.g. $b = 2^n$ $ab = n(2^{n-2})$ or $ab = n\frac{(2^{n-1})}{2}$ $\frac{9}{8}ab = n(n-1)(2^{n-5})$ or $\frac{9}{8}ab = \frac{n(n-1)}{2}\frac{(2^{n-2})}{2^2}$ OR finds e.g. $a = \frac{n}{4}$ and $\frac{9}{8}a = \frac{n(n-1)}{32}$ OR finds e.g. $ab = n \times \frac{b}{2} \times \frac{1}{2}$ and $\frac{9}{8}ab = \frac{n(n-1)}{2} \times \frac{b}{4} \times \frac{1}{4}$	**B3**	**B2** for any two of three correct equations or **B1** for any one of three correct equations **OR B2 for** $a = \frac{n}{4}$ or $\frac{9}{8}a = \frac{n(n-1)}{32}$ oe or $ab = n \times \frac{b}{2} \times \frac{1}{2}$ or $\frac{9}{8}ab = \frac{n(n-1)}{2} \times \frac{b}{4} \times \frac{1}{4}$ oe
	Finds a correct equation in n soi e.g. $n^2 - 10n = 0$ or $n - 1 = 9$ or $10n = n^2$ or $\frac{n}{4} = \frac{n(n-1)}{36}$ OR Finds a correct equation in a soi e.g. $16a^2 - 40a = 0$	**B1**	
	$n = 10$ $a = \frac{5}{2}$ oe $b = 1024$	**B3**	**B1** for each