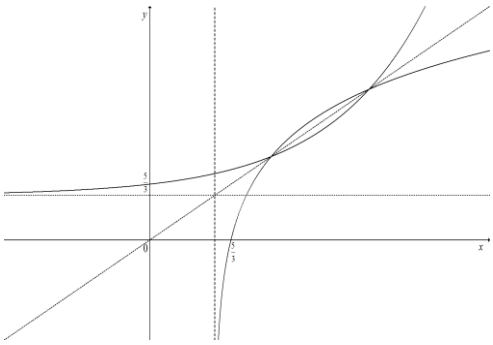


Question	Answer	Marks	Guidance
1	$\pm \frac{1}{3}(x+2)(2x+1)(x-3)$ isw	3	B1 for \pm with 3 factors or expanded out cubic B1 for $\frac{1}{3}$ in a product with 3 factors B1 for $(x+2)(2x+1)(x-3)$ and no extra terms or $2(x+2)(x+0.5)(x-3)$ and no extra terms
2	$x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 (=0)$ oe soi	B1	For multiplying through by $x^{\frac{1}{3}}$ Allow if substitution is used
	$\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} - 2\right) (=0)$ oe	M1	Allow if substitution is used
	$x = -27$	A1	A0 if rejects $x = -27$
	$x = 8$	A1	
3(a)	$5x^2 - 30x + 45 (=0)$ or equivalent 3 term quadratic	2	M1 for attempt to eliminate one variable
	Use of discriminant or solution to show one repeated root or a single solution	A1	Must have conclusion e.g. Discriminant = 0 (so tangent) oe One solution only (so tangent)
	Alternative 1		
	Centre of the circle (5, 2) Equation of radius perpendicular to given line $y - 2 = -\frac{1}{2}(x - 5)$ Intersection at (3, 3)	(2)	M1 for a complete method to obtain the point of intersection
	Show that P lies on the circle or that $CP = \sqrt{5}$	(A1)	
	Alternative 2		
	The line $y = kx + n$ touches the circle $(x - p)^2 + (y - q)^2 = r^2$ if $(-kp + q - n)^2 = r^2(1 + k^2)$ so $(-(2 \times 5) + 2 - 3)^2 = 5(1 + 2^2)$	(2)	M1 for a complete method showing substitution, allow one slip
	$25 = 25$, so a tangent	(A1)	

Question	Answer	Marks	Guidance
3(b)	(3, 3)	2	B1 for each
3(c)	Radius = $3\sqrt{2}$ oe soi	B1	
	$(x-3)^2 + (y-3)^2$ soi	B1	FT on <i>their</i> answer to (b) only
	$(x-3)^2 + (y-3)^2 = 18$ oe	B1	ISW any expansion
4(a)	$[-\cos\theta]_0^\pi$	B1	May be implied by $(-\cos\pi) - (-\cos 0)$
	2	B1	
4(b)	$\frac{1}{\frac{\cos\alpha}{\cos\alpha + \frac{\sin\alpha}{\sin\alpha \cos\alpha}}}$	B1	For ratios in terms of sine and cosine
	$\frac{1}{\frac{\cos\alpha}{\cos^2\alpha + \sin^2\alpha} \sin\alpha \cos\alpha}$	B1	For simplifying the denominator to one term
	$\frac{1}{\frac{\cos\alpha}{\frac{1}{\cos\alpha \sin\alpha}}}$ = $\sin\alpha$	B1	Must show sufficient correct detail
	Alternative		
	$\frac{1}{\frac{\cos\alpha}{\frac{1}{\tan\alpha} + \tan\alpha}} \text{ or } \frac{\sec\alpha}{\frac{1}{\tan\alpha} + \tan\alpha}$	(B1)	For ratios in terms of tangent and cosine or secant and tangent
	$\frac{1}{\frac{\cos\alpha}{\sec^2\alpha} \text{ or } \frac{\sec\alpha}{\sec^2\alpha} \text{ oe } \frac{1}{\tan\alpha}}$	(B1)	For dealing with the denominator
	$\frac{\tan\alpha}{\sec\alpha} = \frac{\sin\alpha}{\cos\alpha} \times \cos\alpha \text{ oe } = \sin\alpha$	(B1)	Must show correct sufficient detail

Question	Answer	Marks	Guidance
5(a)	$p'(x) = 9x^2 - 14x + a$ soi	B1	
	$p'(-1) = 9 + 14 + a = 21$ $a = -2$	B1	
	$p(2) = 24 - 28 + 2a + b = 0$ soi	B1	Each term must be simplified, allow using <i>their a</i>
	$b = 8$	B1	
5(b)	$[x - 2](3x^2 - x - 4)$ soi	2	M1 for quadratic factor with two terms correct A1 must be from correct a and b
	$(x - 2)(x + 1)(3x - 4)$	A1	A1 must be from correct a and b
5(c)	$(e^{2y} - 2) = 0$ soi	B1	
	$y = \frac{1}{2} \ln \frac{4}{3}$ oe, $y = \frac{1}{2} \ln 2$ oe	2	B1 for one correct solution B2 for both solutions and no extra solution.
6(a)	$\ln y = mx^3 + c$ soi	B1	c may be shown as a log term
	$25 = -8m + c$ $5 = 2m + c$	M1	For at least one correct equation, may be used with gradient of -2
	$m = -2, c = 9$	A1	$y = -2x + 9$ seen implies M1A1
	$y = e^{9-2x^3}$ oe	A1	
6(b)	$25 = 9 - 2x^3$	M1	For equating exponential indices and obtaining $x = \dots$ or use of $x = \sqrt[3]{\frac{25 - \text{their } 9}{\text{their } -2}}$
	$x = -2$ only	A1	

Question	Answer	Marks	Guidance
7(a)	$ar^3 = \frac{8k^6}{27}, ar^5 = \frac{32k^{10}}{243}$ soi	B1	May be implied by a correct 5th term $\frac{16k^8}{81}$
	$r = \frac{2k^2}{3}$	2	M1 for solution of <i>their</i> equations to obtain either $r = mk^2$ or $a = c$, where m is an unsimplified numeric constant and c is a non-zero constant e.g. Use of $r^2 = \frac{6\text{th term}}{4\text{th term}}$
	$a = 1$	A1	
7(b)	$\frac{1}{1 - \frac{2k^2}{3}} = 3$ soi	B1	For use of sum to infinity formula with <i>their</i> values of a where a is numeric and r which is in terms of k^2
	$k = \pm 1$	2	B1 for each must be from correct work
8(a)	$\left(\frac{dy}{dx}\right) = \frac{(x+2)\frac{6x}{3x^2+16} - \ln(3x^2+16)}{(x+2)^2}$	3	B1 for $\frac{6x}{3x^2+16}$ M1 for differentiation of a quotient or equivalent product A1 for all other terms apart from $\frac{6x}{3x^2+16}$ correct
	When $x = 0, \frac{dy}{dx} = -\frac{\ln 16}{4}$ $= \ln \frac{1}{2}$ from correct work	2	M1 dep for attempt to substitute in $x = 0$ and obtain a single log term For A1 , all previous marks must have been awarded.
8(b)	Change = $h \ln \frac{1}{2}$ oe	B1	FT on <i>their</i> single log answer to part (a) B0 for $\ln \frac{1}{2}h$ or <i>their</i> $\ln \frac{1}{2}h$
9(a)	$\frac{4}{3}$ or $a = \frac{4}{3}$	B1	Award B1 when a correct answer is seen
9(b)	\mathbb{R} oe	B1	May be in terms of f or y but not x
9(c)	$[f^{-1}(x)] = \frac{4+e^{\frac{x}{3}}}{3}$ oe	2	M1 for a complete valid method, allow a sign error A1 must be using the correct notation

Question	Answer	Marks	Guidance
9(d)		4	<p>B1 for $y = f(x)$ in the 1st and 4th quadrants and appropriate asymptotic behaviour</p> <p>B1 dep on previous B mark for $y = f^{-1}(x)$ intersecting twice with $y = f(x)$, in the 1st and 2nd quadrants and appropriate asymptotic behaviour</p> <p>B1 dep on first B1 for $\frac{5}{3}$ marked correctly on each axis or stated, and no other intercepts</p> <p>B1 for $y = \frac{4}{3}$ and $x = \frac{4}{3}$ (independent) either on the graph or at the side.</p>
10(a)	$\frac{1}{2} \left(\frac{4r}{3} \right)^2 \theta - \frac{1}{2} r^2 \sin \theta \text{ oe}$	3	<p>B1 for $\frac{1}{2} \left(\frac{4r}{3} \right)^2 \theta$</p> <p>B1 for $\frac{1}{2} r^2 \sin \theta$ or $r^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ or $\frac{r^2 \sin^2 \frac{\theta}{2}}{\tan \frac{\theta}{2}}$ oe</p> <p>Allow final answer unsimplified</p>
10(b)	$\text{Arc length} = r \frac{(\pi - \theta)}{2}$	B1	
	$BE = \sqrt{2r^2(1 - \cos \theta)} \text{ or } 2r \sin \frac{\theta}{2} \text{ or } \frac{r \sin \theta}{\sin \left(\frac{\pi - \theta}{2} \right)} \text{ oe}$	2	<p>M1 for complete method to find BE or BE^2, using either cosine rule, sine rule or basic trig using r and a correct angle.</p> <p>A0 if an error in rearranging to obtain BE is made.</p> <p>Allow unsimplified</p>
	$\text{Perimeter} = r(\pi - \theta) + 2r + \sqrt{2r^2(1 - \cos \theta)} \text{ oe}$	2	<p>M1 for a correct plan using <i>their</i> lengths</p> <p>2 arc lengths + $2r$ + <i>their</i> BE</p> <p>Allow A1 for an unsimplified answer</p>

Question	Answer	Marks	Guidance
11(a)	$\overrightarrow{OQ} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{OQ} = \mathbf{b} + \frac{2}{3}(\mathbf{a} - \mathbf{b})$ $\left(\overrightarrow{OQ} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \right)$	2	M1 for attempt to use $\overrightarrow{OQ} = \mathbf{a} + k(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{OQ} = \mathbf{b} + k(\mathbf{a} - \mathbf{b})$, where k can be positive or negative Allow unsimplified
	$\overrightarrow{PQ} = -\frac{3}{4}\mathbf{a} + \text{their } \overrightarrow{OQ}$	M1	
	$\overrightarrow{PQ} = -\frac{3}{4}\mathbf{a} + \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ $\left(\overrightarrow{PQ} = \frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a} \right)$	A1	Allow unsimplified
	$\overrightarrow{OP} = \frac{3}{4}\mathbf{a}$	B1	Allow anywhere
	$\overrightarrow{OR} = \frac{3}{4}\mathbf{a} + \mu \left(\frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a} \right)$ oe	B1	Must be simplified
	Alternative		
	$\overrightarrow{PQ} = \frac{1}{4}\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$ $\left(\overrightarrow{PQ} = \frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a} \right)$	(4)	B1 for $\overrightarrow{PA} = \frac{1}{4}\mathbf{a}$ M1 for $\overrightarrow{AQ} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$, allow a sign error i.e. use of $(\mathbf{a} - \mathbf{b})$ M1 for attempt to use $\overrightarrow{PQ} = \text{their } \frac{1}{4}\mathbf{a} + \text{their } \overrightarrow{AQ}$ Allow unsimplified
	$\overrightarrow{OP} = \frac{3}{4}\mathbf{a}$	(B1)	
$\overrightarrow{OR} = \frac{3}{4}\mathbf{a} + \mu \left(\frac{1}{3}\mathbf{b} - \frac{1}{12}\mathbf{a} \right)$ or $\overrightarrow{OR} = \frac{3}{4}\mathbf{a} - \frac{\mu}{12}\mathbf{a} + \mu \left(\frac{1}{3}\mathbf{b} \right)$ or $\overrightarrow{OR} = \left(\frac{3}{4} - \frac{\mu}{12} \right)\mathbf{a} + \frac{\mu}{3}\mathbf{b}$	(B1)	Must be simplified	

Question	Answer	Marks	Guidance
11(b)	$\lambda = \frac{1}{3}\mu$	B1	For equating b vectors
	$\frac{3}{4} - \frac{1}{12}\mu = 0$ oe	B1	For equating a vectors to zero
	$\mu = 9, \lambda = 3$	B1	
12	When $\frac{dy}{dx} = 0, x = \frac{2}{5}$	B1	Must come from first derivative and not the second derivative
	$(y =) \frac{3}{20}(5x - 2)^{\frac{4}{3}} (+c)$	2	M1 for $k(5x - 2)^{\frac{4}{3}}$
	$\frac{32}{5} = \frac{3}{20}((5 \times 2) - 2)^{\frac{4}{3}} + c$ oe	M1	Dep for attempt to find c using <i>their</i> $k(5x - 2)^{\frac{4}{3}}$
	$c = 4$	A1	Allow an unsimplified fraction
	Stationary point $\left(\frac{2}{5}, 4\right)$	B1	Dep on first B mark