0580/41

May/June 2020

1 **(a)** In 2018, Gretal earned \$32 000.

(i) She paid tax of 24% on these earnings.

Work out the amount she paid in tax in 2018.

$32000 \times 24\% = 7680$

\$ ....7680.................... [2]

(ii) In 2019, Gretal's earnings increased by 7%.

Work out her earnings in 2019.

$32000 + 32000 \times 7\% = 34240$

\$ ....34240.................... [2]

(b) Gretal invests \$5000 at a rate of 2% per year compound interest.

Calculate the value of her investment at the end of 3 years.

$5000\left(1+\frac{2}{100}\right)^3 = 5306.04$

\$ ....5306.04.................... [2]

(c) One month, Gretal spent a total of \$360 on presents.
She spent $\frac{1}{5}$ of this total on presents for her parents.
She spent $\frac{2}{3}$ of the remaining money on presents for her friends.
She spent the rest of the money on presents for her sisters.

Calculate the percentage of the \$360 that she spent on presents for her sisters.

Parents : $360 \times \frac{1}{5} = 72$

Remaining : $360 - 72 = 288$

Friends : $288 \times \frac{2}{3} = 192$

Sisters : $288 - 192 = 96$

$\frac{96}{360} \times 100 \approx 26.7$

....26.7.................... % [4]

(d) Arjun earned \$36 515 in 2019.
This was an increase of 9% on his earnings in 2018.

Work out his earnings in 2018.

$A_{2018} + A_{2018} \times 9\% = A_{2019}$

$1.09\ A_{2018} = 36515$

$A_{2018} = 33500$

\$ ....33500.......... [2]

(e) Arjun and Gretal each pay rent.

In 2018, the ratio of the amount each paid in rent was Arjun : Gretal = 5 : 7.
In 2019, the ratio of the amount each paid in rent was Arjun : Gretal = 9 : 13.

Arjun paid the same amount of rent in both 2018 and 2019. $= A$
Gretal paid \$290 more rent in 2019 than she did in 2018.

Work out the amount Arjun paid in rent in 2019.

$\frac{A}{G_{2018}} = \frac{5}{7} \Rightarrow G_{2018} = \frac{7}{5} A$

$\frac{A}{G_{2019}} = \frac{9}{13} \Rightarrow G_{2019} = \frac{13}{9} A$

$\frac{13}{9} A - \frac{7}{5} A = 290$

$\frac{2}{45} A = 290$

\$ ....6525.......... [4]

2 The heights, h metres, of the 120 boys in an athletics club are recorded.
The table shows information about the heights of the boys.

Mid value	1.35	1.45	1.55	1.65	1.75	1.85
Height (h metres)	$1.3 < h \leqslant 1.4$	$1.4 < h \leqslant 1.5$	$1.5 < h \leqslant 1.6$	$1.6 < h \leqslant 1.7$	$1.7 < h \leqslant 1.8$	$1.8 < h \leqslant 1.9$
Frequency	7	18	30	24	27	14

(a) **(i)** Write down the modal class.

......1.5...... $< h \leqslant$ ......1.6...... [1]

(ii) Calculate an estimate of the mean height.

$$\frac{1.35 \times 7 + 1.45 \times 18 + 1.55 \times 30 + 1.65 \times 24 + 1.75 \times 27 + 1.85 \times 14}{120}$$

......1.62...... m [4]

(b) **(i)** One boy is chosen at random from the club.

Find the probability that this boy has a height greater than 1.8 m.

......$\frac{14}{120}$...... [1]

(ii) Three boys are chosen at random from the club.

Calculate the probability that one of the boys has a height greater than 1.8 m and the other two boys each have a height of 1.4 m or less.

$$\left(\frac{14}{120} \times \frac{7}{119} \times \frac{6}{118}\right) \times 3$$

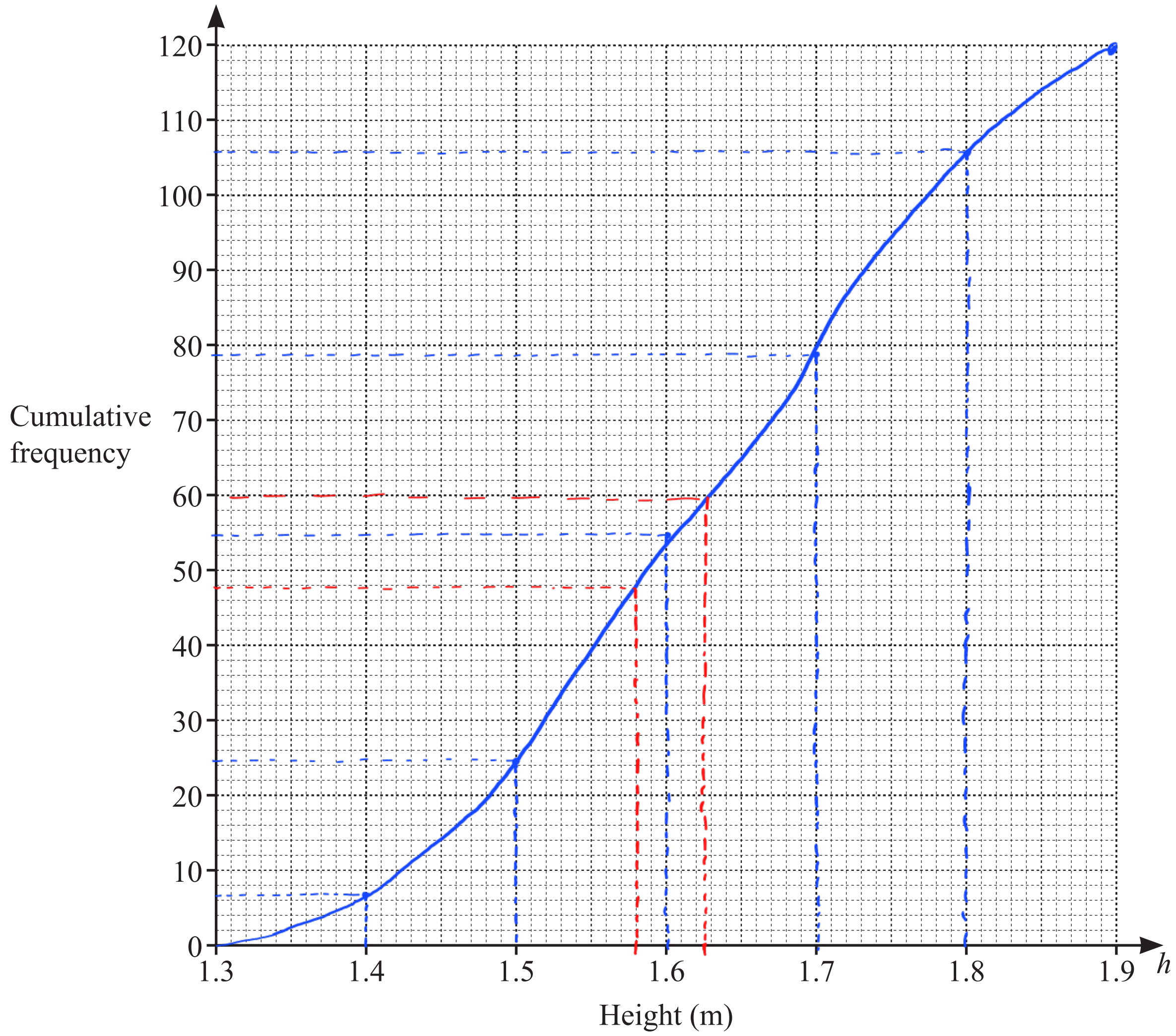
......$\frac{21}{20060}$...... [4]

(c) **(i)** Use the frequency table on page 4 to complete the cumulative frequency table.

Height (h metres)	$h \leqslant 1.4$	$h \leqslant 1.5$	$h \leqslant 1.6$	$h \leqslant 1.7$	$h \leqslant 1.8$	$h \leqslant 1.9$
Cumulative frequency	7	25	55	79	106	120

[2]

(ii) On the grid, draw a cumulative frequency diagram to show this information.



[3]

(d) Use your diagram to find an estimate for

(i) the median height,

..........1.625.......... m [1]

(ii) the 40th percentile.

120 × 40% = 48

..........1.58.......... m [2]

3 **(a)** $s = ut + \frac{1}{2}at^2$

Find the value of s when $u = 5.2$, $t = 7$ and $a = 1.6$.

$s = 5.2 \times 7 + \frac{1}{2} \times 1.6 \times 7^2$

$s = $ ......75.6...... [2]

(b) Simplify.

(i) $3a - 5b - a + 2b$

......$2a - 3b$...... [2]

(ii) $\frac{5}{3x} \times \frac{9x}{20}$

$\frac{3x}{4x}$

......$\frac{3}{4}$...... [2]

(c) Solve.

(i) $\frac{15}{x} = -3$

$x = \frac{15}{-3} = -5$

$x = $ ......-5...... [1]

(ii) $4(5 - 3x) = 23$

$5 - 3x = 5.75$

$3x = 5 - 5.75 = -0.75$

$x = -0.25$

$x = $ ......-0.25...... [3]

(d) Simplify.

$$(27x^9)^{\frac{2}{3}}$$

$27^{\frac{2}{3}} \quad x^{9 \times \frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 \quad x^6 = 3^2 x^6$

$9x^6$ [2]

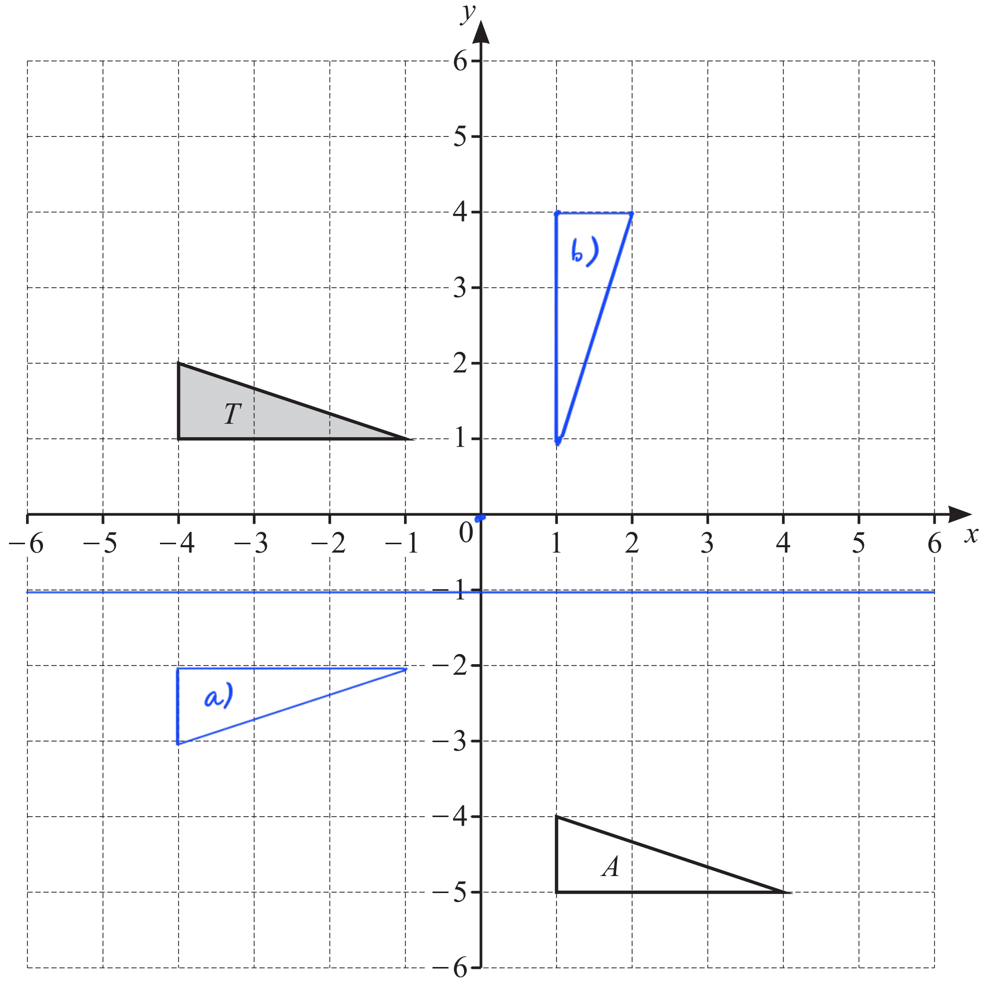
(e) Expand and simplify.

$$(3x - 5y)(2x + y)$$

$6x^2 - 10xy + 3xy - 5y^2$

$6x^2 - 7xy - 5y^2$ [2]

4



(a) Draw the image of triangle T after a reflection in the line $y = -1$. [2]

(b) Draw the image of triangle T after a rotation through 90° clockwise about (0, 0). [2]

(c) Describe fully the **single** transformation that maps triangle T onto triangle A.

Translation by vector $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

.... [2]

5 x is an integer.

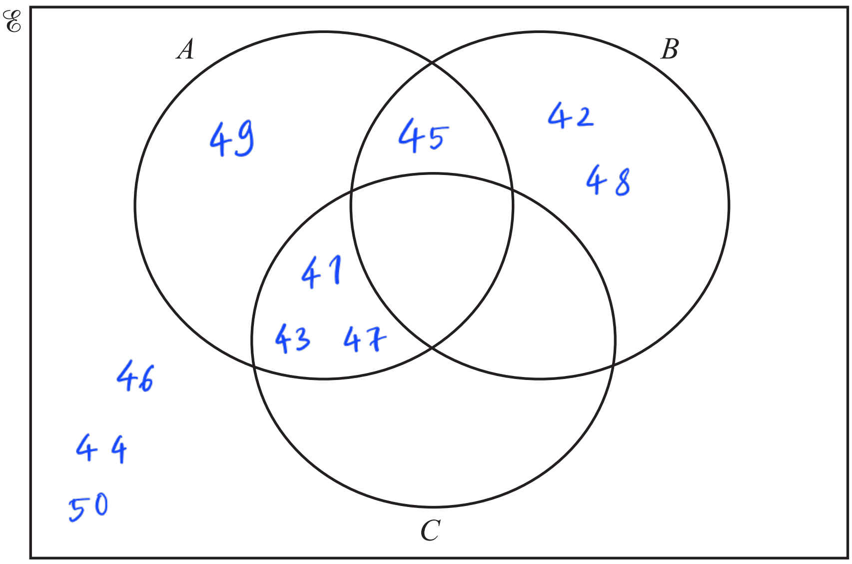
$\mathscr{E} = \{x : 41 \leqslant x \leqslant 50\}$

$A = \{x : x \text{ is an odd number}\}$

$B = \{x : x \text{ is a multiple of } 3\}$

$C = \{x : x \text{ is a prime number}\}$

(a) Complete the Venn diagram to show this information.



[3]

(b) List the elements of

(i) $A \cap C$,

.....41, 43, 47..... [1]

(ii) $(B \cup C)'$.

.....44, 46, 49, 50..... [1]

(c) Find $n(A \cap B \cap C)$.

.....0..... [1]

6 Raheem makes baskets and mats.
Each week he makes x baskets and y mats.

He makes fewer than 10 mats.
The number of mats he makes is greater than or equal to the number of baskets he makes.

(a) One of the inequalities that shows this information is $y < 10$.

Write down the other inequality.

.... $y \geqslant x$ .... [1]

(b) He takes $2\frac{1}{4}$ hours to make a basket and $1\frac{1}{2}$ hours to make a mat.
Each week he works for a maximum of 22.5 hours.

Show that $3x + 2y \leqslant 30$.

$2\frac{1}{4}x + 1\frac{1}{2}y \leq 22.5$

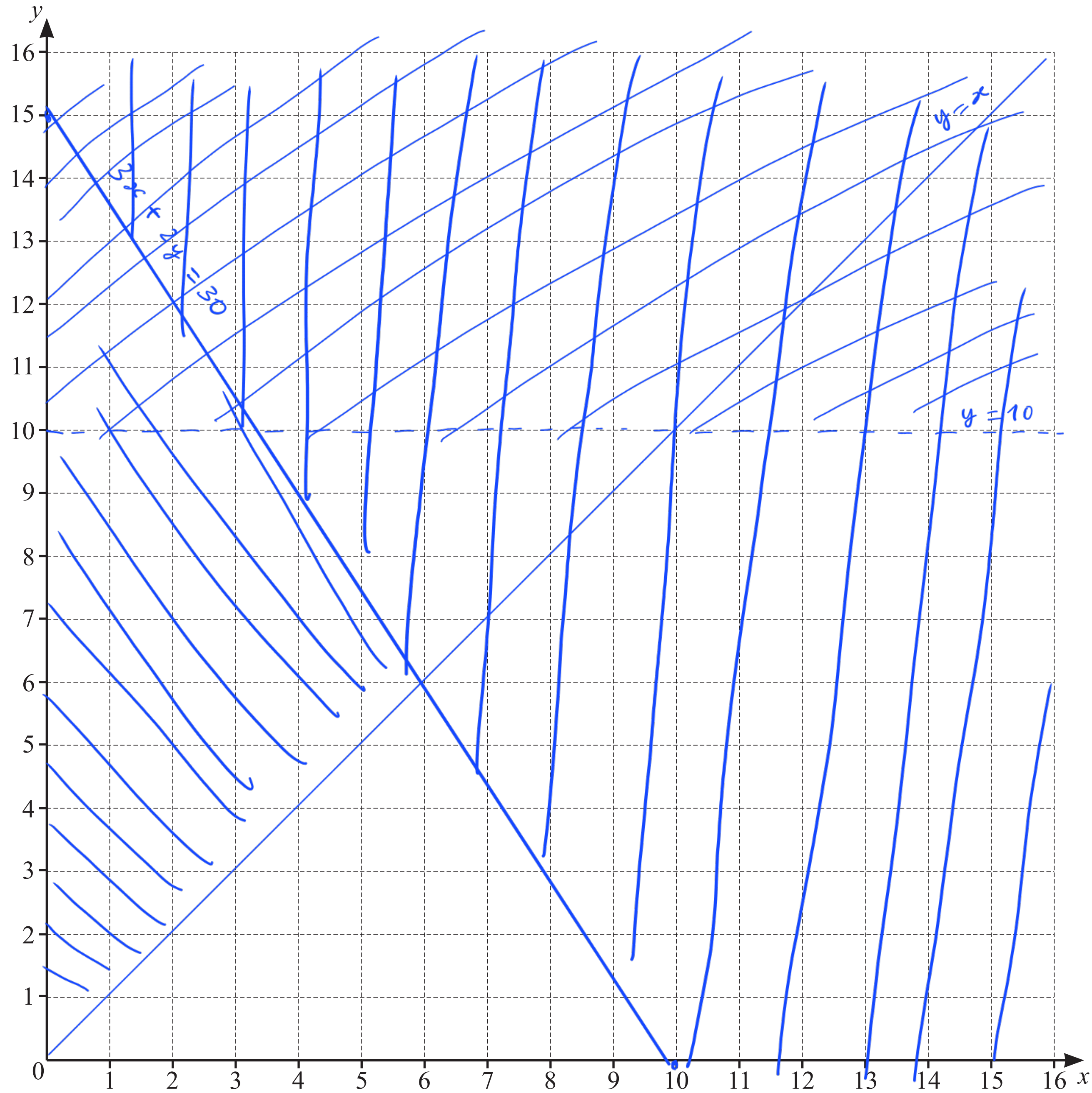
$\frac{9}{4}x + \frac{3}{2}y \leq 22.5$

$9x + 3y \leq 90$

$3x + y \leq 30$

[2]

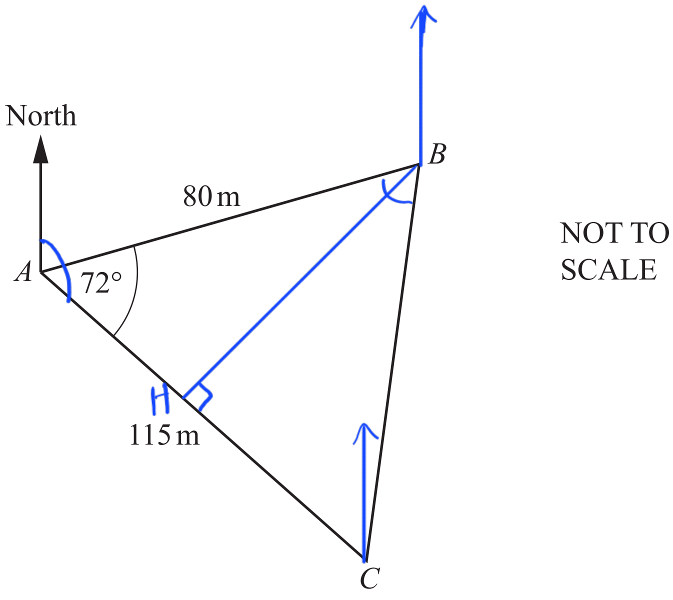
(c) On the grid, draw three straight lines and shade the **unwanted** regions to show these inequalities.



[5]

....

7



The diagram shows the positions of three points *A*, *B* and *C* in a field.

(a) Show that *BC* is 118.1 m, correct to 1 decimal place.

$BC^2 = 80^2 + 115^2 - 2 \times 80 \times 115 \cos 72°$

$BC^2 \simeq 13939$

$BC \simeq 118.064 \simeq 118.1$

[3]

(b) Calculate angle *ABC*.

$$\frac{115}{\sin \widehat{ABC}} = \frac{118.064}{\sin 72°}$$

$$\sin \widehat{ABC} = \frac{115 \sin 72°}{118.064} \simeq 0.926$$

Angle *ABC* = ...67.8°...... [3]

(c) The bearing of C from A is 147°.

Find the bearing of

(i) A from B,

$147° - 72° = 75°$

$180° - 75° = 105°$

$360° - 105° = 255°$

.................255°................ [3]

(ii) B from C.

$105° + 67.8° = 172.8°$

$180° - 172.8° = 7.2°$

.................7.2°................ [2]

(d) Mitchell takes 35 seconds to run from A to C.

Calculate his average running speed in kilometres per hour.

average speed $= \dfrac{115 \text{ m}}{35 \text{ s}} = \dfrac{\frac{115}{1000} \text{ km}}{\frac{35}{3600} \text{ h}} \approx 11.8$

.................11.8................ km/h [3]

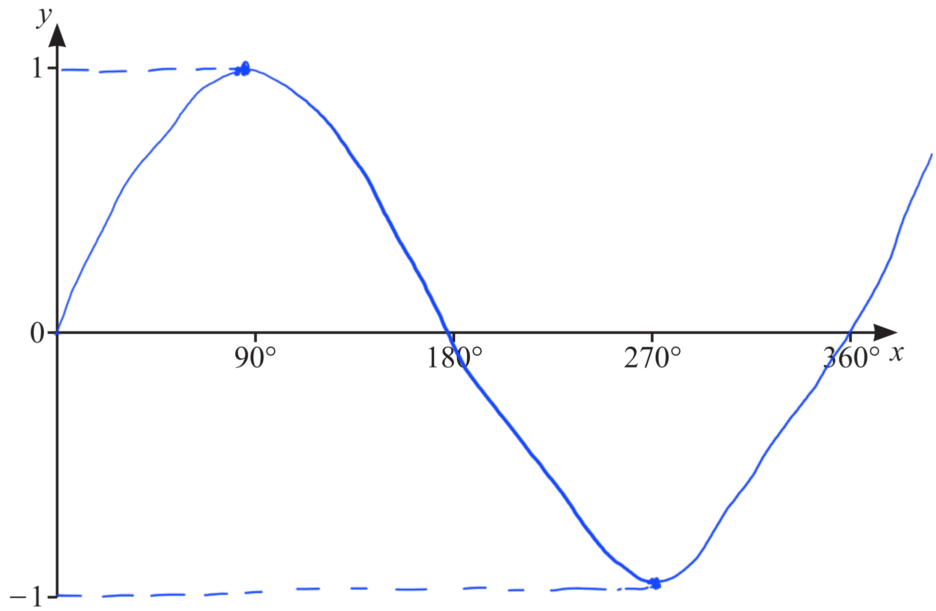
(e) Calculate the shortest distance from point B to AC.

$\sin 72° = \dfrac{BH}{AB} = \dfrac{BH}{80}$

$BH = 80 \sin 72°$

.................76.1................ m [3]

8 **(a)** **(i)** On the axes, sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.



[2]

(ii) Describe fully the symmetry of the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

Rotational symmetry order 2

center (180°, 0) [2]

(b) Solve $4\sin x - 1 = 2$ for $0° \leqslant x \leqslant 360°$.

$4\sin x = 3$

$\sin x = \frac{3}{4}$

$x = 48.6°$ or $x = 180° - 48.6° = 131.4°$

$x =$ ......48.6°...... and $x =$ ......131.4°...... [3]

(c) **(i)** Write $x^2 + 10x + 14$ in the form $(x+a)^2 + b$.

$(x^2 + 2x5 + 5^2) - 11$

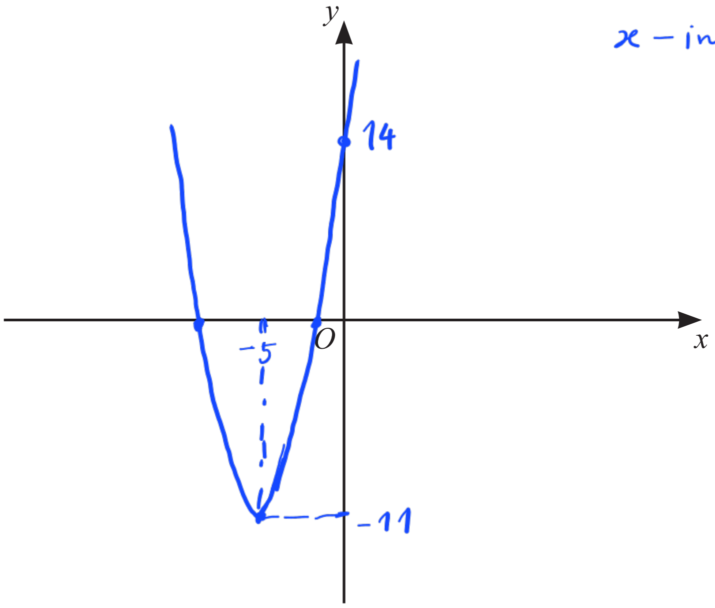
$(x+5)^2 - 11$

$(x+5)^2 - 11$ .... [2]

(ii) On the axes, sketch the graph of $y = x^2 + 10x + 14$, indicating the coordinates of the turning point.

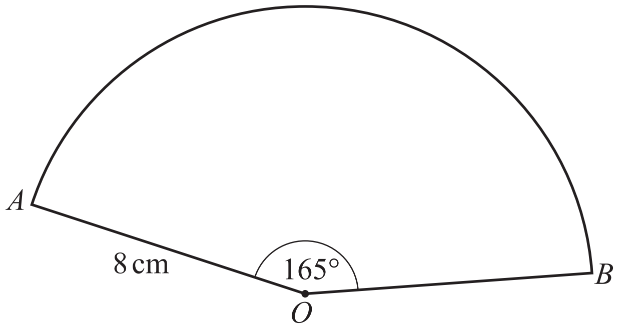
y - intercep: 14

x - intercep: $-5 \pm \sqrt{11}$



[3]

9



The diagram shows a sector of a circle with centre O, radius 8 cm and sector angle 165°.

(a) Calculate the total perimeter of the sector.

$$165° = \frac{165\pi}{180} = \frac{11}{12}\pi \text{ rad}$$

$$\text{Total perimeter} = 8 \times 2 + 8 \times \frac{11}{12}\pi$$

.........39.0......... cm [3]

(b) The surface area of a sphere is the same as the area of the sector.

Calculate the radius of the sphere.

$$\text{Area}_{\text{sector}} = \frac{1}{2} \times 8^2 \times \frac{11}{12}\pi = \frac{88}{3}\pi$$

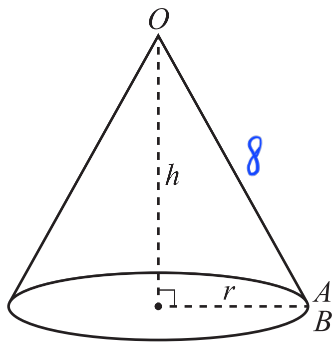
$$4\pi r^2_{\text{sphere}} = \frac{88}{3}\pi$$

$$\Rightarrow \quad r^2_{\text{sphere}} = \frac{22}{3}$$

$$r_{\text{sphere}} = \sqrt{\frac{22}{3}}$$

.........2.71......... cm [4]

(c)



NOT TO SCALE

A cone is made from the sector by joining *OA* to *OB*.

(i) Calculate the radius, *r*, of the cone.

$$\text{Arc length}_{\text{sector}} = 2\pi r_{\text{cone}}$$

$$8 \times \frac{11}{12}\pi = 2\pi r_{\text{cone}}$$

$$r_{\text{cone}} = \frac{11}{3}$$

r = ...3.67...... cm [2]

(ii) Calculate the volume of the cone.

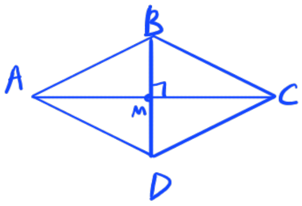
$$h = \sqrt{8^2 - \left(\frac{11}{3}\right)^2} = \frac{\sqrt{455}}{3}$$

$$V = \frac{1}{3}\pi\, 3.67^2\, \frac{\sqrt{455}}{3} \simeq 100.$$

......100,...... cm^3 [4]

10 (a) A rhombus *ABCD* has a diagonal *AC* where *A* is the point (−3, 10) and *C* is the point (4, −4).

(i) Calculate the length *AC*.



$AC = \sqrt{(-4-10)^2 + (4 - -3)^2}$

$= 7\sqrt{5}$

.........15.7......... [3]

(ii) Show that the equation of the line *AC* is $y = -2x + 4$.

$m_{AC} = \frac{-4-10}{4-(-3)} = \frac{-14}{7} = -2$

$AC: \quad y - 10 = -2(x+3)$

$y - 10 = -2x - 6$

$y = -2x + 4$

[2]

(iii) Find the equation of the line *BD*.

$BD \perp AC$

$\Rightarrow m_{BD} = \frac{1}{2}$

Mid point of AC: $M = \left(\frac{-3+4}{2}, \frac{10-4}{2}\right) = \left(\frac{1}{2}, 3\right)$

$BD: \quad y - 3 = \frac{1}{2}\left(x - \frac{1}{2}\right)$

$y - 3 = \frac{1}{2}\left(x - \frac{1}{2}\right)$ [4]

(b) A curve has the equation $y = x^3 + 8x^2 + 5x$.

(i) Work out the coordinates of the two turning points.

$$\frac{dy}{dx} = 3x^2 + 16x + 5 = 0$$

$$(x+5)(3x+1) = 0$$

$$x = -5 \quad \text{or} \quad x = -\frac{1}{3}$$

when $x = -5$, $y = (-5)^3 + 8(-5)^2 + 5(-5) = 50$

When $x = \frac{-1}{3}$, $y = \left(\frac{-1}{3}\right)^3 + 8\left(-\frac{1}{3}\right)^2 + 5\left(\frac{-1}{3}\right) = \frac{-22}{27}$

(........ -5 .... , ... 50 ....) and (........ $\frac{-1}{3}$ .... , ... $\frac{-22}{27}$ ....) [6]

(ii) Determine whether each of the turning points is a maximum or a minimum.
Give reasons for your answers.

$$\frac{d^2y}{dx^2} = 6x + 16$$

When $x = -5$: $6(-5) + 16 < 0 \Rightarrow (-5, 50)$ is a maximum

When $x = -\frac{1}{3}$: $6\left(-\frac{1}{3}\right) + 16 > 0 \Rightarrow \left(-\frac{1}{3}, \frac{-22}{27}\right)$ is a minimum

[3]