1 **(a)** **(i)** Divide \$24 in the ratio 7 : 5.

$\frac{24}{7+5} \times 7 = 14$

$24 - 14 = 10$

\$ 14 , \$ 10 [2]

(ii) Write \$24.60 as a fraction of \$2870.
Give your answer in its lowest terms.

$\frac{24.60}{2870} = \frac{3}{350}$

$\frac{3}{350}$ [2]

(iii) Write \$1.92 as a percentage of \$1.60 .

$\frac{1.92}{1.60} \times 100$

120 % [1]

(b) In a sale the original prices are reduced by 15%.

(i) Calculate the sale price of a book that has an original price of \$12.

$12 - 12 \times 15\%$

\$ 10.2 [2]

(ii) Calculate the original price of a jacket that has a sale price of \$38.25 .

j

$j - 15\% j = 38.25$

$0.85 j = 38.25$

$j = 45$

\$ 45 [2]

(c) (i) Dean invests \$500 for 10 years at a rate of 1.7% per year simple interest.

Calculate the total interest earned during the 10 years.

$500 \times \frac{1.7}{100} \times 10$

\$ ....85..... [2]

(ii) Ollie invests \$200 at a rate of 0.0035% **per day** compound interest.

Calculate the value of Ollie's investment at the end of 1 year.
[1 year = 365 days.]

$200\left(1+\frac{0.0035}{100}\right)^{365}$

\$ .....203..... [2]

(iii) Edna invests \$500 at a rate of r% per year compound interest.
At the end of 6 years, the value of Edna's investment is \$559.78 .

Find the value of r.

$559.78 = 500\left(1+\frac{r}{100}\right)^{6}$

$1+\frac{r}{100} = \sqrt[6]{\frac{559.78}{500}}$

$\frac{r}{100} = 0.019$

$r = 1.9$

r = ...1.9..... [3]

2 **(a)** $\mathbf{p} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

(i) Find $2\mathbf{p} + \mathbf{q}$.

$2\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

$\begin{pmatrix} 6 \\ 17 \end{pmatrix}$ [2]

(ii) Find $|\mathbf{p}|$.

$\sqrt{4^2 + 5^2} = \sqrt{41}$

6.40 [2]

(b) A is the point (4, 1) and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

Find the coordinates of B.

$x_B - x_A = -3 \Rightarrow x_B = -3 + 4 = 1$

$y_B - y_A = 1 \Rightarrow y_B = 1 + 1 = 2$

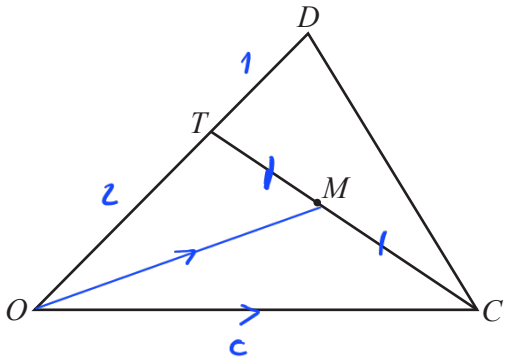
(1 , 2) [1]

(c) The line $y = 3x - 2$ crosses the y-axis at G.

Write down the coordinates of G.

(0 , −2) [1]

(d)



NOT TO SCALE

In the diagram, O is the origin, $OT = 2TD$ and M is the midpoint of TC.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.

Find the position vector of M.
Give your answer in terms of $\mathbf{c}$ and $\mathbf{d}$ in its simplest form.

$\overrightarrow{OT} = \frac{2}{3}\overrightarrow{OD} = \frac{2}{3}\mathbf{d}$

$\overrightarrow{TC} = \overrightarrow{TO} + \overrightarrow{OC} = -\frac{2}{3}\mathbf{d} + \mathbf{c}$

$\overrightarrow{TM} = \frac{1}{2}\overrightarrow{TC} = -\frac{1}{3}\mathbf{d} + \frac{1}{2}\mathbf{c}$

$\overrightarrow{OM} = \overrightarrow{OT} + \overrightarrow{TM} = \frac{2}{3}\mathbf{d} - \frac{1}{3}\mathbf{d} + \frac{1}{2}\mathbf{c}$

$= \frac{1}{3}\mathbf{d} + \frac{1}{2}\mathbf{c}$

$\frac{1}{3}\mathbf{d} + \frac{1}{2}\mathbf{c}$ [3]

3 The speed, v km/h, of each of 200 cars passing a building is measured.
The table shows the results.

Mid value	10	30	42.5	47.5	55	70
Speed (v km/h)	$0 < v \leqslant 20$	$20 < v \leqslant 40$	$40 < v \leqslant 45$	$45 < v \leqslant 50$	$50 < v \leqslant 60$	$60 < v \leqslant 80$
Frequency	16	34	62	58	26	4

(a) Calculate an estimate of the mean.

$$\frac{10 \times 16 + 30 \times 34 + 42.5 \times 62 + 47.5 \times 58 + 55 \times 26 + 70 \times 4}{200}$$

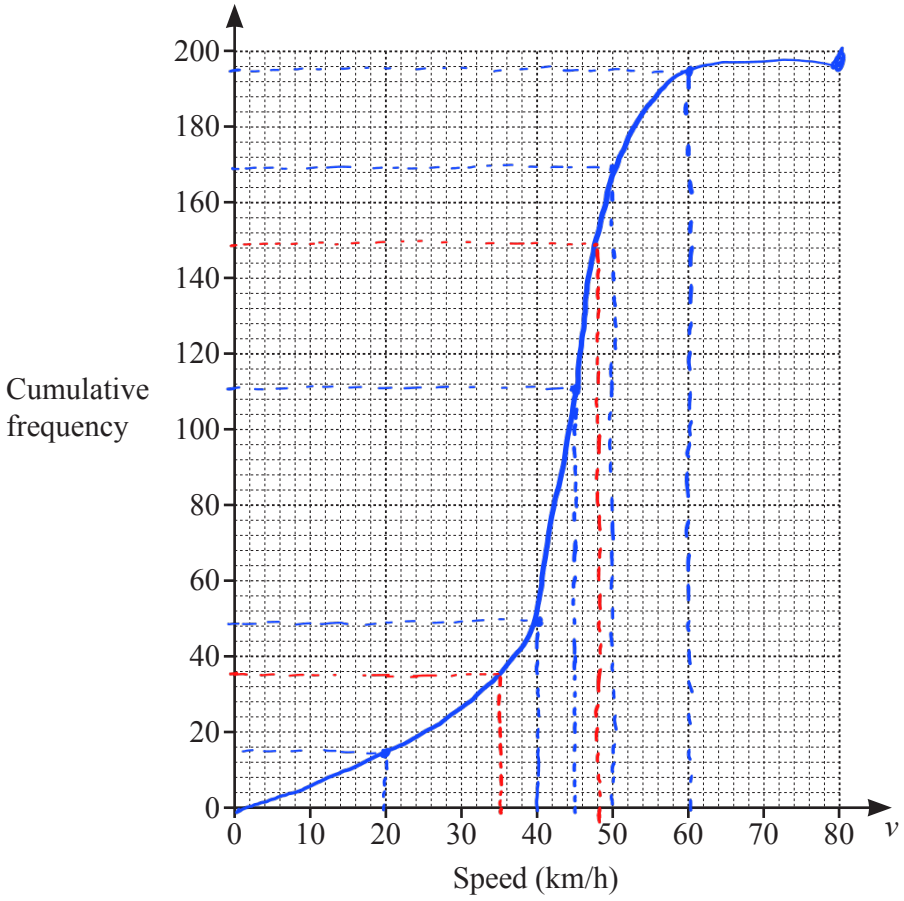
.....41.4..... km/h [4]

(b) (i) Use the frequency table to complete the cumulative frequency table.

Speed (v km/h)	$v \leqslant 20$	$v \leqslant 40$	$v \leqslant 45$	$v \leqslant 50$	$v \leqslant 60$	$v \leqslant 80$
Cumulative frequency	16	50	112	170	196	200

[1]

(ii) On the grid, draw a cumulative frequency diagram.



[3]

(iii) Use your diagram to find an estimate of

(a) the upper quartile,

200 × 75% = 150

.....48..... km/h [1]

(b) the number of cars with a speed greater than 35 km/h.

.....3.6..... [2]

(c) Two of the 200 cars are chosen at random.

Find the probability that they both have a speed greater than 50 km/h.

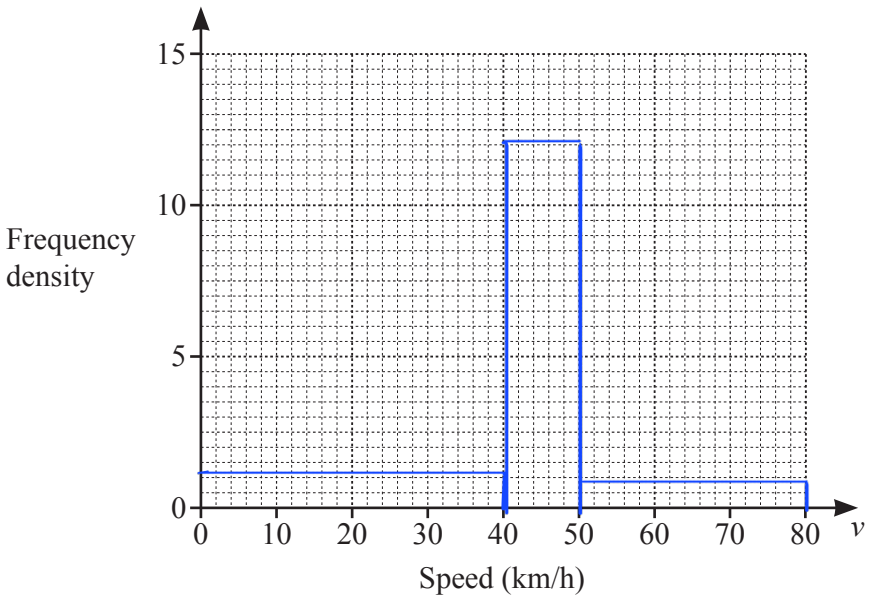
$\frac{30}{200} \times \frac{29}{199}$

.....$\frac{87}{3980}$..... [2]

(d) A new frequency table is made by combining intervals.

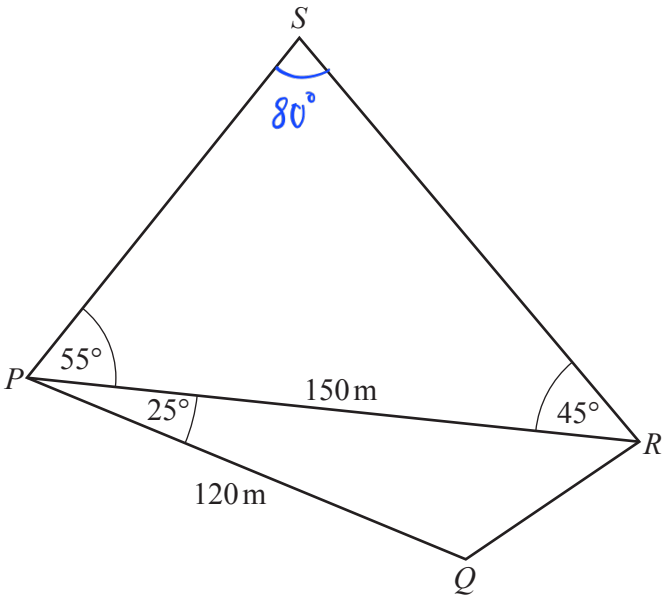
Class interval	40	10	30
Speed (v km/h)	$0 < v \leqslant 40$	$40 < v \leqslant 50$	$50 < v \leqslant 80$
Frequency	50	120	30
Freq density	1.25	12	1

On the grid, draw a histogram to show the information in this table.



[3]

4



NOT TO SCALE

The diagram shows two triangles.

(a) Calculate QR.

$QR^2 = 120^2 + 150^2 - 2 \times 120 \times 150 \times \cos 25°$

$QR^2 = 4272.92$

$QR \approx 65.4$

$QR =$ ..... 65.4 ..... m [3]

(b) Calculate RS.

$\hat{PSR} = 180° - 55° - 45° = 80°$

$\frac{150}{\sin 80°} = \frac{RS}{\sin 55°}$

$RS = \frac{150 \sin 55°}{\sin 80°} \approx 124.77$

$RS =$ ..... 125 ..... m [4]

(c) Calculate the total area of the two triangles.

$A_{\Delta PRS} + A_{\Delta PRQ}$

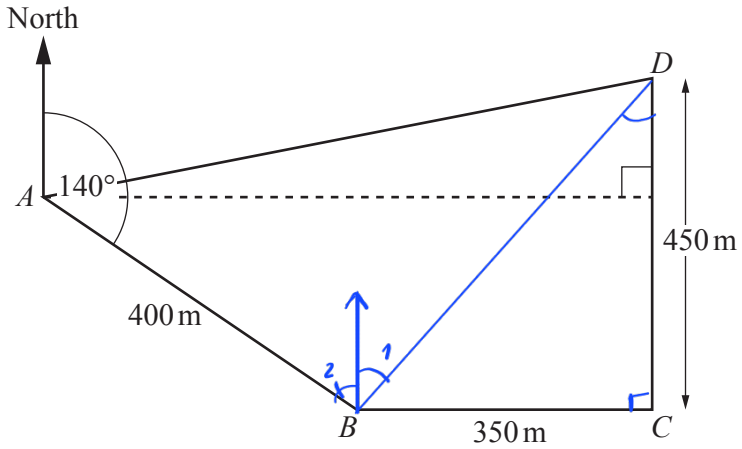
$= \frac{1}{2} SR \cdot PR \cdot \sin 45° + \frac{1}{2} PR \cdot PQ \cdot \sin 25°$

$= \frac{1}{2} \times 124.77 \times 150 \sin 45° + \frac{1}{2} 150 \times 120 \sin 25°$

≈ 10420.5

10400 m² [3]

5



NOT TO SCALE

The diagram shows a field $ABCD$.
The bearing of B from A is 140°.
C is due east of B and D is due north of C.
$AB = 400$ m, $BC = 350$ m and $CD = 450$ m.

(a) Find the bearing of D from B.

$\tan \widehat{BDC} = \frac{350}{450}$

$\widehat{BDC} \approx 37.87°$

$\Rightarrow$ bearing $_{B \to D} = 038°$

038° [2]

(b) Calculate the distance from D to A.

$\widehat{B_2} = 180° - 140° = 40°$

$\widehat{ABD} = \widehat{B_1} + \widehat{B_2} = 37.87 + 40° = 77.87°$

$BD^2 = 350^2 + 450^2 = 325000 \Rightarrow BD = 50\sqrt{130}$

$AD^2 = AB^2 + BD^2 - 2\,AB.BD\cos\widehat{ABD}$

$AD^2 = 400^2 + 325000 - 2 \times 400 \times 50\sqrt{130}\cos 77.87°$

$AD^2 = 389166$

$AD \approx 624$

.....624..... m [6]

(c) Jono runs around the field from A to B, B to C, C to D and D to A.
He runs at a speed of 3 m/s.

Calculate the total time Jono takes to run around the field.
Give your answer in minutes and seconds, correct to the nearest second.

$\text{Total time} = \dfrac{AB + BC + CD + DA}{3}$

$= \dfrac{400 + 350 + 450 + 624}{3}$

$= 608\ s$

$608 : 60 = 10 \text{ r } 8$

.....10..... min .....8..... s [4]

6 $f(x) = 3x + 2$ $g(x) = x^2 + 1$ $h(x) = 4^x$

(a) Find h(4).

4^4

256 [1]

(b) Find fg(1).

$g(1) = 1^2 + 1 = 2$

$f(2) = 3 \times 2 + 2 = 8$

8 [2]

(c) Find gf(x) in the form $ax^2 + bx + c$.

$(3x + 2)^2 + 1$

$9x^2 + 12x + 4 + 1$

$9x^2 + 12x + 5$ [3]

(d) Find x when $f(x) = g(7)$.

$3x + 2 = 7^2 + 1$

$3x = 48$

$x = 16$

$x =$ 16 [2]

(e) Find $f^{-1}(x)$.

$\times 3 \rightarrow + 2$

$: 3 \leftarrow - 2$

$f^{-1}(x) = \dfrac{x - 2}{3}$ [2]

(f) Find $\frac{g(x)}{f(x)} + x$.

Give your answer as a single fraction, in terms of x, in its simplest form.

$\frac{x^2+1}{3x+2} + x$

$\frac{x^2 + 1 + x(3x+2)}{3x+2}$

$\frac{x^2 + 1 + 3x^2 + 2x}{3x+2}$

$\frac{4x^2 + 2x + 1}{3x+2}$ [3]

(g) Find x when $h^{-1}(x) = 2$.

$y = 4^x$

Swap $x = 4^y$ (y here is $h^{-1}(x)$)

$x = 4^2$

$x =$ 16 [1]

7 Tanya plants some seeds.
The probability that a seed will produce flowers is 0.8 .
When a seed produces flowers, the probability that the flowers are red is 0.6 and the probability that the flowers are yellow is 0.3 .

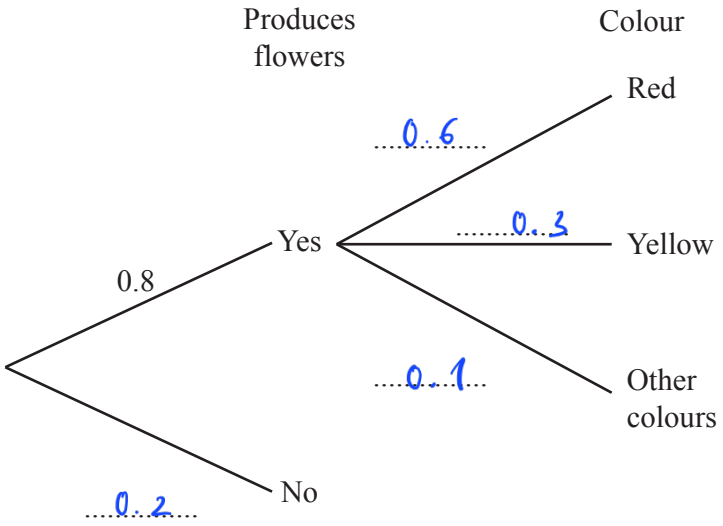
(a) Tanya has a seed that produces flowers.

Find the probability that the flowers are not red and not yellow.

$1 - 0.6 - 0.3 = 0.1$

.....0.1..... [1]

(b) **(i)** Complete the tree diagram.



[2]

(ii) Find the probability that a seed chosen at random produces red flowers.

0.8×0.6

.....0.48..... [2]

(iii) Tanya chooses a seed at random.

Find the probability that this seed does not produce red flowers and does not produce yellow flowers.

P (not produce flower) + P (produce other colors)

0.2 + 0.8 × 0.1

..... 0.28 ..... [3]

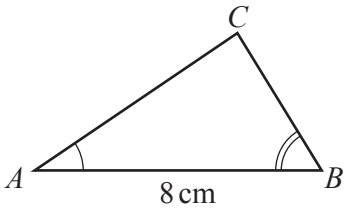
(c) Two of the seeds are chosen at random.

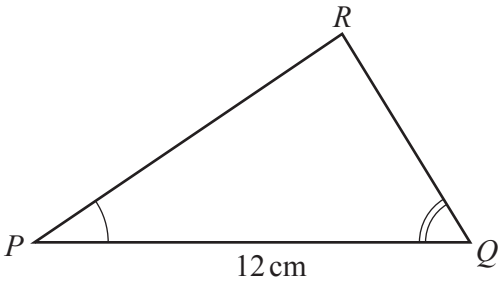
Find the probability that one produces flowers and one does not produce flowers.

(0.8 × 0.2) × 2

..... 0.32 ..... [3]

8 (a)





NOT TO SCALE

Triangle ABC is mathematically similar to triangle PQR.
The area of triangle ABC is $16\,cm^2$.

(i) Calculate the area of triangle PQR.

$$\frac{A_{\Delta PQR}}{A_{\Delta ABC}} = \left(\frac{PQ}{AB}\right)^2 = \left(\frac{12}{8}\right)^2 = \frac{9}{4}$$

$$A_{\Delta PQR} = \frac{9}{4} \times 16$$

.....36..... cm^2 [2]

(ii) The triangles are the cross-sections of prisms which are also mathematically similar.
The volume of the smaller prism is $320\,cm^3$.

Calculate the length of the larger prism.

$$\text{Ratio}_{\text{volume}} = (\text{Ratio}_{\text{side}})^3$$

$$\frac{V_{small}}{V_{large}} = \left(\frac{side_{small}}{side_{large}}\right)^3 = \left(\frac{8}{12}\right)^3 = \frac{8}{27}$$

$$\Rightarrow V_{large} = 320 : \frac{8}{27} = 1080$$

$$\Rightarrow length_{large} = \frac{1080}{A_{\Delta PQR}} = \frac{1080}{36}$$

.....30..... cm [3]

(b) A cylinder with radius 6 cm and height h cm has the same volume as a sphere with radius 4.5 cm.

Find the value of h.

$\pi 6^2 h = \frac{4}{3} \pi 4.5^3$

$36h = 121.5$

$h = 3.375$

$h =$ 3.375 [3]

(c) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres, each of radius r cm.

Find the value of r.

$V_{cube} = 20^3$

$V_{sphere} = \frac{20^3}{40} = 200$

$\frac{4}{3} \pi r^3 = 200$

$r = \sqrt[3]{\frac{200}{\frac{4}{3}\pi}}$

$r =$ 3.63 [3]

(d) A solid cylinder has radius x cm and height $\frac{7x}{2}$ cm.

The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder.

Find an expression for R in terms of x.

$4\pi R^2 = 2\pi x^2 + 2\pi x h$

$2R^2 = x^2 + x \cdot \frac{7x}{2}$

$2R^2 = \frac{9}{2} x^2$

$R^2 = \frac{9}{4} x^2 = \left(\frac{3}{2} x\right)^2$

$R =$ $\frac{3}{2}x$ [3]

9 **(a)** **(i)** Write x^2+8x-9 in the form $(x+k)^2+h$.

$(x^2 + 2 \times 4 + 4^2) - 4^2 - 9$

$(x+4)^2 - 25$

$(x+4)^2 - 25$ [2]

(ii) Use your answer to **part (a)(i)** to solve the equation $x^2+8x-9=0$.

$(x+4)^2 - 25 = 0$

$(x+4)^2 = 25$

$x + 4 = -5$ or $x + 4 = 5$

$x = -9$ or $x = 1$ [2]

(b) The solutions of the equation $x^2+bx+c=0$ are $\frac{-7+\sqrt{61}}{2}$ and $\frac{-7-\sqrt{61}}{2}$.

Find the value of b and the value of c.

$-b = -7 \Rightarrow b = 7$

$b^2 - 4ac = 61$

$\Rightarrow 7^2 - 4c = 61$

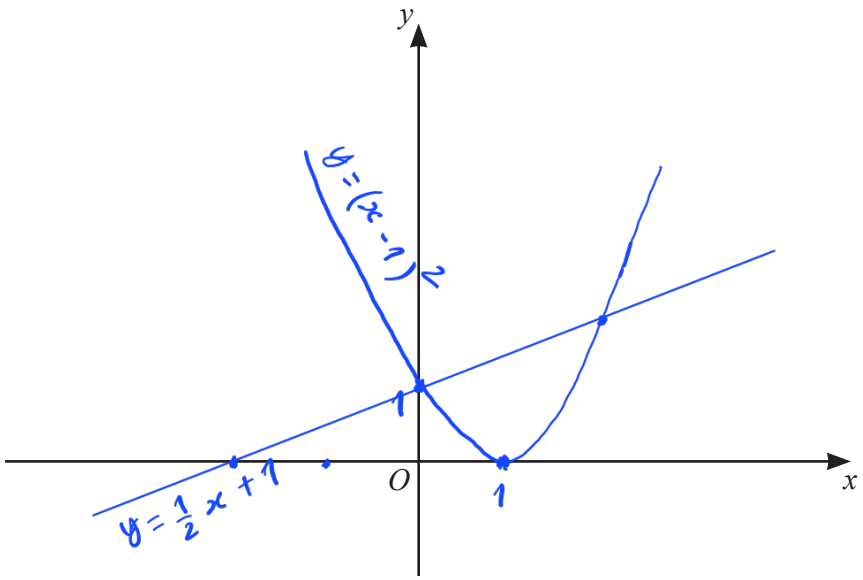
$4c = -12$

$c = -3$

$b = 7$

$c = -3$ [3]

(c) (i)



On the diagram,

(a) sketch the graph of $y = (x-1)^2$, [2]

(b) sketch the graph of $y = \frac{1}{2}x + 1$. [2]

(ii) The graphs of $y = (x-1)^2$ and $y = \frac{1}{2}x + 1$ intersect at A and B.

Find the length of AB.

$(x-1)^2 = 0.5x + 1$

$x^2 - 2x + 1 = 0.5x + 1$

$x^2 - 2.5x = 0$

$x(x - 2.5) = 0$

$x = 0$ or $x = 2.5$

When $x = 0$, $y = (0-1)^2 = 1$ $\Rightarrow$ point $(0, 1)$

When $x = 2.5$, $y = (2.5-1)^2 = 2.25$ $\Rightarrow$ point $(2.5, 2.25)$

$AB = \sqrt{(2.5-0)^2 + (2.25-1)^2} = \frac{5\sqrt{5}}{4}$

$AB =$ ..... 2.80 ..... [7]

10 **(a)** $y = x^4 - 4x^3$

(i) Find the value of y when $x = -1$.

$(-1)^4 - 4(-1)^3$

$y =$ 5 [2]

(ii) Find the two stationary points on the graph of $y = x^4 - 4x^3$.

$\frac{dy}{dx} = 4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$x = 0$ or $x = 3$

When $x = 0$, $y = 0^4 - 4 \times 0^3 = 0$

When $x = 3$, $y = 3^4 - 4 \times 3^3 = -27$

(0 , 0)

(3 , −27) [6]

(b) $y = x^p + 2x^q$

$\frac{dy}{dx} = 11x^{10} + 10x^4$, where $\frac{dy}{dx}$ is the derived function.

Find the value of p and the value of q.

$\frac{dy}{dx} = px^{p-1} + 2qx^{q-1}$

$= 11x^{10} + 10x^4$

$\Rightarrow \begin{cases} p = 11 \\ 2q = 10 \end{cases}$

$p =$ 11

$q =$ 5 [2]