0580/43

May/June 2020

1 **(a)**

Campsite fees (per day)
Tent \$15.00
Caravan \$25.00

The sign shows the fees charged at a campsite.
Today there are 54 tents and 18 caravans on the site.

Calculate the fees charged today.

$54 \times 15 + 18 \times 25$

\$ ....1260........ [2]

(b) In September the total income at the campsite was \$37 054.
This was a decrease of 4.5% on the total income in August.

Calculate the total income in August.

$I_{Aug} - I_{Aug} \times 4.5\% = 37054$

$0.955\ I_{Aug} = 37054$

\$ ....38800........ [2]

(c) The visitors to the campsite today are in the ratio

men : women = 5 : 4 and women : children = 3 : 7.

(i) Calculate the ratio men : women : children in its simplest form.

Men : $5a$ women : $4a$

$\frac{4a}{\text{children}} = \frac{3}{7} \Rightarrow \text{children} = \frac{28}{3}a$

$5 : 4 : \frac{28}{3}$

........15........ : ....12........ : ....28........ [2]

(ii) Today there are 224 children at the campsite.

Calculate the total number of men and women.

$\text{Women} = \frac{3}{7} \times 224 = 96$

$\text{Men} = \frac{5}{4} \times 96 = 120$

........216........ [3]

(d) The space allowed for each tent is a rectangle measuring 8 m by 6 m, each correct to the nearest metre.

Calculate the upper bound for the area of the space allowed for each tent.

$$\text{Area}_{(\max)} = \text{length}_{(\max)} \times \text{width}_{(\max)}$$

$$= \left(8 + \tfrac{1}{2}\right)\left(6 + \tfrac{1}{2}\right)$$

.................55.25................ m^2 [2]

(e) The value of the campsite has increased exponentially by 1.5% every year since it opened 30 years ago.

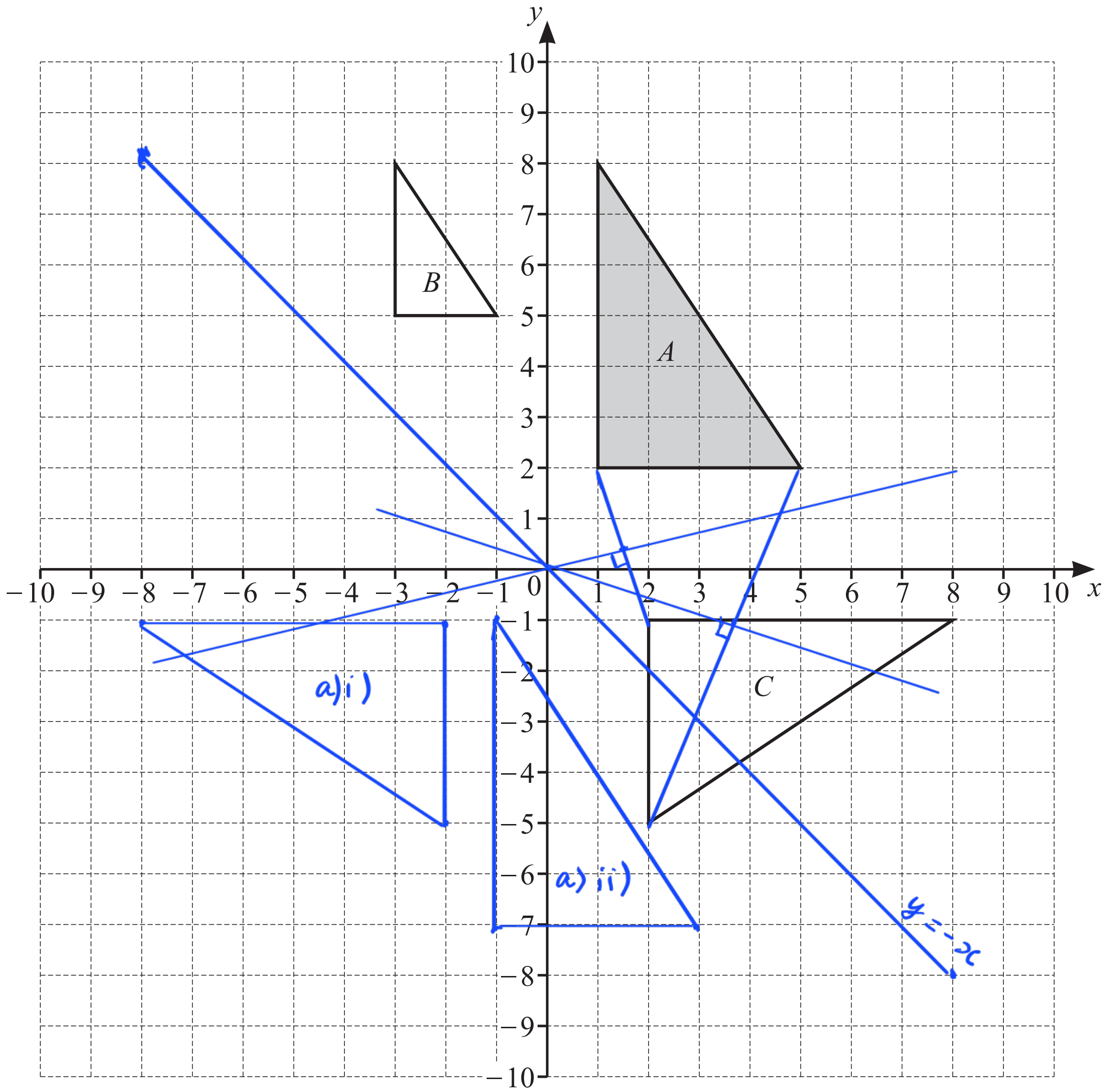
Calculate the value of the campsite now as a percentage of its value 30 years ago.

$$\text{Value}_{\text{now}} = V_{30 \text{ years ago}} \times \left(1 + \frac{1.5}{100}\right)^{30}$$

$$\Rightarrow \frac{\text{Value}_{\text{now}}}{\text{Value}_{30 \text{ years ago}}} \simeq 1.5631$$

.................156................ % [2]

2



(a) **(i)** Draw the image of triangle A after a reflection in the line $y = -x$. [2]

(ii) Draw the image of triangle A after a translation by the vector $\begin{pmatrix} -2 \\ -9 \end{pmatrix}$. [2]

(b) Describe fully the **single** transformation that maps

(i) triangle A onto triangle B,

Enlargement centre (-7, 8), scale factor of $\frac{1}{2}$

.... [3]

(ii) triangle A onto triangle C.

Rotation center (0, 0), clock wise, 90°

.... [3]

(b) The table shows the marks scored by some students in a test.

Mark	5	6	7	8	9	10
Frequency	8	2	12	2	0	1

Calculate the mean mark.

$$\frac{5\times 8 + 6\times 2 + 7\times 12 + 8\times 2 + 9\times 0 + 10\times 1}{8+2+12+2+0+1}$$

................6.48................ [3]

4 **(a)** Solve the inequality.

$$3m + 12 \leqslant 8m - 5$$

$12 + 5 \leq 8m - 3m$

$17 \leq 5m$

$\frac{17}{5} \leq m$

................$\frac{17}{5} \leq m$................ [2]

(b) Solve the equation. $\dfrac{2x+5}{3-x} = \dfrac{14}{15}$

$15(2x+5) = 14(3-x)$

$30x + 75 = 42 - 14x$

$44x = -33$

$x = -0.75$

$x =$ ....-0.75................ [3]

(c) Solve the simultaneous equations.
You must show all your working.

$$y = 4 - x$$
$$x^2 + 2y^2 = 67$$

$x^2 + 2(4-x)^2 = 67$

$x^2 + 2(16 - 8x + x^2) = 67$

$x^2 + 32 - 16x + 2x^2 = 67$

$3x^2 - 16x - 35 = 0$

$(3x + 5)(x - 7) = 0$

$3x + 5 = 0$ or $x - 7 = 0$

$x = -\frac{5}{3}$ or $x = 7$

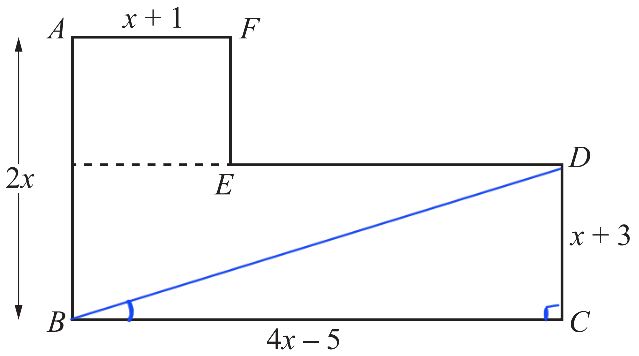
When $x = \frac{-5}{3}$, $y = 4 - \left(\frac{-5}{3}\right) = \frac{17}{3}$

When $x = 7$, $y = 4 - 7 = -3$

$x =$ ....7........ , $y =$ ....-3........

$x =$ ....$\frac{-5}{3}$........ , $y =$ ....$\frac{17}{3}$........ [6]

5 All the lengths in this question are in centimetres.



NOT TO SCALE

The diagram shows a shape *ABCDEF* made from two rectangles.
The total area of the shape is 342 cm^2.

(a) Show that $x^2 + x - 72 = 0$.

$EF = 2x - (x+3) = 2x - x - 3 = x - 3$

Total area $= (x+1)(x-3) + (x+3)(4x-5) = 342$

$x^2 + x - 3x - 3 + 4x^2 + 12x - 5x - 15 = 342$

$5x^2 + 5x - 360 = 0$

$x^2 + x - 72 = 0$

[5]

(b) Solve by factorisation.

$$x^2 + x - 72 = 0$$

$(x+9)(x-8) = 0$

$x + 9 = 0$ or $x - 8 = 0$

$x = -9$ or $x = 8$

$x =$ ....-9........ or $x =$ ....8........ [3]

(c) Work out the perimeter of the shape *ABCDEF*.

When $x = -9$, $AF = -9 + 1 = -8 < 0$ (nonsense)

$\Rightarrow x = 8$

$ED = BC - AF = (4 \times 8 - 5) - (8 + 1) = 18$

$\text{Perimeter}_{ABCDEF} = AB + BC + CD + DE + EF + FA$

$= (2 \times 8) + (4 \times 8 - 5) + (8 + 3) + 18 + (8 - 3) + (8 + 1)$

$= 86$

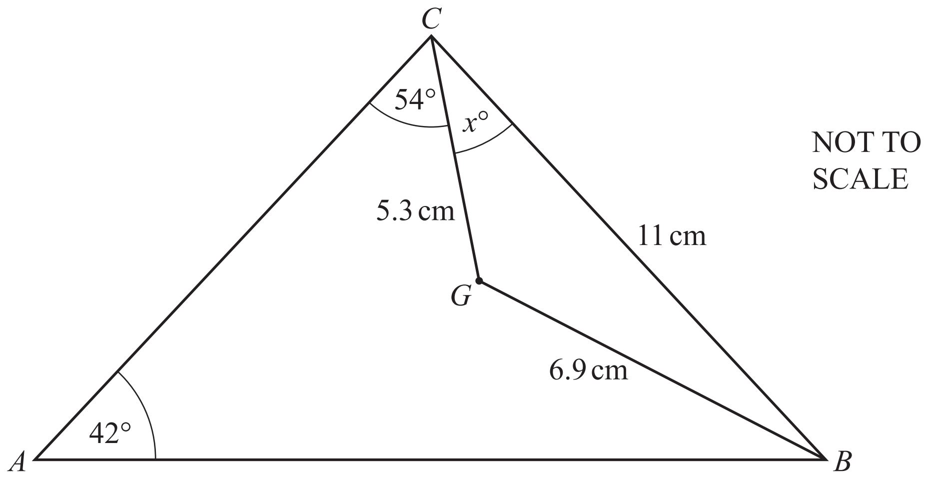
..........86.......... cm [2]

(d) Calculate angle *DBC*.

$\tan \widehat{DBC} = \dfrac{x + 3}{4x - 5} = \dfrac{8 + 3}{4 \times 8 - 5} = \dfrac{11}{27}$

Angle *DBC* = ...22.2°.......... [2]

6 (a)



The diagram shows triangle ABC with point G inside.
$CB = 11$ cm, $CG = 5.3$ cm and $BG = 6.9$ cm.
Angle $CAB = 42°$ and angle $ACG = 54°$.

(i) Calculate the value of x.

$6.9^2 = 5.3^2 + 11^2 - 2 \times 5.3 \times 11 \cos x$

$-101.48 = -116.6 \cos x$

$\cos x = \frac{2537}{2915}$

$x =$ ....29.5°.... [4]

(ii) Calculate AC.

$\widehat{ACB} = 54° + 29.5° = 83.5°$

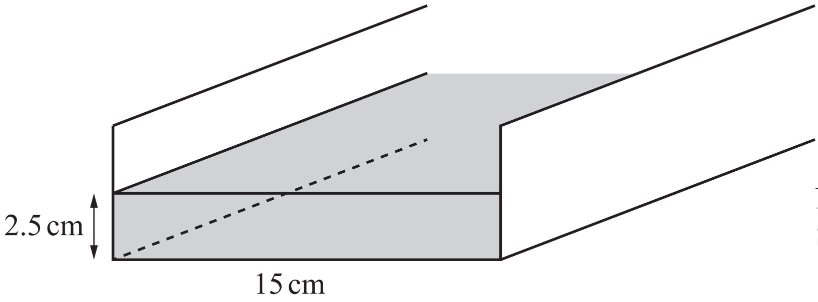
$\widehat{ABC} = 180° - 83.5° - 42° = 54.5°$

$\frac{AC}{\sin 54.5°} = \frac{11}{\sin 42°}$

$AC = \frac{11 \sin 54.5°}{\sin 42°}$

$AC =$ ....13.4.... cm [4]

(b)



Water flows at a speed of 20 cm/s along a rectangular channel into a lake.
The width of the channel is 15 cm.
The depth of the water is 2.5 cm.

Calculate the amount of water that flows from the channel into the lake in 1 hour.
Give your answer in litres.

The length of water flows into the lake in 1 hour:

$20 \times 3600 = 72\,000$

Volume of water flows into the lake in 1 hour:

$15 \times 2.5 \times 72\,000 = 2\,700\,000 \text{ cm}^3$

$1 \text{ dm} = 10 \text{ cm}$

$1 \text{ dm}^3 (1 \ell) = 10^3 \text{ cm}^3$

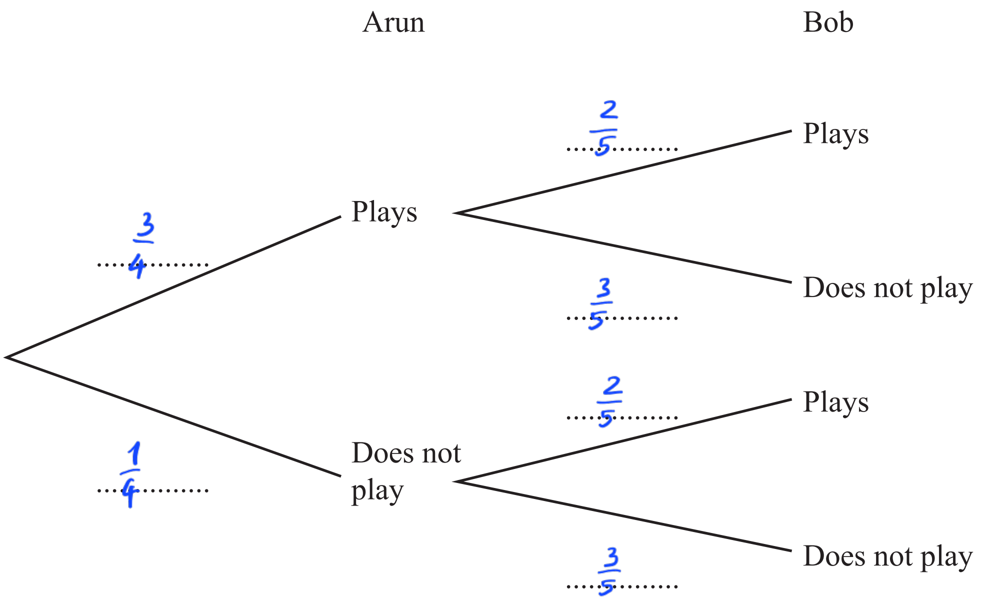
$2700 \ \ell = 2\,700\,000 \text{ cm}^3$

..........2700.......... litres [4]

7 On any Saturday, the probability that Arun plays football is $\frac{3}{4}$.

On any Saturday, the probability that Bob plays football is $\frac{2}{5}$.

(a) **(i)** Complete the tree diagram.



[2]

(ii) Calculate the probability that, one Saturday, Arun and Bob both play football.

$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$

$\frac{6}{20}$ [2]

(iii) Calculate the probability that, one Saturday, either Arun plays football or Bob plays football, but not both.

$\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{2}{5}$

$\frac{11}{20}$ [3]

(b) Calculate the probability that Bob plays football for 2 of the next 3 Saturdays.

$$\left(\frac{2}{5} \times \frac{2}{5} \times \frac{3}{5}\right) \times 3$$

$\frac{36}{125}$ .... [3]

(c) When Arun plays football, the probability that he scores the winning goal is $\frac{1}{7}$.

Calculate the probability that Arun scores the winning goal one Saturday.

$$\frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$$

$\frac{3}{28}$ .... [2]

8 **(a)** The interior angle of a regular polygon with n sides is 150°.

Calculate the value of n.

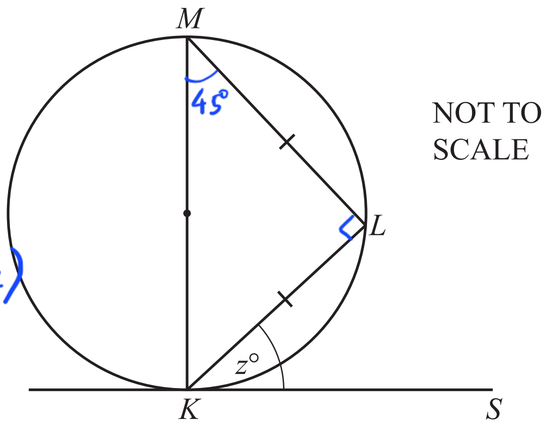
$$\frac{(n-2)\,180}{n} = 150$$

$$180n - 360 = 150n$$

$$30n = 360$$

n = ...12...... [2]

(b) **(i)** K, L and M are points on the circle.
KS is a tangent to the circle at K.
KM is a diameter and
triangle KLM is isosceles.



Find the value of z.

$\widehat{MLK} = 90°$ (angle in a semicircle)

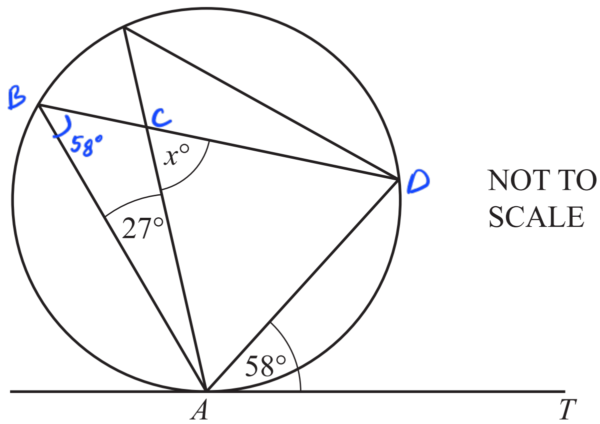
$\widehat{KML} = 45°$

$\widehat{LKS} = \widehat{KML} = 45°$

(alternate segment theorem)

z = ...45°...... [2]

(ii) AT is a tangent to the circle at A.



Find the value of x.

$\widehat{ABC} = \widehat{DAT} = 58°$

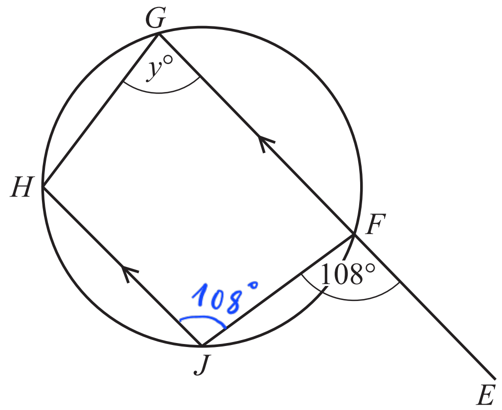
(alternate segment theorem)

$\widehat{ACB} = 180° - 58° - 27° = 95°$

$x = 180° - 95° = 85°$

x = ...85°...... [2]

(iii)



NOT TO SCALE

F, G, H and J are points on the circle.
EFG is a straight line parallel to JH.

Find the value of y.

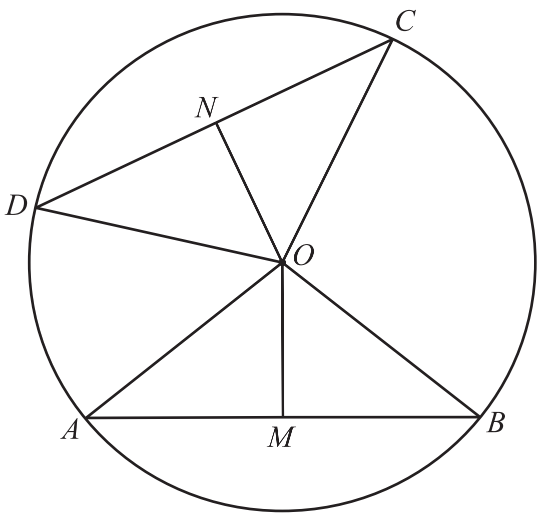
$\widehat{HJF} = \widehat{JFE} = 108°$ (alternating)

$\widehat{HGF} = 180° - 108° = 72°$

(2 opposite angles in a cyclic quadrilateral add up to 180°)

$y =$ ....72°........ [2]

(c)



NOT TO SCALE

A, B, C and D are points on the circle, centre O.
M is the midpoint of AB and N is the midpoint of CD.
$OM = ON$

Explain, giving reasons, why triangle OAB is congruent to triangle OCD.

....

....

....

.... [3]

9 **(a)** The equation of line L is $3x-8y+20=0$.

(i) Find the gradient of line L.

$8y = 3x + 20$

$y = \frac{3}{8}x + \frac{20}{8}$

$\frac{3}{8}$ .... [2]

(ii) Find the coordinates of the point where line L cuts the y-axis.

Sub $x = 0$ into equation of L:

$3 \times 0 - 8y + 20 = 0$

$8y = 20$

$y = 2.5$

(........0........ , ...2.5........) [1]

(b) The coordinates of P are $(-3, 8)$ and the coordinates of Q are $(9, -2)$.

(i) Calculate the length PQ.

$PQ = \sqrt{(-2-8)^2 + [9-(-3)]^2}$

$= 2\sqrt{61}$

15.6 [3]

(ii) Find the equation of the line parallel to PQ that passes through the point $(6, -1)$.

ℓ

$m_{PQ} = \frac{-2-8}{9-(-3)} = \frac{-5}{6}$

$m_\ell = m_{PQ} = \frac{-5}{6}$

Equation of ℓ: $y-(-1) = \frac{-5}{6}(x-6)$

$y+1 = \frac{-5}{6}(x-6)$ [3]

(iii) Find the equation of the perpendicular bisector of PQ.

p

Midpoint of $PQ = \left(\frac{-3+9}{2}, \frac{8-2}{2}\right) = (3, 3)$

$m_p = -1 : \left(\frac{-5}{6}\right) = 1.2$

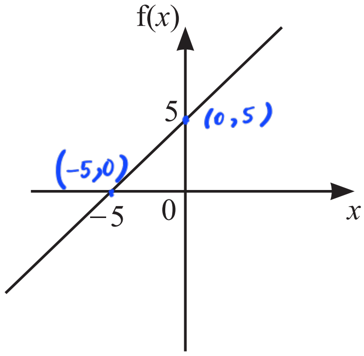
Equation of line p: $y-3 = 1.2(x-3)$

$y-3 = 1.2(x-3)$ [4]

10 **(a)** The diagrams show the graphs of two functions.

Write down each function.

(i)



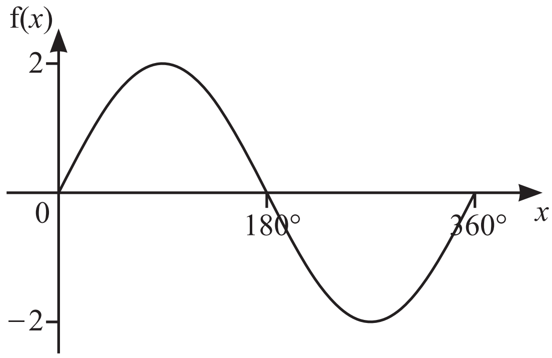
$m = \frac{5-0}{0-(-5)} = 1$

$y - 5 = 1(x - 0)$

$y - 5 = x$

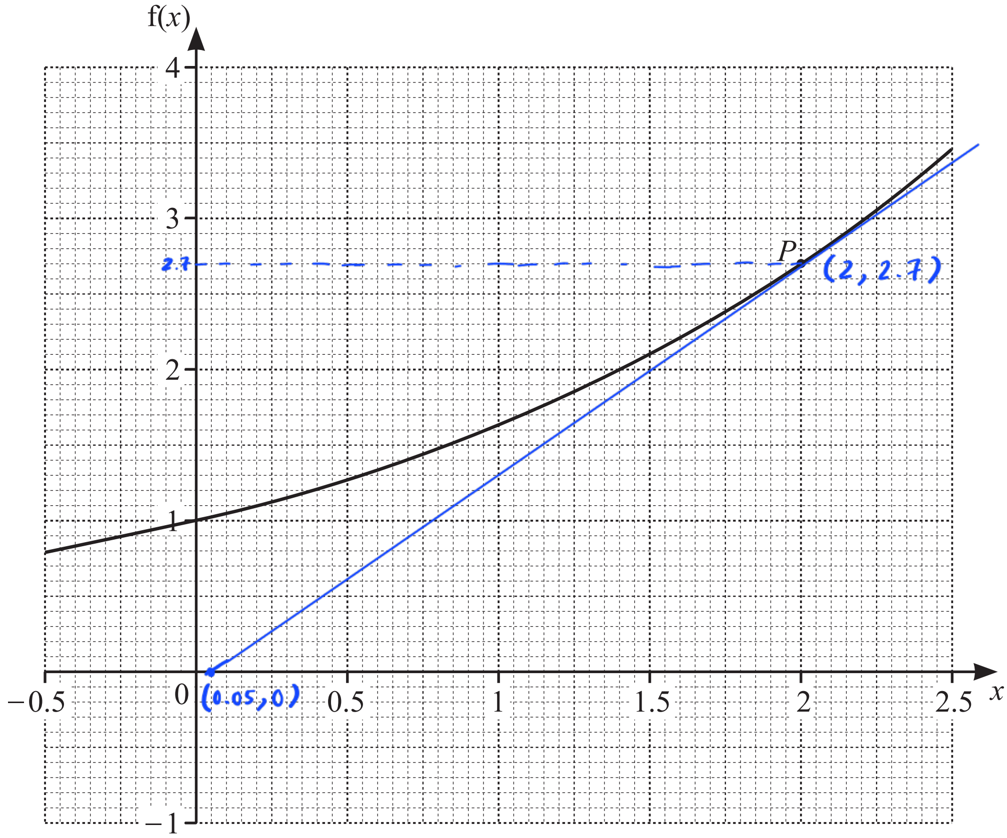
f(x) = ...$y = x + 5$...... [2]

(ii)



f(x) = ...$2 \sin x$...... [2]

(b)



The diagram shows the graph of another function.

By drawing a suitable tangent, find an estimate for the gradient of the function at the point *P*.

$$m = \frac{2.7 - 0}{2 - 0.05}$$

.... 1.3 .... [3]

11 $\mathrm{f}(x) = 7x - 4$ $\mathrm{g}(x) = \dfrac{2x}{x-3},\ x \neq 3$ $\mathrm{h}(x) = x^2$

(a) Find g(6).

$$\frac{2 \times 6}{6-3}$$

4 [1]

(b) Find fg(4).

$$g(4) = \frac{2 \times 4}{4-3} = 8$$

$$f(8) = 7 \times 8 - 4$$

52 [2]

(c) Find fh(x).

$$7x^2 - 4$$

$7x^2 - 4$ [1]

(d) Find $\dfrac{\mathrm{f}(x)}{2} + \mathrm{g}(x)$.

Give your answer as a single fraction, in terms of x, in its simplest form.

$$\frac{7x-4}{2} + \frac{2x}{x-3}$$

$$\frac{(7x-4)(x-3) + 2x \times 2}{2(x-3)}$$

$$\frac{7x^2 - 4x - 21x + 12 + 4x}{2x-6}$$

$\dfrac{7x^2 - 21x + 12}{2x - 6}$ [3]

(e) Find the value of x when $f(x+2) = -11$.

$7(x+2) - 4 = -11$

$7x + 14 - 4 = -11$

$7x = -21$

$x =$ ......-3...... [2]

(f) Find the values of p that satisfy $h(p) = p$.

$p^2 = p$

$p^2 - p = 0$

$p(p-1) = 0$

$p = 0$ or $p = 1$

......$0, 1$...... [2]

12 (a) A curve has equation $y = 4x^3 - 3x + 3$.

(i) Find the coordinates of the two stationary points.

$\frac{dy}{dx} = 12x^2 - 3 = 0$

$x^2 = \frac{3}{12} = \frac{1}{4}$

$x = \pm 0.5$

When $x = 0.5$, $y = 4(0.5)^3 - 3 \times 0.5 + 3 = 2$

When $x = -0.5$, $y = 4(-0.5)^3 - 3 \times (-0.5) + 3 = 4$

(......0.5...... , ...2......) and (......-0.5...... , ...4......) [5]

(ii) Determine whether each of the stationary points is a maximum or a minimum.
Give reasons for your answers.

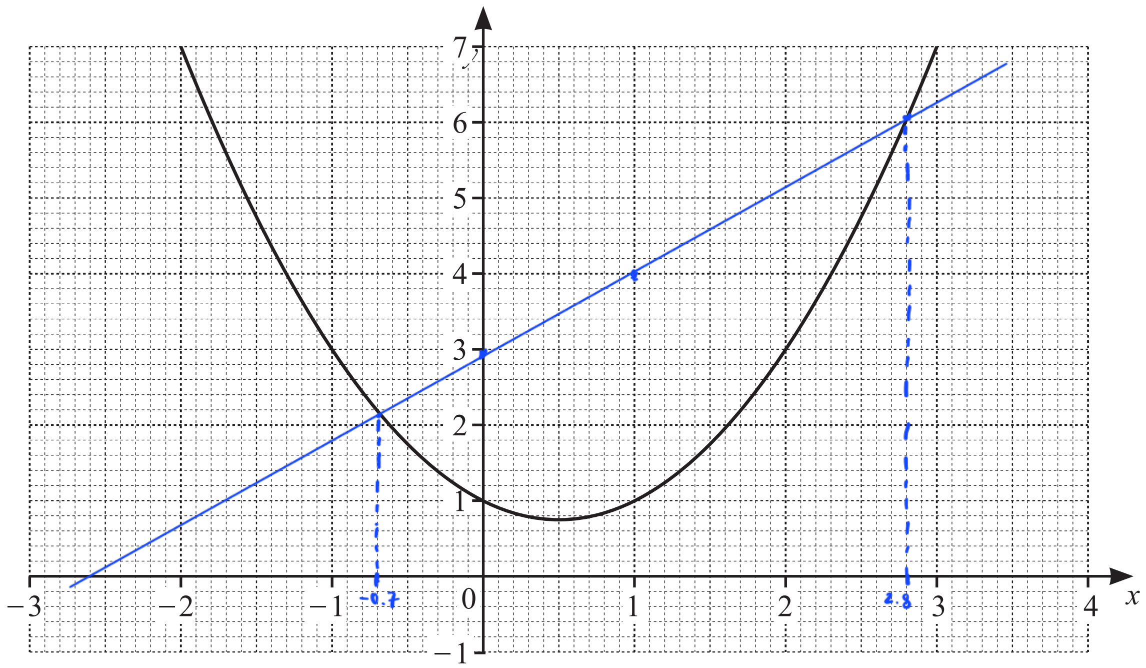
$$\frac{d^2y}{dx^2} = 24x$$

When $x = 0.5$, $24 \times 0.5 = 12 > 0 \Rightarrow (0.5, 2)$ is a minimum

When $x = -0.5$, $24 \times (-0.5) = -12 < 0 \Rightarrow (-0.5, 4)$ is a maximum

[3]

(b) The graph of $y = x^2 - x + 1$ is shown on the grid.



By drawing a suitable line on the grid, solve the equation $x^2 - 2x - 2 = 0$.

$x^2 - x + 1 - x - 3 = 0$

$x^2 - x + 1 = x + 3$

$x =$ ....-0.7.... or $x =$ ....2.8.... [3]