0580/43

October/November 2020

1 (a) The Earth has a surface area of approximately 510 100 000 km^2.

(i) Write this surface area in standard form.

5.101×10^8 km^2 [1]

(ii) Water covers 70.8% of the Earth's surface.

Work out the area of the Earth's surface covered by water.

$510\,100\,000 \times 70.8\%$

361 150 800 km^2 [2]

(b) The table shows the surface area of some countries and their estimated population in 2017.

Country	Surface area (km^2)	Estimated population in 2017
Brunei	5.77×10^3	433 100
China	9.60×10^6	1 388 000 000
France	6.41×10^5	67 000 000
Maldives	3.00×10^2	374 600

(i) Find the total surface area of Brunei and the Maldives.

$5.77 \times 10^3 + 3.00 \times 10^2$

6070 km^2 [1]

(ii) The ratio surface area of the Maldives : surface area of China can be written in the form 1 : n.

Find the value of n.

$$\frac{3.00 \times 10^2}{9.6 \times 10^6} = \frac{1}{32\,000}$$

n = 1 : 32 000 [2]

(iii) Find the surface area of France as a percentage of the surface area of China.

$$\frac{6.41 \times 10^5}{9.60 \times 10^6} \times 100$$

6.68 % [2]

(iv) Find the population density of the Maldives.
[Population density = population ÷ surface area]

$$\frac{374\ 600}{3.00 \times 10^2} \approx 1248.67$$

..........1250.......... people/km^2 [2]

(c) The population of the Earth in 2017 was estimated to be 7.53×10^9.

The population of the Earth in 2000 was estimated to be 6.02×10^9.

(i) Work out the percentage increase in the Earth's estimated population from 2000 to 2017.

$$\frac{7.53 \times 10^9 - 6.02 \times 10^9}{6.02 \times 10^9} \times 100 \approx 25.08\ \%$$

..........25.1.......... % [2]

(ii) Assume that the population of the Earth increased exponentially by y% each year for these 17 years.

Find the value of y.

$$7.53 \times 10^9 = 6.02 \times 10^9 \left(1 + \frac{y}{100}\right)^{17}$$

$$1 + \frac{y}{100} = \sqrt[17]{\frac{753}{602}}$$

$$\frac{y}{100} = 0.01325$$

y = ...1.33.......... [3]

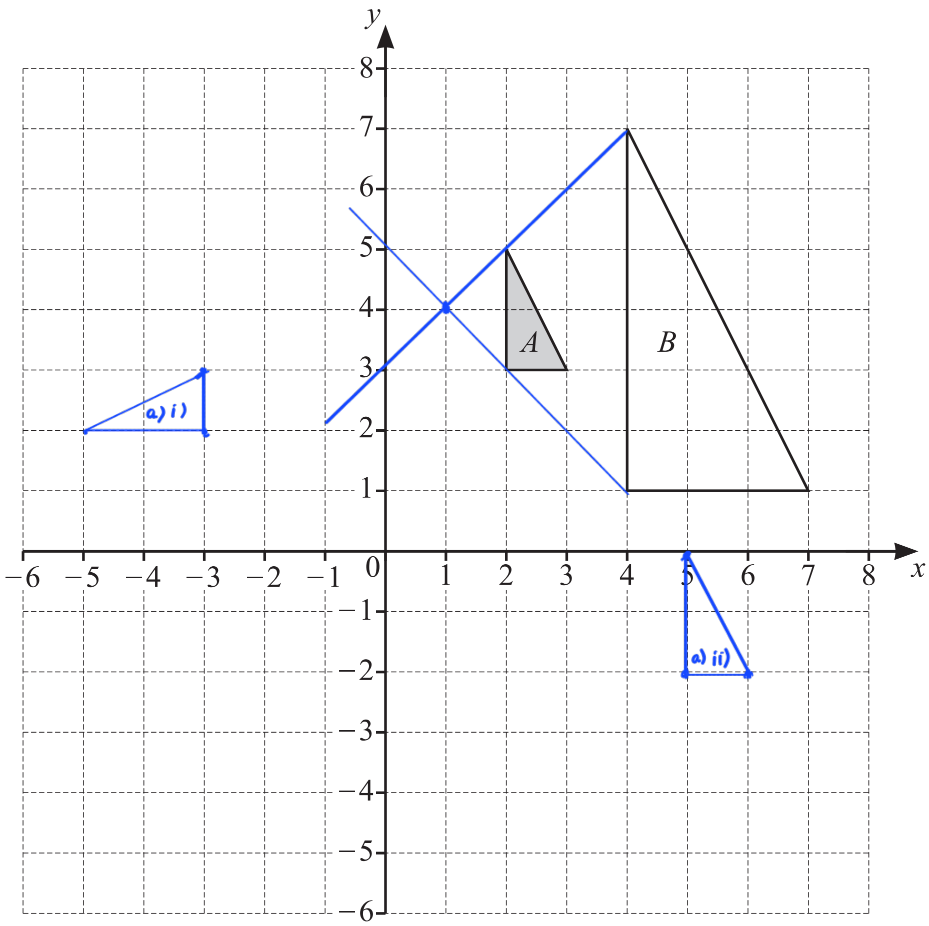
(d) A car takes 18 seconds to travel 400 m along this road.

Calculate the average speed of this car in km/h.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{400\text{ m}}{18\text{ s}} = \frac{0.4\text{ km}}{\frac{18}{3600}\text{ s}} = 80$$

..........80.......... km/h [3]

2



(a) On the grid, draw the image of

(i) triangle A after a rotation of 90° anticlockwise about (0, 0), [2]

(ii) triangle A after a translation by the vector $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$. [2]

(b) Describe fully the **single** transformation that maps triangle A onto triangle B.

Enlargement , center (1, 4) , scale factor = 3

[3]

4

P	O	S	S	I	B	I	L	I	T	Y

Morgan picks two of these letters, at random, **without** replacement.

(a) Find the probability that he picks

(i) the letter Y first,

$\frac{1}{11} \times \frac{10}{10}$

$\frac{1}{11}$ .... [1]

(ii) the letter B then the letter Y,

$\frac{1}{11} \times \frac{1}{10}$

$\frac{1}{110}$ .... [2]

(iii) two letters that are the same.

$(S \cap S)$ or $(I \cap I)$

$\frac{2}{11} \times \frac{1}{10} + \frac{3}{11} \times \frac{2}{10}$

$\frac{4}{55}$ .... [3]

(b) Morgan now picks a third letter at random.

Find the probability that

(i) all three letters are the same,

$I \cap I \cap I$

$\frac{3}{11} \times \frac{2}{10} \times \frac{1}{9}$

$\frac{1}{165}$ .... [2]

(ii) exactly two of the three letters are the same,

$(S \cap S)$ or $(I \cap I)$

$\left(\frac{2}{11} \times \frac{1}{10} \times \frac{9}{9}\right) \times 3 + \left(\frac{3}{11} \times \frac{2}{10} \times \frac{8}{9}\right) \times 3$

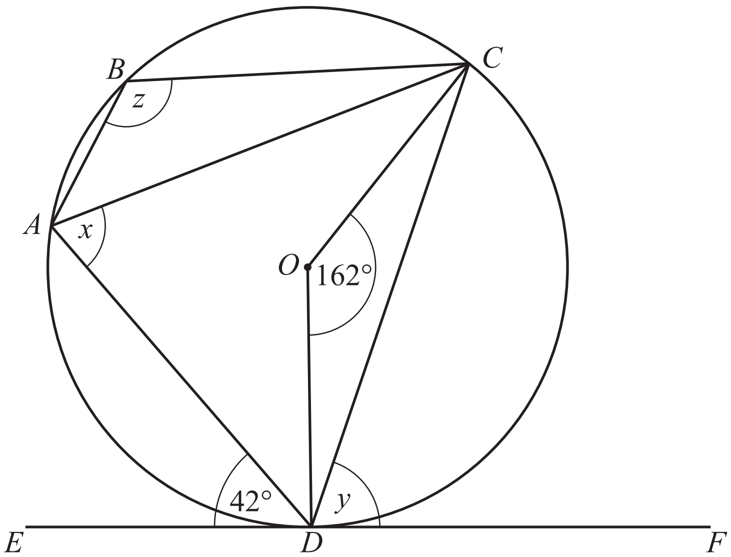
$\frac{1}{5}$ .... [5]

(iii) all three letters are different.

$1 - \frac{1}{165} - \frac{1}{5}$

$\frac{131}{165}$ .... [2]

5 **(a)**



NOT TO SCALE

A, B, C and D are points on the circle, centre O.
EF is a tangent to the circle at D.
Angle $ADE = 42°$ and angle $COD = 162°$.

Find the following angles, giving reasons for each of your answers.

(i) Angle x

$x =$ $\frac{162}{2} = 81°$ because angle at circumference is half angle at center [2]

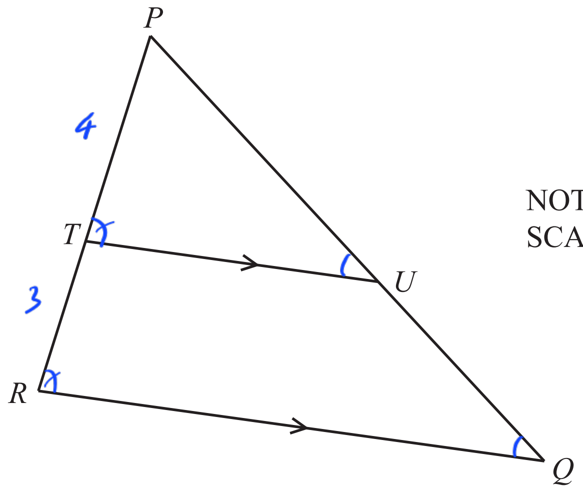
(ii) Angle y

$y =$ $81°$ because alternate segment theorem [2]

(iii) Angle z

$z =$ $123°$ because $\widehat{ADC} = 180° - 42° - 81° = 57°$ (angles on a straight line add up to $180°$), $z = 180° - 57° = 123°$ (2 opposite angles in a cyclic quadrilateral add up to $180°$) [3]

(b)



PQR is a triangle.
T is a point on *PR* and *U* is a point on *PQ*.
RQ is parallel to *TU*.

(i) Explain why triangle *PQR* is similar to triangle *PUT*.
Give a reason for each statement you make.

+ common angle $\hat{P}$

+ $\widehat{PTU} = \widehat{PRQ}$ (corresponding angles)

+ $\widehat{PUT} = \widehat{PQR}$ (corresponding angles)

Corresponding angles are equal [3]

(ii) $PT : TR = 4 : 3$

(a) Find the ratio $PU : PQ$.

$$\frac{PU}{PQ} = \frac{PT}{PR} = \frac{4}{4+3} = \frac{4}{7}$$

4 : 7 [1]

(b) The area of triangle *PUT* is $20\,\text{cm}^2$.

Find the area of the quadrilateral *QRTU*.

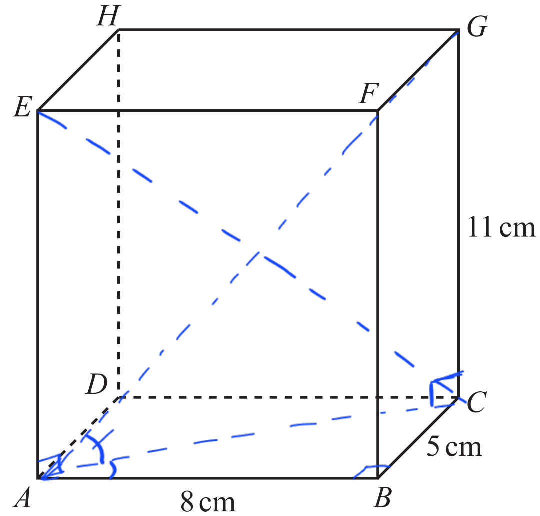
$$\frac{A_{\Delta PUT}}{A_{\Delta PQR}} = \left(\frac{PU}{PQ}\right)^2 = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$$

$$\Rightarrow A_{\Delta PQR} = 20 : \frac{16}{49} = \frac{245}{4}$$

$$A_{QRTU} = \frac{245}{4} - 20 = 41.25$$

41.25 cm^2 [3]

6 *ABCDEFGH* is a cuboid.
$AB = 8$ cm, $BC = 5$ cm and $CG = 11$ cm.



NOT TO SCALE

(a) Work out the volume of the cuboid.

$V = 8 \times 5 \times 11$

.............440............. cm^3 [2]

(b) Ivana has a pencil of length 13 cm.
Does this pencil fit completely inside the cuboid?
Show how you decide.

$AC^2 = 8^2 + 5^2 = 89$

$EC = \sqrt{89 + 11^2} = \sqrt{210} > 13$

$\Rightarrow$ This pencil fits completely inside the cuboid

[4]

(c) **(i)** Calculate angle *CAB*.

$\tan \widehat{CAB} = \frac{5}{8}$

Angle *CAB* = ...32.0°....... [2]

(ii) Calculate angle *GAC*.

$\tan \widehat{GAC} = \frac{11}{\sqrt{89}}$

Angle *GAC* = ...49.4°....... [2]

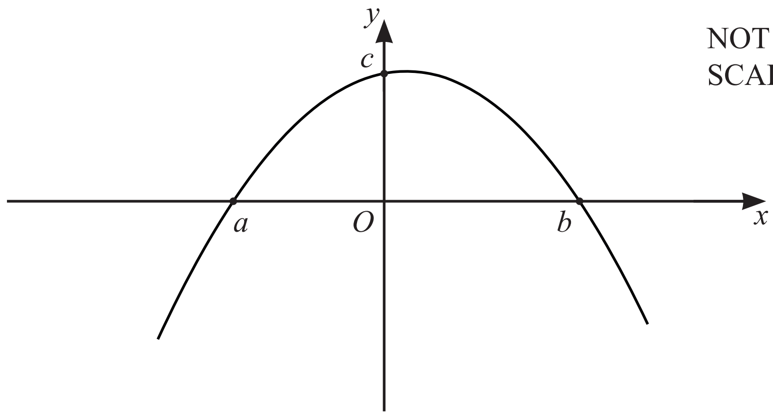
7 **(a)** **(i)** Factorise $24+5x-x^2$.

$24 - 3x + 8x - x^2$

$3(8-x) + x(8-x)$

$(3+x)(8-x)$ [2]

(ii) The diagram shows a sketch of $y = 24+5x-x^2$.



Work out the values of a, b and c.

sub $x = 0$: $24 + 5 \times 0 - 0^2 = 24 \Rightarrow c = 24$

$(3+x)(8-x) = 0$

$\Rightarrow x = -3$ or $x = 8$

$\Rightarrow a = -3,\ b = 8$

$a =$ -3

$b =$ 8

$c =$ 24 [3]

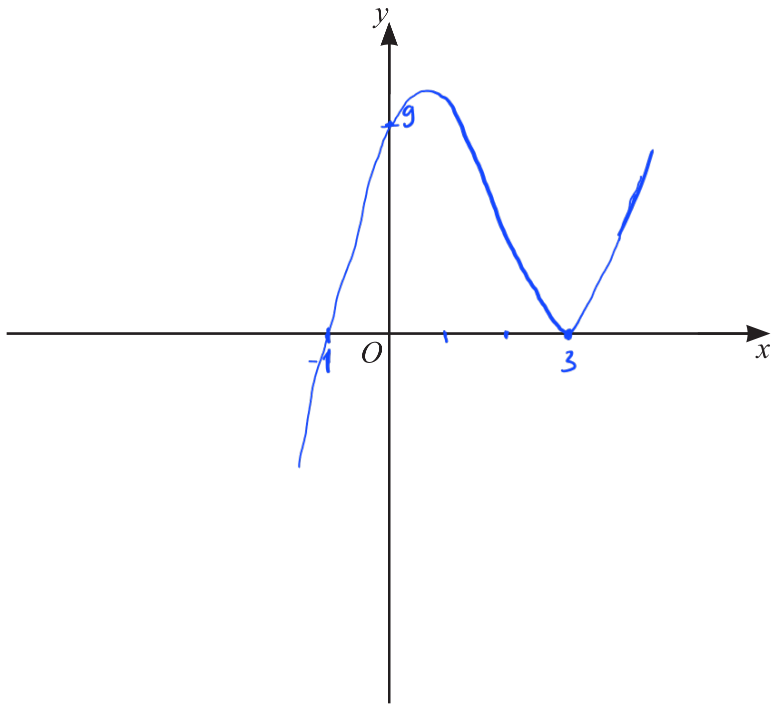
(iii) Calculate the gradient of $y = 24+5x-x^2$ at $x = -1.5$.

$\frac{dy}{dx} = 5 - 2x$

When $x = -1.5$: gradient $= 5 - 2(-1.5) = 8$

8 [3]

(b) **(i)** On the diagram, sketch the graph of $y = (x+1)(x-3)^2$.
Label the values where the graph meets the x-axis and the y-axis.



[4]

(ii) Write $(x+1)(x-3)^2$ in the form $ax^3 + bx^2 + cx + d$.

$(x+1)(x^2 - 6x + 9)$

$x^3 - 6x^2 + 9x + x^2 - 6x + 9$

$x^3 - 5x^2 + 3x + 9$ [3]

8 **(a)** $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Find

(i) $\overrightarrow{AC}$,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ [2]

(ii) $\overrightarrow{BD}$,

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

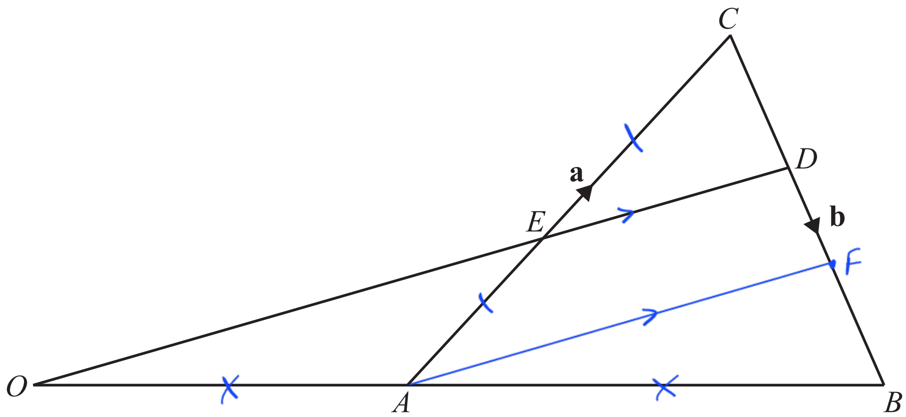
$\overrightarrow{BD} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$ [2]

(iii) $|\overrightarrow{BC}|$.

$\sqrt{(-2)^2 + 5^2} \approx 5.39$

..........5.39.......... [2]

(b)



NOT TO SCALE

In the diagram, OAB and OED are straight lines.
O is the origin, A is the midpoint of OB and E is the midpoint of AC.
$\overrightarrow{AC} = \mathbf{a}$ and $\overrightarrow{CB} = \mathbf{b}$.

Find, in terms of **a** and **b**, in its simplest form

(i) $\overrightarrow{AB}$,

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} = a + b$

$\overrightarrow{AB} =$ ....$a + b$........ [1]

(ii) $\overrightarrow{OE}$,

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$

$= a + b + \frac{1}{2} a$

$\overrightarrow{OE} =$ ....$\frac{3}{2} a + b$........ [2]

(iii) the position vector of D.

Draw $AF \parallel OD$

$\Delta BAF \sim \Delta BOD$

$\frac{BF}{BD} = \frac{BA}{BO} = \frac{1}{2} \Rightarrow DF = FB \quad (1)$

$\Delta CED \sim \Delta CAF$

$\frac{CE}{CA} = \frac{CD}{CF} = \frac{1}{2} \Rightarrow CD = DF \quad (2)$

........$2a + \frac{4}{3} b$........ [3]

$(1), (2) \Rightarrow \overrightarrow{DB} = \frac{2}{3}\overrightarrow{CB} = \frac{2}{3} b$

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= 2(a+b) - \frac{2}{3} b = 2a + \frac{4}{3} b$

9 **(a)** Find the integer values that satisfy the inequality $2 < 2x \leqslant 10$.

Divide by 2 : $1 < x \leq 5$

$2, 3, 4, 5$ .... [2]

(b) Factorise completely.

(i) $6y^2 - 15xy$

$3y(2y - 5x)$ .... [2]

(ii) $y^2 - 9x^2$

$y^2 - (3x)^2$

$(y - 3x)(y + 3x)$ .... [2]

(c) Simplify.

$$\frac{3}{x-1} - \frac{2}{2x+1}$$

$$= \frac{3(2x+1) - 2(x-1)}{(x-1)(2x+1)}$$

$$= \frac{6x + 3 - 2x + 2}{(x-1)(2x+1)}$$

$\dfrac{4x + 5}{(x-1)(2x+1)}$ .... [3]

(d) The straight line $y = 3x + 2$ intersects the curve $y = 2x^2 + 7x - 11$ at two points.

Find the coordinates of these two points.
Give your answers correct to 2 decimal places.

$3x + 2 = 2x^2 + 7x - 11$

$2x^2 + 4x - 13 = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 2 \times (-13)}}{2 \times 2}$

$x = 1.739$ or $x = -3.739$

when $x = 1.739$, $y = 3 \times 1.739 + 2 = 7.217$

when $x = -3.739$, $y = 3 \times (-3.739) + 2 = -9.217$

(1.74 , 7.22)

(−3.74 , −9.22) [6]

10 $f(x) = 4 - 3x$ $g(x) = x^2 + x$ $h(x) = 3^x$

(a) Find fh(2).

$h(2) = 3^2 = 9$

$f(9) = 4 - 3 \times 9 = -23$

.................-23................. [2]

(b) Find $f^{-1}(x)$.

$\times(-3) \longrightarrow +4$

$\div(-3) \longleftarrow -4$

$f^{-1}(x) =$ ....$\frac{x-4}{-3}$................. [2]

(c) Simplify.

(i) $f(1 - 2x)$

$4 - 3(1 - 2x)$

$4 - 3 + 6x$

.................$6x + 1$................. [2]

(ii) $gf(x) - 9g(x)$

$(4 - 3x)^2 + (4 - 3x) - 9(x^2 + x)$

$16 - 24x + 9x^2 + 4 - 3x - 9x^2 - 9x$

.................$-36x + 20$................. [4]

(d) $\dfrac{1}{h(x)} = 9^{kx}$

Find the value of k.

$\frac{1}{3^x} = 9^{kx} = (3^2)^{kx} = 3^{2kx}$

$3^{-x} = 3^{2kx}$

$\Rightarrow -x = 2kx$

$k =$ ....$-\frac{1}{2}$................. [2]

11 The table shows the first four terms in sequences *A*, *B*, and *C*.

Sequence	1st term	2nd term	3rd term	4th term	5th term		*n*th term
A	4	9	14	19	24		$5n-1$
B	3	10	29	66	127		n^3+2
C	1	4	16	64	256		4^{n-1}

Complete the table.

[9]