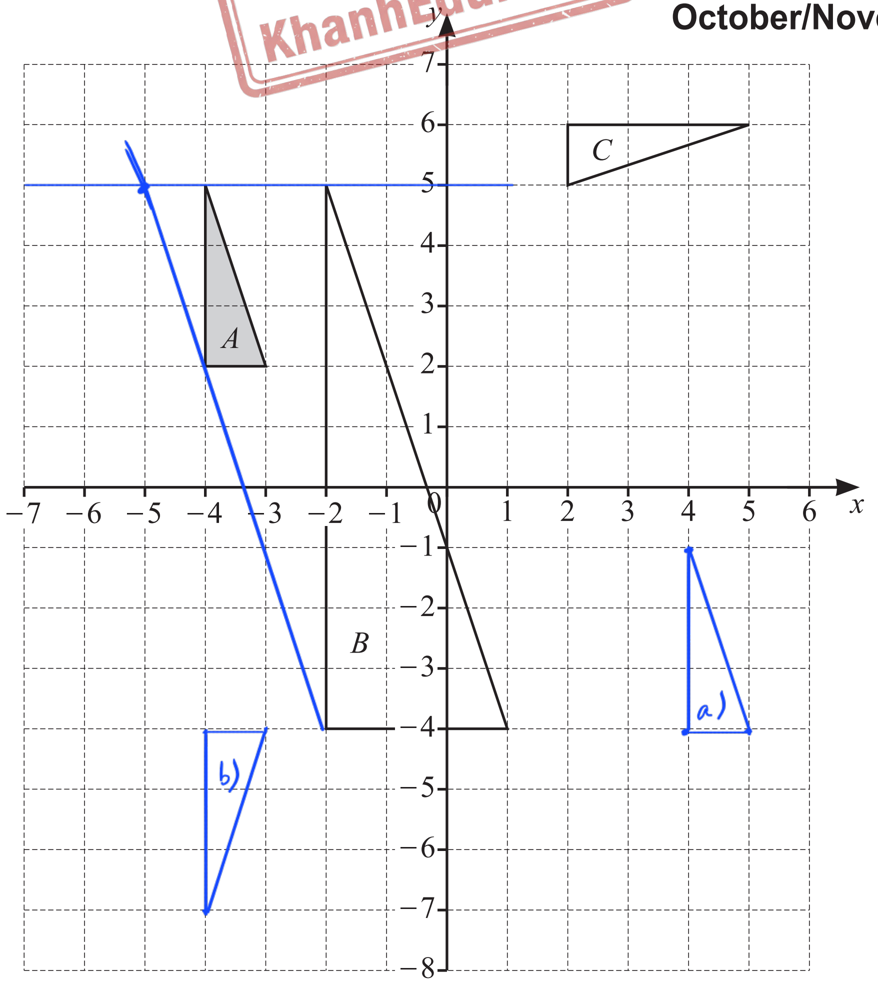
1



(a) Draw the image of shape A after a translation by the vector $\begin{pmatrix} 8 \\ -6 \end{pmatrix}$. [2]

(b) Draw the image of shape A after a reflection in the line $y = -1$. [2]

(c) Describe fully the **single** transformation that maps shape A onto shape B.

Enlargement, center $(-5, 5)$, scale factor $= 3$

.... [3]

(d) Describe fully the **single** transformation that maps shape A onto shape C.

Rotation, center $(1, 1)$, clockwise 90°

.... [3]

2 **(a)** A plane has 14 First Class seats, 70 Premium seats and 168 Economy seats.

Find the ratio First Class seats : Premium seats : Economy seats.
Give your answer in its simplest form.

$\frac{14}{14} : \frac{70}{14} : \frac{168}{14}$

.......1...... : ....5....... : ....12....... [2]

(b) **(i)** For a morning flight, the costs of tickets are in the ratio

First Class : Premium : Economy = 14 : 6 : 5.

The cost of a Premium ticket is \$114.

Calculate the cost of a First Class ticket and the cost of an Economy ticket.

$\frac{F}{114} = \frac{14}{6} \Rightarrow F = \frac{114 \times 14}{6} = 266$

$\frac{114}{E} = \frac{6}{5} \Rightarrow E = \frac{114 \times 5}{6} = 95$

First Class \$ ....266...........

Economy \$ ....95........... [3]

(ii) For an afternoon flight, the cost of a Premium ticket is reduced from \$114 to \$96.90 .

Calculate the percentage reduction in the cost of a ticket.

$\frac{114 - 96.9}{114} \times 100$

...........15........... % [2]

(c) When the local time in Athens is 09 00, the local time in Berlin is 08 00.
A plane leaves Athens at 13 15.
It arrives in Berlin at 15 05 local time. 16:05 Athens time

(i) Find the flight time from Athens to Berlin.

16:05 − 13:15

...........2...... h .....50............. min [1]

(ii) The distance the plane flies from Athens to Berlin is 1802 km.

Calculate the average speed of the plane.
Give your answer in kilometres per hour.

$2h\ 50' = 2\frac{50}{60}\ h$

$\frac{1802}{2\frac{5}{6}}$

...........636........... km/h [2]

(b) The frequency table shows the times, t minutes, each of 100 children spent exercising in one week.

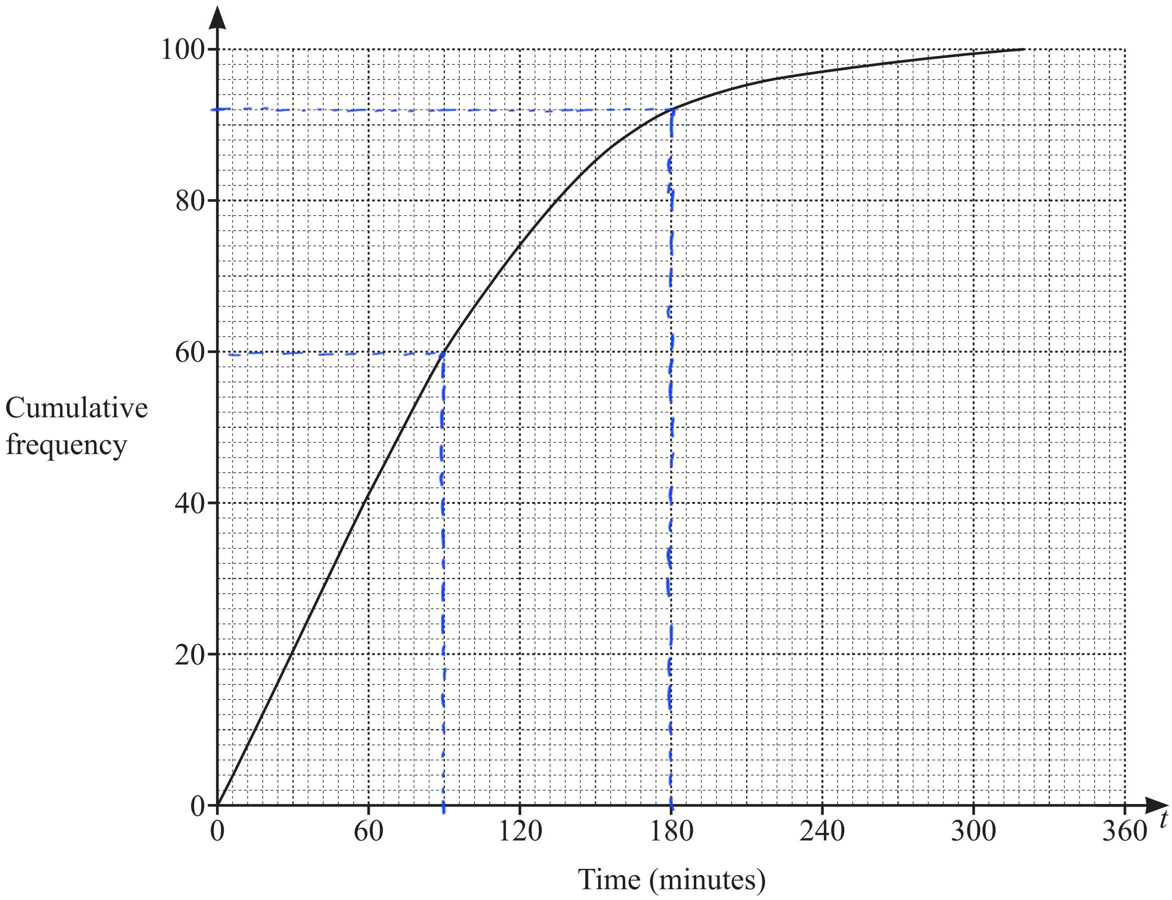
Mid value	30	80	130	190	270
Time (t minutes)	$0 < t \leqslant 60$	$60 < t \leqslant 100$	$100 < t \leqslant 160$	$160 < t \leqslant 220$	$220 < t \leqslant 320$
Frequency	41	24	23	8	4
Freq. density		0.6		2/15	

(i) Calculate an estimate of the mean time.

$$\frac{30 \times 41 + 80 \times 24 + 130 \times 23 + 190 \times 8 + 270 \times 4}{100}$$

.........87.4......... min [4]

(ii) The information in the frequency table is shown in this cumulative frequency diagram.



Use the cumulative frequency diagram to find an estimate of

(a) the 60th percentile,

..........90.......... min [1]

(b) the number of children who spent more than 3 hours exercising.

100 − 92

..........8.......... [2]

(iii) A histogram is drawn to show the information in the frequency table.
The height of the bar for the interval $60 < t \leqslant 100$ is 10.8 cm.

Calculate the height of the bar for the interval $160 < t \leqslant 220$.

$$\frac{10.8 \times \frac{2}{15}}{0.6} = 2.4$$

..........2.4.......... cm [2]

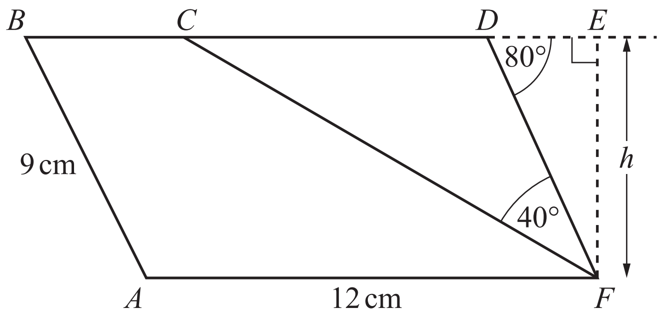
4 **(a)** A rectangle measures 8.5 cm by 10.7 cm, both correct to 1 decimal place. 0·1

Calculate the upper bound of the perimeter of the rectangle.

$2\left(8.5 + \frac{0.1}{2} + 10.7 + \frac{0.1}{2}\right)$

.......38.6....... cm [3]

(b)



NOT TO SCALE

ABDF is a parallelogram and *BCDE* is a straight line.
AF = 12 cm, *AB* = 9 cm, angle *CFD* = 40° and angle *FDE* = 80°.

(i) Calculate the height, *h*, of the parallelogram.

$\sin 80^\circ = \frac{h}{DF} = \frac{h}{9}$

$h = 9 \sin 80^\circ$

h = ...8.86....... cm [2]

(ii) Explain why triangle *CDF* is isosceles.

$\widehat{CDF} = 180^\circ - 80^\circ = 100^\circ \Rightarrow \widehat{DCF} = 180^\circ - 100^\circ - 40^\circ = 40^\circ$

$\widehat{DCF} = \widehat{DFC} \Rightarrow \Delta CDF$ is isosceles [2]

(iii) Calculate the area of the **trapezium** *ABCF*.

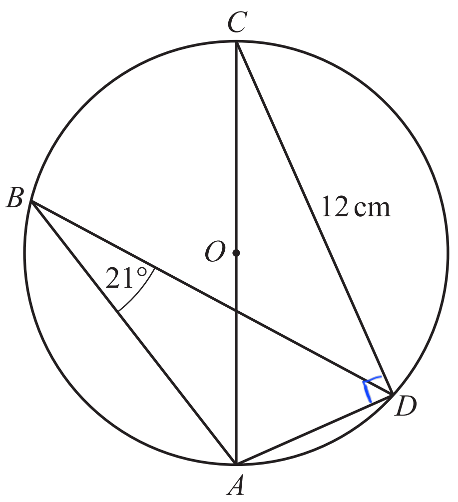
$DC = DF = 9$

$BC = 12 - 9 = 3$

$\text{Area}_{ABCF} = \frac{3+12}{2} \times 9 \sin 80^\circ$

.......66.5....... cm^2 [3]

(c)



NOT TO SCALE

A, B, C and D are points on the circle, centre O.
Angle $ABD = 21°$ and $CD = 12$ cm.

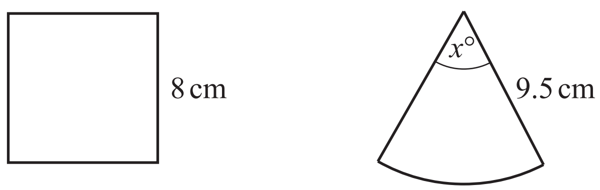
Calculate the area of the circle.

$\widehat{CDA} = 90°$ (angle at semicircle)
$\widehat{ACD} = \widehat{ABD} = 21°$ (subtend the same arc)
$\cos 21° = \frac{12}{AC} \Rightarrow AC = \frac{12}{\cos 21°} \Rightarrow OA = \frac{6}{\cos 21°}$

$\text{Area}_{\text{circle}} = \pi\left(\frac{6}{\cos 21°}\right)^2$

......130...... cm^2 [5]

(d)



NOT TO SCALE

The diagram shows a square with side length 8 cm and a sector of a circle with radius 9.5 cm and sector angle $x°$.
The perimeter of the square is equal to the perimeter of the sector.

Calculate the value of x.

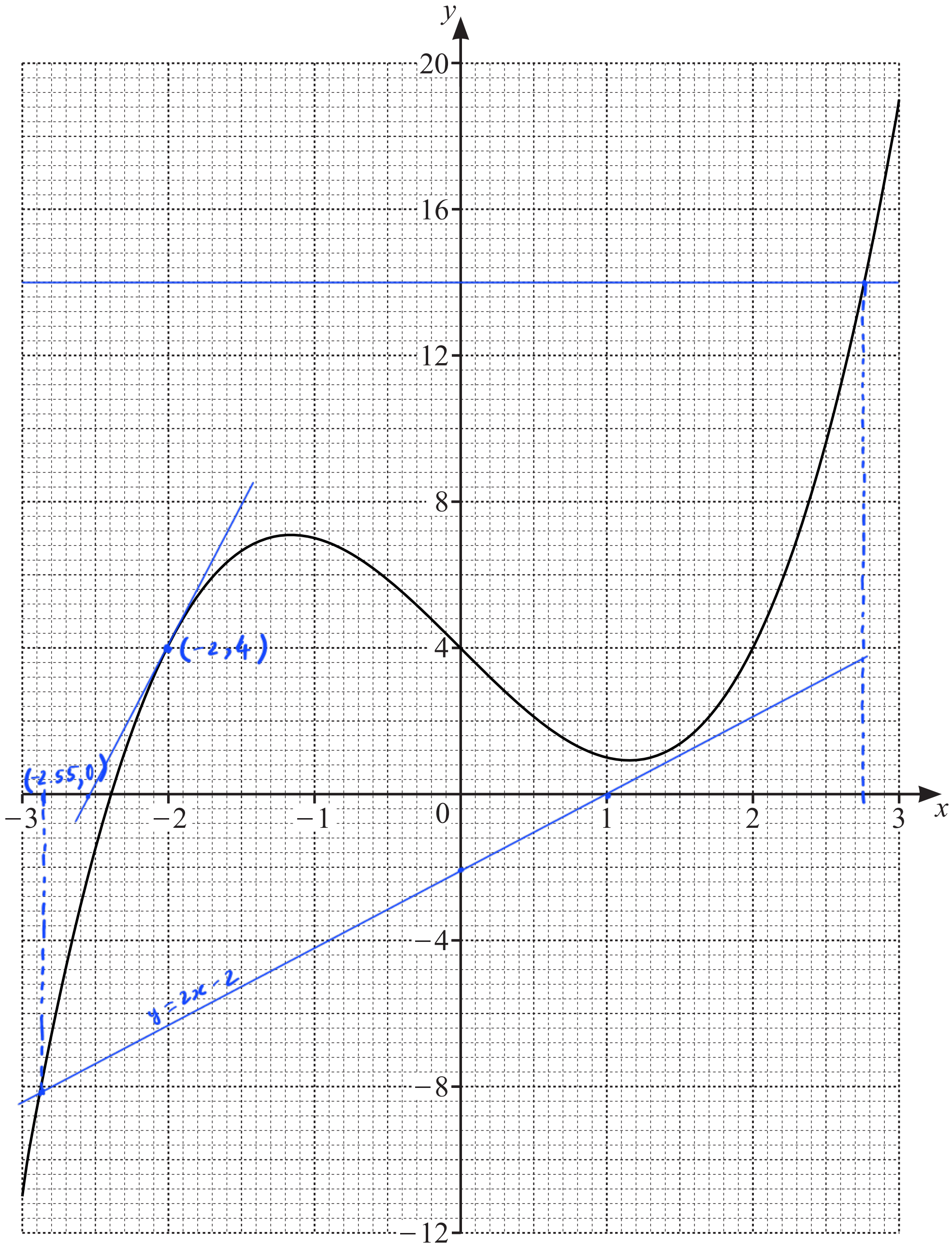
$8 \times 4 = 9.5 \times 2 + 9.5 \frac{x\pi}{180}$

$13 = \frac{19\pi}{360} x$

$x = 13 : \frac{19\pi}{360}$

$x =$ ......78.4...... [3]

5 **(a)** The diagram shows the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3$.



(i) Solve $f(x) = 14$.

$x =$ 2.75 [1]

(ii) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point $(-2, 4)$.

$$\frac{4-0}{-2-(-2.55)}$$

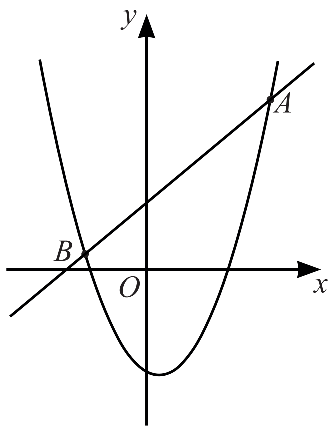
$\frac{80}{11}$ [3]

(iii) By drawing a suitable straight line on the grid, solve $f(x) = 2x - 2$ for $-3 \leqslant x \leqslant 3$.

.

$x =$ ....-2.85.......... [3]

(b)



NOT TO SCALE

The diagram shows a curve with equation $y = 2x^2 - 2x - 7$.
The straight line with equation $y = 3x + 5$ intersects the curve at the points A and B.

Find the coordinates of the points A and B.

$2x^2 - 2x - 7 = 3x + 5$

$2x^2 - 5x - 12 = 0$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-12)}}{2 \times 2}$

$x = 4$ or $x = -1.5$

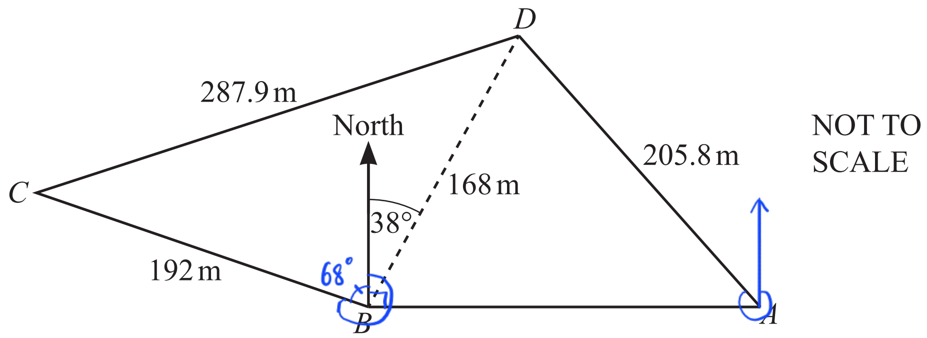
When $x = 4$ (point A), $y = 3 \times 4 + 5 = 17$

When $x = -1.5$ (point B), $y = 3 \times (-1.5) + 5 = 0.5$

A (.....4..... , ...17.....)

B (.....-1.5..... , ...0.5.....) [5]

6



The diagram shows a field, *ABCD*, on horizontal ground.
BC = 192 m, *CD* = 287.9 m, *BD* = 168 m and *AD* = 205.8 m.

(a) **(i)** Calculate angle *CBD* and show that it rounds to 106.0°, correct to 1 decimal place.

$287.9^2 = 192^2 + 168^2 - 2 \times 192 \times 168 \cos \widehat{CBD}$

$17798.41 = -64512 \cos \widehat{CBD}$

$\cos \widehat{CBD} = -0.2759$

$\widehat{CBD} = 106.016°$

$\widehat{CBD} \approx 106.0°$

[4]

(ii) The bearing of *D* from *B* is 038°.

Find the bearing of *C* from *B*.

$106.016° - 38° = 68.016°$

$360° - 68.016° = 291.984$

.........292.0......................... [1]

(iii) *A* is **due east** of *B*.

Calculate the bearing of *D* from *A*.

$\widehat{DBA} = 90° - 38° = 52°$

$\dfrac{168}{\sin \widehat{DAB}} = \dfrac{205.8}{\sin 52°}$

$\sin \widehat{DAB} = \dfrac{168 \sin 52°}{205.8} = 0.643$

$\widehat{DAB} = 40.016°$

$\Rightarrow \text{bearing}_{A \to D} = 270° + 40.016°$

.........310.0°......................... [5]

(b) **(i)** Calculate the area of triangle *BCD*.

$\frac{1}{2} \times 192 \times 168 \times \sin 106.016°$

$= 15502$

.........15500......... m^2 [2]

(ii) Tomas buys the triangular part of the field, *BCD*.
The cost is $35 750 per hectare.

Calculate the amount he pays.
Give your answer correct to the nearest $100.
[1 hectare = 10 000 m^2]

$15500 \, m^2 = 1.55$ hectare

Amount he pays: $35750 \times 1.55 = 55412.5$

$ ....55400......... [2]

7

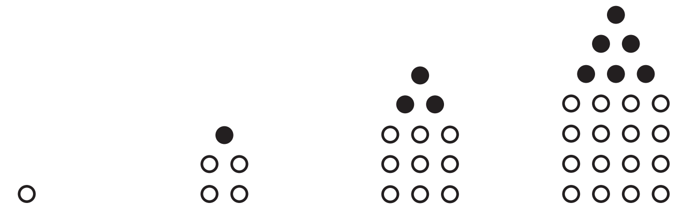


Diagram 1 Diagram 2 Diagram 3 Diagram 4

These are the first four diagrams of a sequence.
The diagrams are made from white dots and black dots.

(a) Complete the table for Diagram 5 and Diagram 6.

Diagram	1	2	3	4	5	6
Number of white dots	1	4	9	16	25	36
Number of black dots	0	1	3	6	10	15
Total number of dots	1	5	12	22	35	51

[2]

(b) Write an expression, in terms of n, for the number of white dots in Diagram n.

.... n^2 .... [1]

(c) The expression for the total number of dots in Diagram n is $\frac{1}{2}(3n^2 - n)$.

(i) Find the total number of dots in Diagram 8.

$\frac{1}{2}(3 \times 8^2 - 8)$

.... 92 .... [1]

(ii) Find an expression for the number of black dots in Diagram n.
Give your answer in its simplest form.

$an^2 + bn + c$

$a + b + c = 0$
$4a + 2b + c = 1$
$9a + 3b + c = 3$

$\Rightarrow \begin{cases} a = 0.5 \\ b = -0.5 \\ c = 0 \end{cases}$

.... $0.5n^2 - 0.5n$ .... [2]

(d) T is the total number of dots used to make **all** of the first n diagrams.

$$T = an^3 + bn^2$$

Find the value of a and the value of b.
You must show all your working.

$n = 1,\ T = 1 \Rightarrow 1 = a + b \quad (1)$

$n = 2,\ T = 1 + 5 = 6 \Rightarrow 6 = 8a + 4b$

multiply equation (1) by 4, $4 = 4a + 4b$

$\Rightarrow 6 - 4 = 8a - 4a$

$\Rightarrow 2 = 4a$

$\Rightarrow a = 0.5$

$b = 1 - 0.5 = 0.5$

$a =$ ......0.5......

$b =$ ......0.5...... [5]

8 **(a)** Factorise completely.

$$3a^2b - ab^2$$

$ab(3a-b)$ .... [2]

(b) Solve the inequality.

$$3x + 12 < 5x - 3$$

$12 + 3 < 5x - 3x$

$15 < 2x$

$\frac{15}{2} < x$

$x > \frac{15}{2}$ .... [2]

(c) Simplify.

$$(3x^2y^4)^3$$

$3^3(x^2)^3(y^4)^3$

$27x^6y^{12}$ .... [2]

(d) Solve.

$$\frac{2}{x} = \frac{6}{2-x}$$

$2(2-x) = 6x$

$4 - 2x = 6x$

$4 = 8x$

$x =$ 0.5 .... [3]

(e) Expand and simplify.

$$(x-2)(x+5)(2x-1)$$

$(x^2 - 2x + 5x - 10)(2x-1)$

$(x^2 + 3x - 10)(2x-1)$

$2x^3 + 6x^2 - 20x - x^2 - 3x + 10$

$2x^3 + 5x^2 - 23x + 10$ .... [3]

(f) Alan invests \$200 at a rate of r% per year compound interest.
After 2 years the value of his investment is \$206.46 .

(i) Show that $r^2 + 200r - 323 = 0$.

$$206.46 = 200\left(1 + \frac{r}{100}\right)^2$$

$$1.0323 = \left(1 + \frac{r}{100}\right)^2$$

$$10323 = 100^2 \times \left(1 + \frac{r}{100}\right)^2$$

$$10323 = (100 + r)^2$$

$$10323 = 100^2 + 200r + r^2$$

$$r^2 + 200r - 323 = 0$$

[3]

(ii) Solve the equation $r^2 + 200r - 323 = 0$ to find the rate of interest.
Show all your working and give your answer correct to 2 decimal places.

$$r = \frac{-200 \pm \sqrt{200^2 - 4 \times 1 \times (-323)}}{2 \times 1}$$

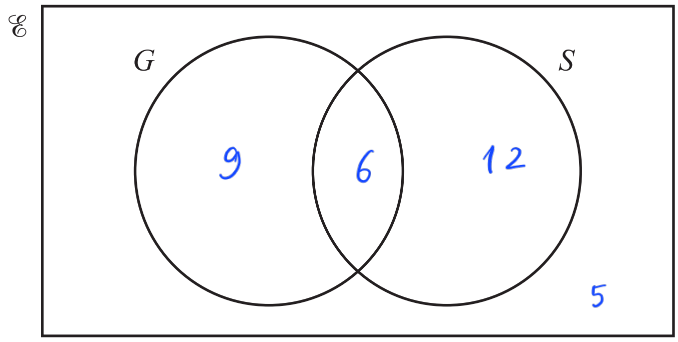
$r = -201.60$ or $r = 1.60$

Because $r > 0$ so $r = 1.60$

$r =$ ......1.60...................................... [3]

9 **(a)** There are 32 students in a class.

5 do not study any languages.
15 study German (G).
18 study Spanish (S).



(i) Complete the Venn diagram to show this information. [2]

(ii) A student is chosen at random.

Find the probability that the student studies Spanish but not German.

$\frac{12}{32}$ .... [1]

(iii) A student who studies German is chosen at random.

Find the probability that this student also studies Spanish.

$\frac{6}{15}$ .... [1]

(b) A bag contains 54 red marbles and some blue marbles.
36% of the marbles in the bag are red.

Find the number of blue marbles in the bag.

64 % of the marbles are blue

54 : 36 %

? 64 %

$\frac{64 \times 54}{36} = 96$

.................96.................. [2]

(c) Another bag contains 15 red beads and 10 yellow beads.
Ariana picks a bead at random, records its colour and replaces it in the bag.
She then picks another bead at random.

(i) Find the probability that she picks two red beads.

$\frac{15}{25} \times \frac{15}{25} = \frac{9}{25}$

.................$\frac{9}{25}$.................. [2]

(ii) Find the probability that she does not pick two red beads.

$1 - \frac{9}{25}$

.................$\frac{16}{25}$.................. [1]

(d) A box contains 15 red pencils, 8 yellow pencils and 2 green pencils.
Two pencils are picked at random without replacement.

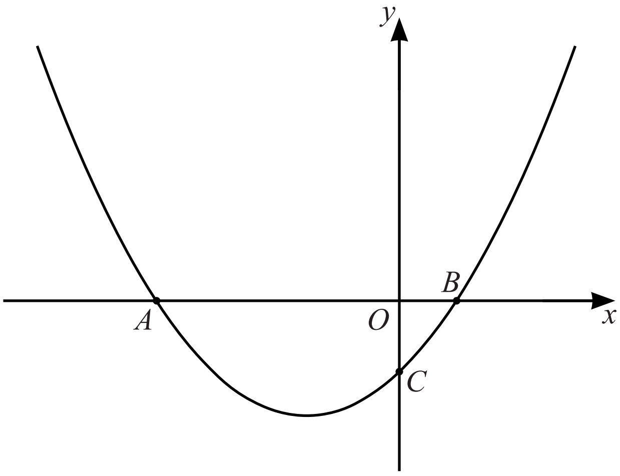
Find the probability that at least one pencil is red.

1st	2nd		
R	Y	$\rightarrow$	$15/25 \times 8/24$
Y	R	$\rightarrow$	$8/25 \times 15/24$
R	G	$\rightarrow$	$15/25 \times 2/24$
G	R	$\rightarrow$	$2/25 \times 15/24$
R	R	$\rightarrow$	$15/25 \times 14/24$

Add them up

.................$\frac{17}{20}$.................. [3]

10 (a)



NOT TO SCALE

The diagram shows a sketch of the curve $y = x^2 + 3x - 4$.

(i) Find the coordinates of the points A, B and C.

sub $x = 0$: $y_C = 0^2 + 3 \times 0 - 4 = -4$

$x^2 + 3x - 4 = 0$

$\Rightarrow \quad x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 1(-4)}}{2 \times 1}$

$x_A = -4 \quad , \quad x_B = 1$

A (......-4...... , ...0......)

B (......1...... , ...0......)

C (......0...... , ...-4......) [4]

(ii) Differentiate $x^2 + 3x - 4$.

......$2x + 3$...... [2]

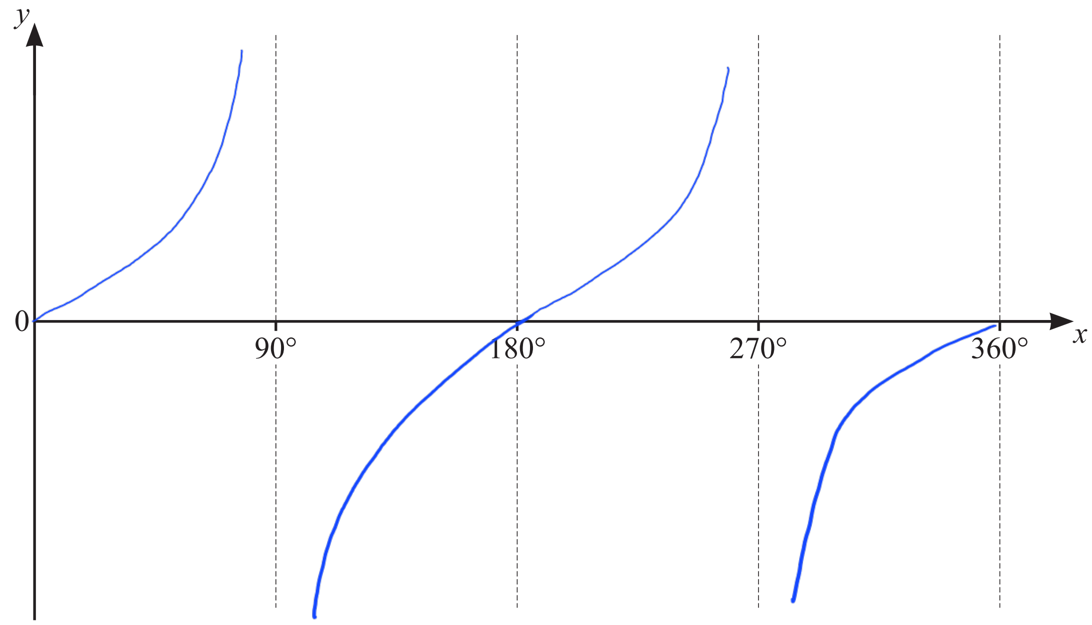
(iii) Find the equation of the tangent to the curve at the point (2, 6).

gradient of tangent $= 2 \times 2 + 3 = 7$

$\Rightarrow$ Equation of tangent : $y - 6 = 7(x - 2)$

......$y - 6 = 7(x - 2)$...... [3]

(b)



(i) On the diagram, sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. [2]

(ii) Solve the equation $5\tan x = -7$ for $0° \leqslant x \leqslant 360°$.

$\tan x = \frac{-7}{5}$

$x = -54.5°$ or $x = -54° + 180° = 125.5°$

or $x = -54.5° + 360° = 305.5°$

$x =$ ....125.5°.... or $x =$ ....305.5°.... [3]