1 **(a)** A 2.5-litre tin of paint costs \$13.50 .
In a sale, the cost is reduced by 14%.

(i) Work out the sale price of this tin of paint.

$13.50 - 13.50 \times 14\%$

\$ ....11.61.... [2]

(ii) Work out the cost of buying 42.5 litres of paint at this sale price.

$\frac{42.5 \times 11.61}{2.5}$

\$ ....197.37.... [2]

(b) Henri buys some paint in the ratio red paint : white paint : green paint = 2 : 8 : 5.

(i) Find the percentage of this paint that is white.

$\frac{8}{2+8+5} \times 100$

....53.3.... % [1]

(ii) Henri buys a total of 22.5 litres of paint.

Find the number of litres of green paint he buys.

$\frac{22.5}{2+8+5} \times 5$

....7.5.... litres [2]

(c) Maria paints a rectangular wall.
The length of the wall is 20.5 m and the height is 2.4 m, both correct to 1 decimal place.
0.1
One litre of paint covers an area of exactly 10 m^2.
Calculate the smallest number of 2.5-litre tins of paint she will need to be sure all the wall is painted.

$\text{Area}_{max} = \left(20.5 + \frac{0.1}{2}\right)\left(2.4 + \frac{0.1}{2}\right) = 50.3475 \text{ m}^2$

$1\ell : 10 \text{m}^2$

$5.03475\ell \quad 50.3475 \text{m}^2$

$\frac{5.03475}{2.5} = 2.0139$ round up to 3

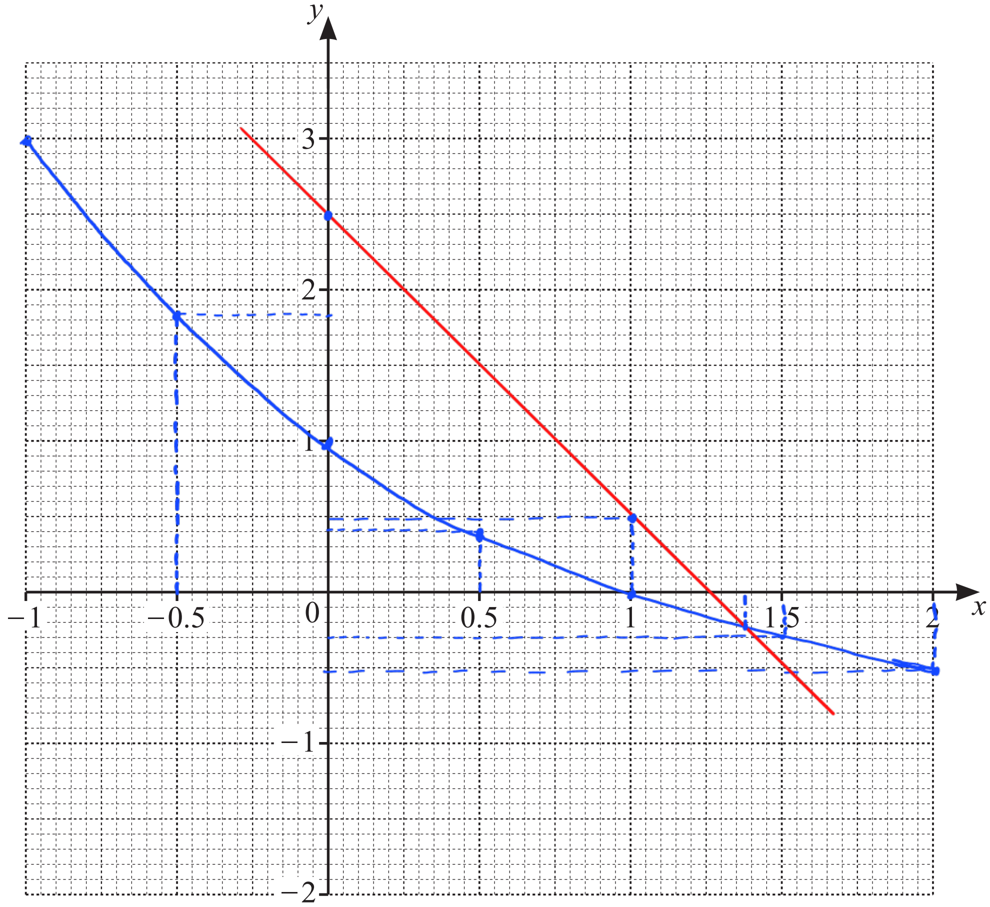
....3.... [4]

2 The table shows some values for $y = 2 \times 0.5^x - 1$.

x	-1	-0.5	0	0.5	1	1.5	2
y	3	1.83	1	0.41	0	-0.29	-0.5

(a) (i) Complete the table. [2]

(ii) On the grid, draw the graph of $y = 2 \times 0.5^x - 1$ for $-1 \leqslant x \leqslant 2$.



[4]

(b) By drawing a suitable straight line, solve the equation $2 \times 0.5^x + 2x - 3.5 = 0$ for $-1 \leqslant x \leqslant 2$.

$(2 \times 0.5^x - 1) + 2x - 2.5 = 0$

$2 \times 0.5^x - 1 = -2x + 2.5$

$x =$ ......1.37...... [3]

(c) There are no solutions to the equation $2 \times 0.5^x - 1 = k$ where k is an integer.

Complete the following statements.

The highest possible value of k is ......-1......

The equation of the asymptote to the graph of $y = 2 \times 0.5^x - 1$ is ......$y = -1$...... [2]

3 **(a)** Simplify, giving your answer as a single power of 7.

(i) $7^5 \times 7^6$

7^{11} .... [1]

(ii) $7^{15} \div 7^5$

7^{10} .... [1]

(iii) $42 \div 7$

7^2 .... [1]

(b) Simplify.

$(5x^2 \times 2xy^4)^3$

$(5x^2)^3 \times (2xy^4)^3$

$5^3(x^2)^3 \times 2^3 x^3 (y^4)^3$

$(5^3 \times 2^3)(x^6 x^3) y^{12}$

$1000x^9y^{12}$ .... [3]

(c) $P = 2^5 \times 3^3 \times 7$ $\quad Q = 540 = 2^2 \times 3^2 \times 5$

(i) Find the highest common factor (HCF) of P and Q.

$HCF(P, Q) = 2^2 \times 3^3$

108 .... [2]

(ii) Find the lowest common multiple (LCM) of P and Q.

$LCM(P, Q) = 2^5 \times 3^3 \times 5 \times 7$

30240 .... [2]

(iii) $P \times R$ is a cube number, where R is an integer.

Find the smallest possible value of R.

$R_{smallest} = 2^1 \times 7^2$

98 .... [2]

(d) Factorise the following completely.

(i) $x^2 - 3x - 28$

$x^2 - 7x + 4x - 28$

$x(x-7) + 4(x-7)$

$(x+4)(x-7)$ [2]

(ii) $7(a+2b)^2 + 4a(a+2b)$

$(a+2b)[7(a+2b) + 4a]$

$(a+2b)(7a + 14b + 4a)$

$(a+2b)(11a+14b)$ [2]

(e) $3^{2x-1} = \frac{1}{9^x} \times 3^{2y-x}$

Find an expression for y in terms of x.

$3^{2x-1} = 9^{-x} \times 3^{2y-x}$

$3^{2x-1} = 3^{-2x} \times 3^{2y-x}$

$3^{2x-1} = 3^{-2x+2y-x}$

$\Rightarrow 2x - 1 = -3x + 2y$

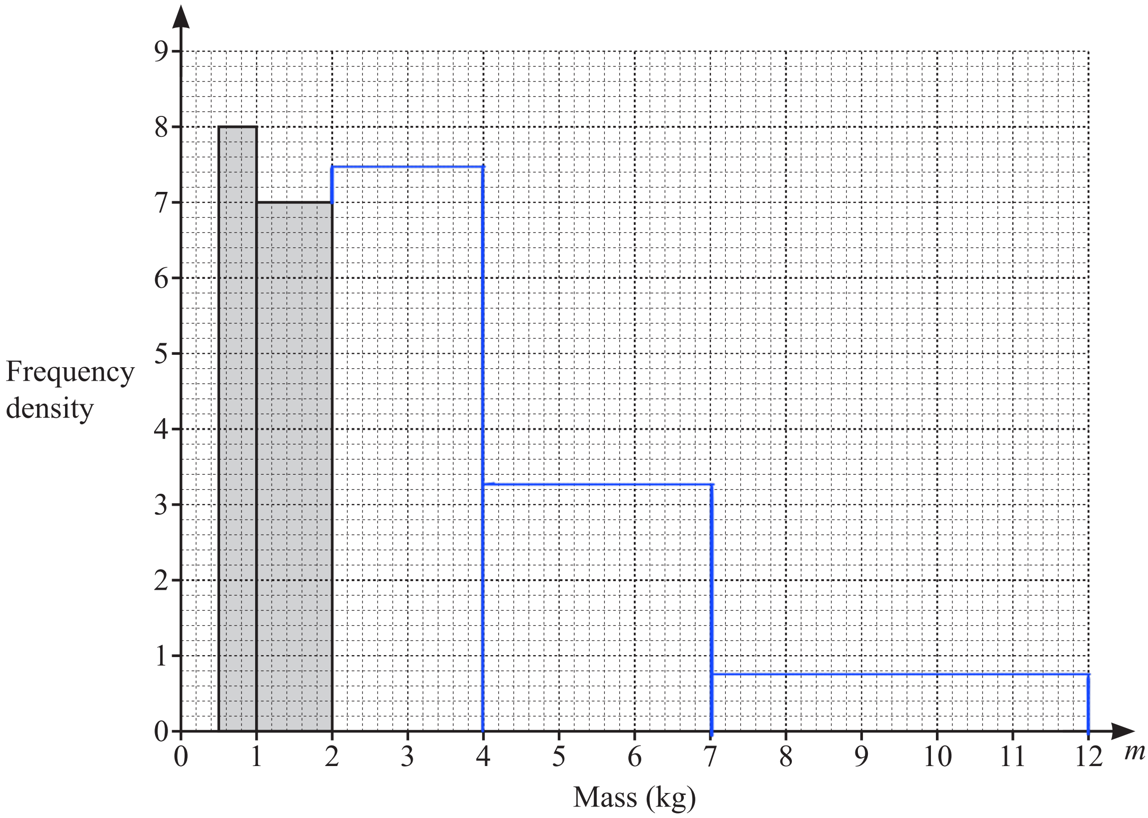
$5x - 1 = 2y$

$y = \frac{5x-1}{2}$ [4]

4 (a) The mass, m kg, of each of 40 parcels in a warehouse is recorded.
The table shows information about the masses of these parcels.

Class width	0.5	1	2	3	5
Mass (m kg)	$0.5 < m \leqslant 1$	$1 < m \leqslant 2$	$2 < m \leqslant 4$	$4 < m \leqslant 7$	$7 < m \leqslant 12$
Frequency	4	7	15	10	4
Freq. density	8	7	7.5	$\frac{10}{3}$	0.8

(i) Complete the histogram to show this information.



[3]

(ii) Calculate an estimate of the mean mass of the parcels.

Mid value	0.75	1.5	3	5.5	9.5

$$\text{Mean} = \frac{(0.75 \times 4) + (1.5 \times 7) + (3 \times 15) + (5.5 \times 10) + (9.5 \times 4)}{40}$$

..........3.7875.......... kg [4]

(iii) A parcel is picked at random from the 40 parcels.

Find the probability that this parcel has a mass of 2 kg or less.

$$\frac{4+7}{40}$$

.... $\frac{11}{40}$ .... [1]

(iv) Two parcels are picked at random without replacement from those with a mass **greater than** 2 kg.

Work out the probability that one of them has a mass greater than 7 kg and the other has a mass of 4 kg or less.

There are 29 parcels with mass > 2 kg

$2 \times P(> 7\,kg) \times P(< 4\,kg)$

$= 2 \times \frac{4}{29} \times \frac{15}{28}$

$\frac{30}{203}$ .... [3]

5 **(a)** $\mathbf{a} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

(i) Find

(a) $\mathbf{b} - \mathbf{a}$,

$\begin{pmatrix} 2-(-3) \\ -5-8 \end{pmatrix}$

$\begin{pmatrix} 5 \\ -13 \end{pmatrix}$ [1]

(b) $2\mathbf{a} + \mathbf{b}$,

$\begin{pmatrix} 2(-3)+2 \\ 2\times 8-5 \end{pmatrix}$

$\begin{pmatrix} -4 \\ 11 \end{pmatrix}$ [2]

(c) $|\mathbf{b}|$.

$\sqrt{2^2+(-5)^2} \approx 5.39$

........5.39........ [2]

(ii) $\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 13 \\ m \end{pmatrix}$, where k and m are integers.

Find the value of k and the value of m.

$\begin{pmatrix} -3 \\ 8 \end{pmatrix} + k\begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 13 \\ m \end{pmatrix}$

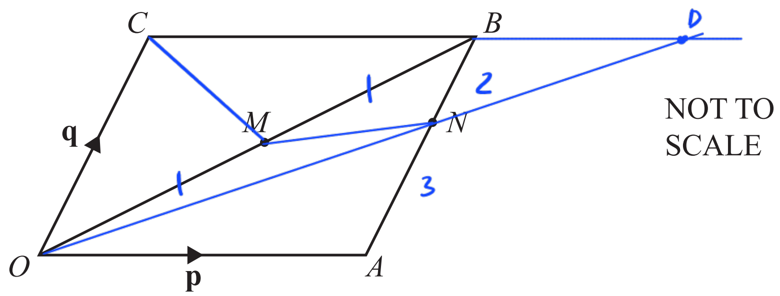
$\Rightarrow \begin{cases} -3+2k = 13 \\ 8-5k = m \end{cases}$

$\Rightarrow \begin{cases} k = 8 \\ m = -32 \end{cases}$

$k =$ ....8........

$m =$ ....−32........ [3]

(b)



OABC is a parallelogram and *O* is the origin.
M is the midpoint of *OB*.
N is the point on *AB* such that *AN* : *NB* = 3 : 2.
$\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$.

(i) Find, in terms of **p** and **q**, in its simplest form.

(a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC}$

$\overrightarrow{OB} =$ ......$p + q$...... [1]

(b) $\overrightarrow{CM} = \overrightarrow{CO} + \overrightarrow{OM}$

$= -q + \frac{1}{2}\overrightarrow{OB}$

$= -q + \frac{1}{2}(p+q)$

$\overrightarrow{CM} =$ ......$\frac{1}{2}p - \frac{1}{2}q$...... [2]

(c) $\overrightarrow{MN}$

$\overrightarrow{BN} = \frac{2}{5}\overrightarrow{BA} = -\frac{2}{5}q$

$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN} = \frac{1}{2}\overrightarrow{OB} - \frac{2}{5}q$

$= \frac{1}{2}(p+q) - \frac{2}{5}q$

$\overrightarrow{MN} =$ ......$\frac{1}{2}p + \frac{1}{10}q$...... [2]

(ii) *CB* and *ON* are extended to meet at *D*.

Find the position vector of *D* in terms of **p** and **q**.
Give your answer in its simplest form.

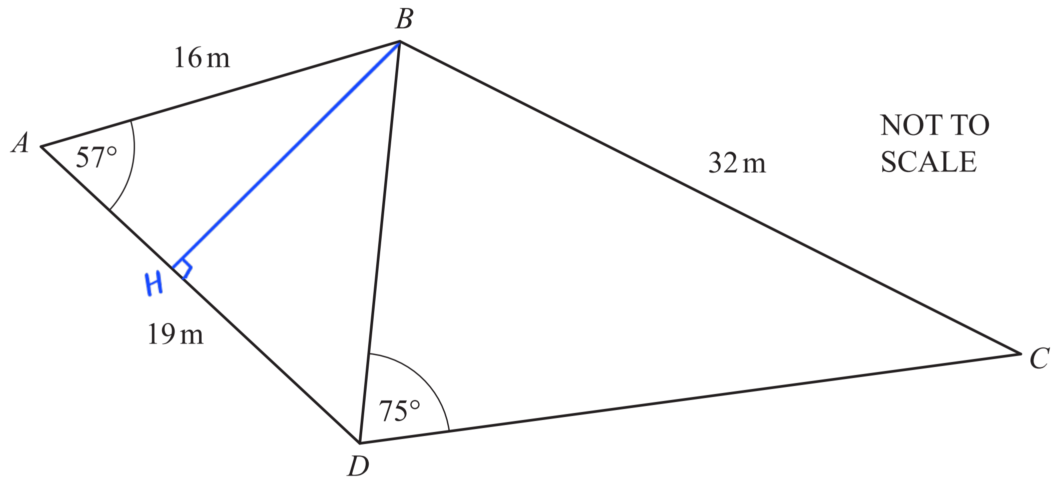
$\Delta BND \sim \Delta ANO$

$\frac{BN}{AN} = \frac{BD}{AO} = \frac{2}{3} \Rightarrow \overrightarrow{BD} = \frac{2}{3}\overrightarrow{OA} = \frac{2}{3}p$

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD} = p + q + \frac{2}{3}p$

......$\frac{5}{3}p + q$...... [3]

6



The diagram shows a quadrilateral *ABCD* made from two triangles, *ABD* and *BCD*.

(a) Show that *BD* = 16.9 m, correct to 1 decimal place.

$BD^2 = 16^2 + 19^2 - 2 \times 16 \times 19 \times \cos 57° = 285.860$

$BD = \sqrt{285.860} \approx 16.9$

[3]

(b) Calculate angle *CBD*.

$\frac{\sqrt{285.86}}{\sin \widehat{BCD}} = \frac{32}{\sin 75°} \Rightarrow \sin \widehat{BCD} = \frac{\sin 75° \sqrt{285.6}}{32} \approx 0.51012$

$\widehat{BCD} = 30.67°$

$\widehat{CBD} = 180° - 30.67° - 75° = 74.33°$

Angle *CBD* = ...74.3°.......... [4]

(c) Find the area of the quadrilateral *ABCD*.

$A_{ABCD} = A_{\Delta ABD} + A_{\Delta BCD}$

$= \frac{1}{2} \times 16 \times 19 \times \sin 57° + \frac{1}{2} \sqrt{285.86} \times 32 \times \sin 74.33°$

≈ 387.94

..........388.......... m^2 [3]

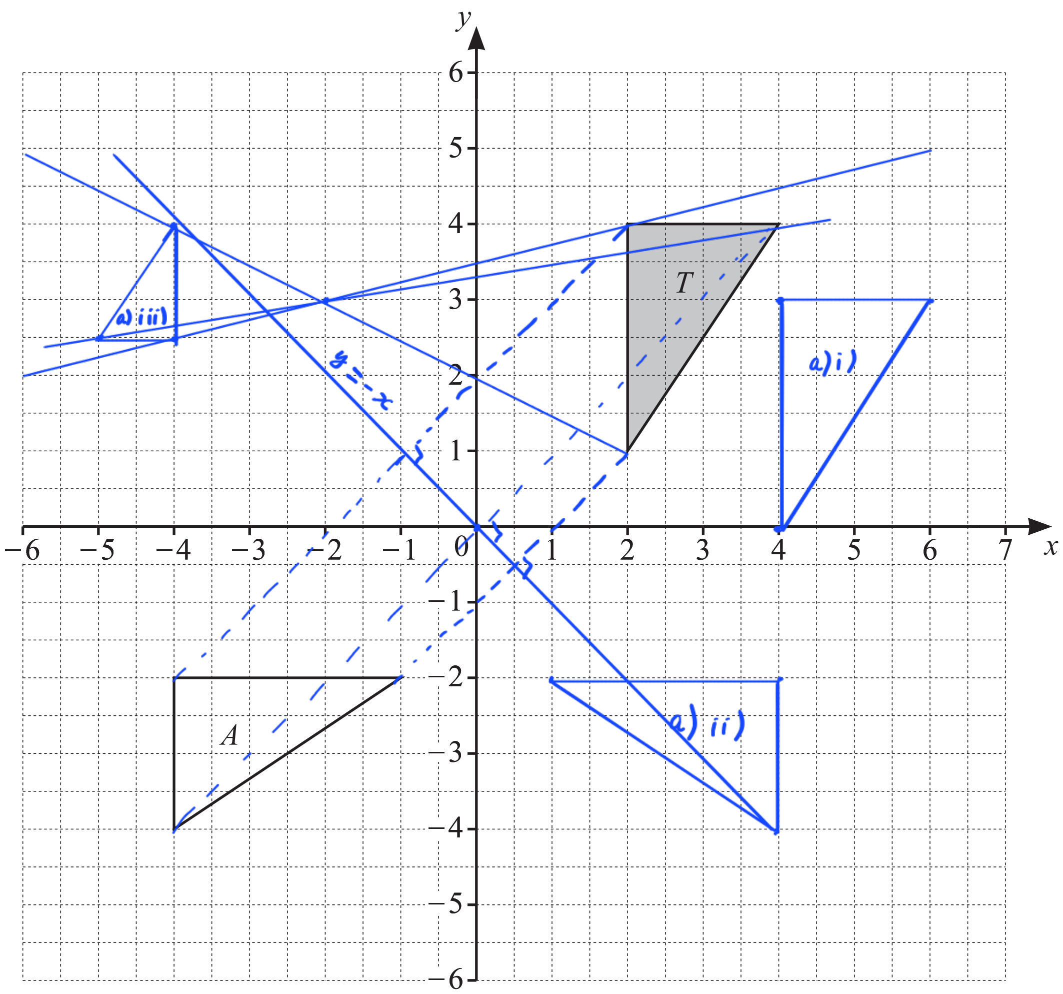
(d) Find the shortest distance from *B* to *AD*.

$\sin 57° = \frac{BH}{16}$

$\Rightarrow BH = 16 \sin 57° \approx 13.42$

..........13.4.......... m [3]

7



(a) On the grid, draw the image of

(i) triangle T after a translation by the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, [2]

(ii) triangle T after a rotation, 90° clockwise, about the origin, [2]

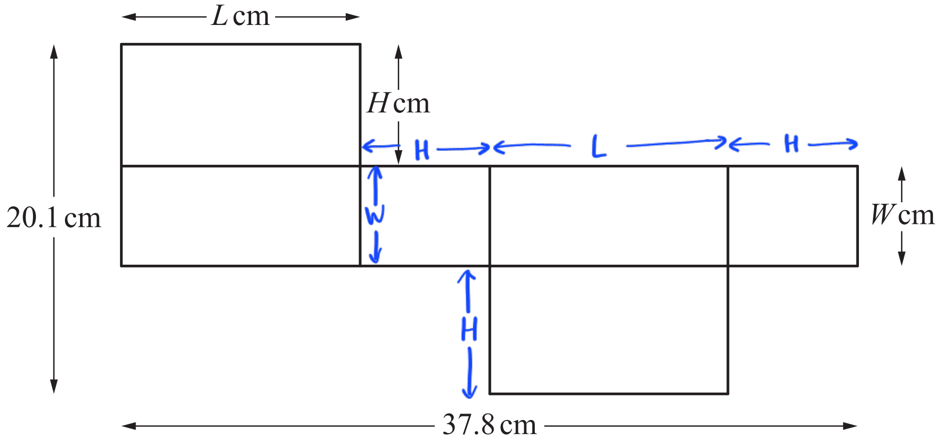
(iii) triangle T after an enlargement, scale factor $-\frac{1}{2}$, centre (−2, 3). [2]

(b) Describe fully the **single** transformation that maps triangle T onto triangle A.

Reflection over $y = -x$

.... [2]

8 **(a)** A cuboid has length L cm, width W cm and height H cm.



NOT TO SCALE

The diagram shows the net of this cuboid.
The ratio $W : L = 1 : 2$.

Find the value of L, the value of W and the value of H.

$L = 2W$

$H + W + H = 20.1 \Rightarrow 2H + W = 20.1 \quad (1)$

$L + H + L + H = 37.8 \Rightarrow 2H + 2L = 37.8 \quad (2)$

$(2) - (1): \quad 2L - W = 37.8 - 20.1 = 17.7 \quad (3)$

Sub $L = 2W$ into (3):

$2(2W) - W = 17.7$

$3W = 17.7$

$W = 5.9$

$\Rightarrow L = 2 \times 5.9 = 11.8$

Sub $L = 1.8$ into (2):

$2H + 2 \times 11.8 = 37.8$

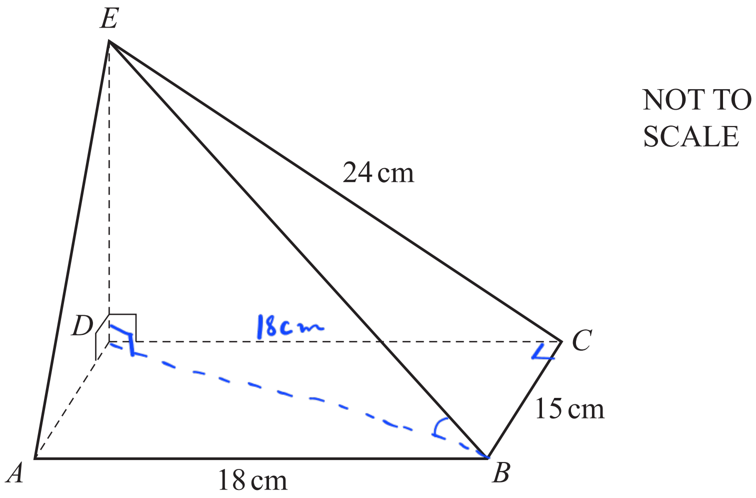
$\Rightarrow H = 7.1$

$L =$ ......11.8......

$W =$ ......5.9......

$H =$ ......7.1...... [5]

(b)



The diagram shows a solid pyramid with a rectangular base *ABCD*.
E is vertically above *D*.
Angle *EDC* = angle *EDA* = 90°.
AB = 18 cm, *BC* = 15 cm and *EC* = 24 cm.

(i) The pyramid is made of wood and has a mass of 800 g.

Calculate the density of the wood.
Give the units of your answer.

[The volume, V, of a pyramid is $V = \frac{1}{3} \times$ area of base $\times$ height.]
[Density = mass ÷ volume]

$ED = \sqrt{24^2 - 18^2} = 6\sqrt{7}$

$V_{pyramid} = \frac{1}{3} \times (18 \times 15) \times 6\sqrt{7} \approx 1428.71 \text{ cm}^3$

$\text{Density} = \frac{800 \text{ g}}{1428.71 \text{ cm}^3} \approx 0.560 \text{ g/cm}^3$

..........0.560.......... g/cm³ [5]

(ii) Calculate the angle between *BE* and the base of the pyramid.

$BD = \sqrt{15^2 + 18^2} = 3\sqrt{61}$

$\tan \widehat{EBD} = \frac{ED}{BD} = \frac{6\sqrt{7}}{3\sqrt{61}}$

$\widehat{EBD} = \tan^{-1}\left(\frac{6\sqrt{7}}{3\sqrt{61}}\right) \approx 34.118°$

..........34.1°.. [4]

9 **(a)** **(i)** The equation $y = x^3 - 4x^2 + 4x$ can be written as $y = x(x-a)^2$.

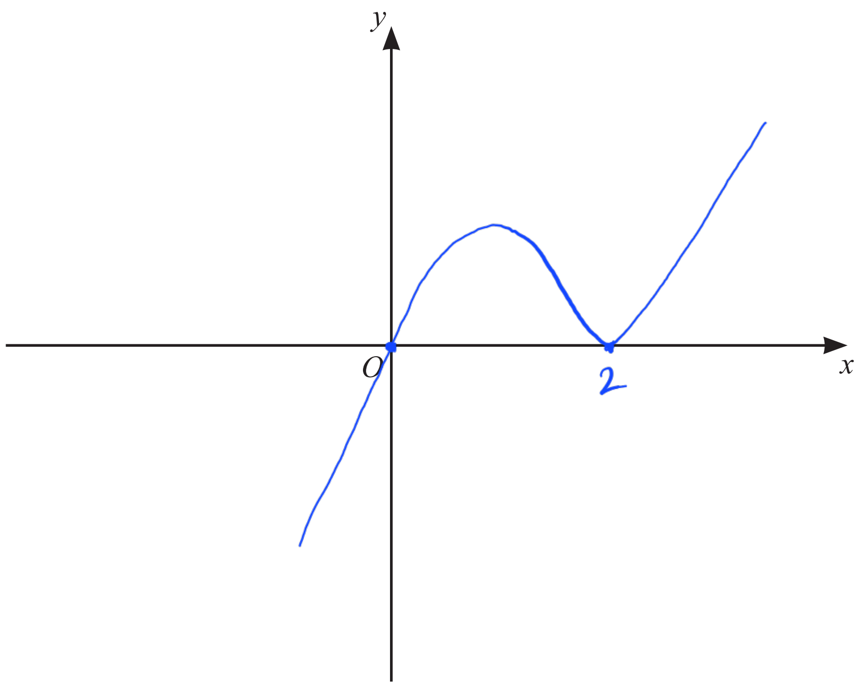
Find the value of a.

$x^3 - 4x^2 + 4x = x(x-a)^2$
$x^3 - 4x^2 + 4x = x(x^2 - 2ax + a^2)$
$x^3 - 4x^2 + 4x = x^3 - 2ax^2 + a^2 x$

$a =$ ....2.. [2]

(ii) On the axes, sketch the graph of $y = x^3 - 4x^2 + 4x$, indicating the values where the graph meets the axes.

double roots at $x = 2$



[4]

(b) Find the equation of the tangent to the graph of $y = x^3 - 4x^2 + 4x$ at $x = 4$.
Give your answer in the form $y = mx + c$.

$\frac{dy}{dx} = 3x^2 - 8x + 4$

$\frac{dy}{dx}$ (when $x = 4$) $= 3 \times 4^2 - 8 \times 4 + 4 = 20$

$\Rightarrow$ gradient$_{\text{tangent}} = 20$

When $x = 4$, $y = 4^3 - 4 \times 4^2 + 4 \times 4 = 16$

Equation of tangent : $y - 16 = 20(x - 4)$

$y - 16 = 20x - 80$

$y = 20x - 64$

$y =$ $20x - 64$ [7]

10 The table shows four sequences A, B, C and D.

Sequence	1st term	2nd term	3rd term	4th term	5th term		nth term
A	1	8	27	64	125		n^3
B	5	11	17	23	29		$6n - 1$
C	0.25	0.5	1	2	4		2^{n-3}
D	4.75	10.5	16	21	25.25		$-\frac{1}{24}n^3 + \frac{1}{8}n^2 + \frac{17}{3}n - 1$

Complete the table.

$+5.75$ $+5.5$ $+5$ $+4.25$

-0.25 -0.5 -0.75

-0.25 -0.25

D : $a = \frac{-0.25}{6} = \frac{-1}{24}$

$12 \times \left(-\frac{1}{24}\right) + 2b = -0.25 \Rightarrow b = \frac{1}{8}$

$7a + 3b + c = 5.75 \Rightarrow c = \frac{17}{3}$

$a + b + c + d = 4.75 \Rightarrow d = -1$

[9]