1 (a) The total cost of a taxi journey is calculated as

- \$0.50 per kilometre

plus

- \$0.40 per minute.

(i) Calculate the total cost of a journey of 32 km that takes 30 minutes.

$32 \times 0.5 + 30 \times 0.4$

\$ ....28........ [2]

(ii) The total cost of a journey of 100 km is \$98.

Show that the time taken is 2 hours.

t

$100 \times 0.5 + 0.4t = 98$

$0.4t = 48$

$t = 120$ minutes $= 2$ hours

[3]

(b) Three taxi drivers travel a total of 8190 km in the ratio 5 : 2 : 7.

Calculate the distance each driver travels.

$\dfrac{8190}{5+2+7} = 585$

Driver 1: 585×5

Driver 2: 585×2

Driver 3: 585×7

Driver 1 ....2925........ km

Driver 2 ....1170........ km

Driver 3 ....4095........ km [3]

(c) After midnight, the cost of any taxi journey increases by 45%.
One journey costs \$84.10 after midnight.

Calculate the cost of the same journey before midnight.

b

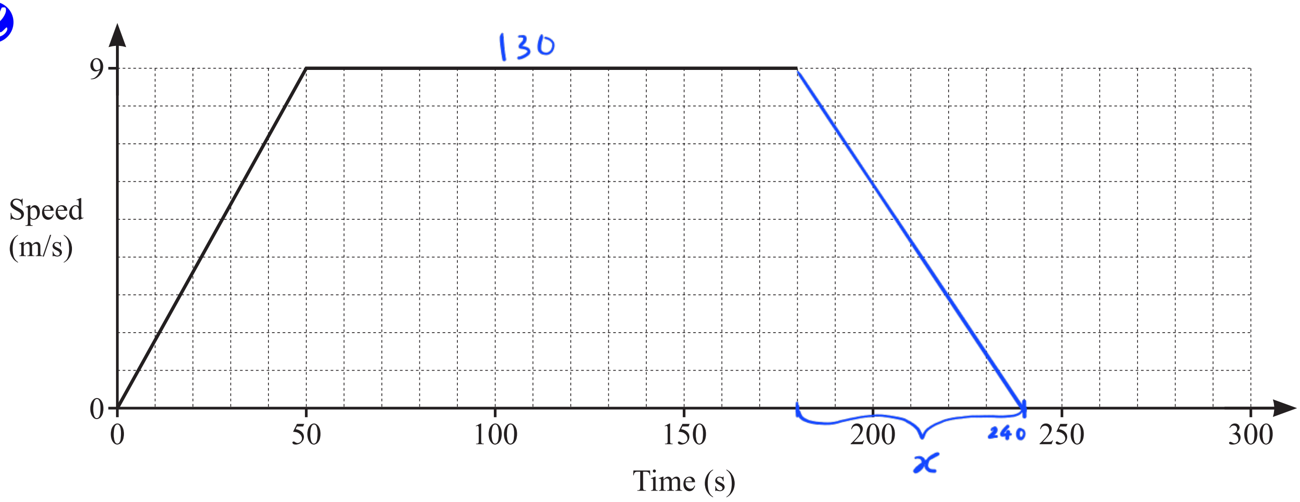
$b + b \times 45\% = 84.10$

$1.45b = 84.10$

$b = 58$

\$ ....58........ [2]

2 The diagram shows the speed–time graph for the first 180 seconds of a train journey.



(a) Find the acceleration, in m/s^2, of the train during the first 50 seconds.

.......$\frac{9}{50}$....... m/s^2 [1]

(b) After 180 seconds, the train decelerates at a constant rate of 1944 **km/h²**.

Show that the train decelerates for 60 seconds until it stops.

$$\frac{1944\ km}{h^2} = \frac{1944\,000\ m}{(3600\ s)^2} = \frac{1944\,000\ m}{3600^2\ s^2} = 0.15\ m/s^2$$

$$\frac{9-0}{x} = 0.15 \quad \Rightarrow x = \frac{9}{0.15} = 60$$

[2]

(c) Complete the speed–time graph. [1]

(d) Calculate the average speed of the train for the whole journey.

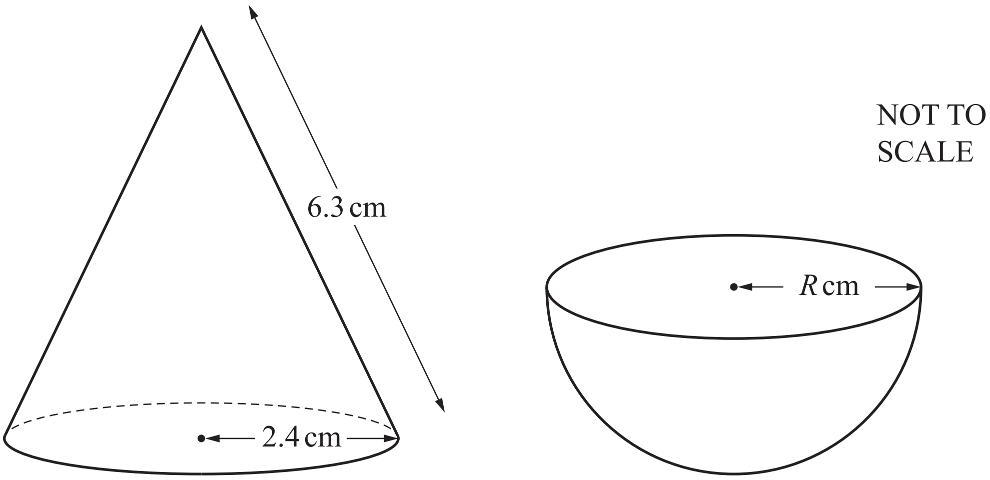
Distance = $\text{Area}_{\text{trapezium}}$

$$= \frac{1}{2}(130 + 240)\,9 = 1665$$

$$\text{Average speed} = \frac{1665}{240} = 6.9375$$

.......6.9375....... m/s [4]

3 (a)



The diagram shows a solid cone and a solid hemisphere.
The cone has radius 2.4 cm and slant height 6.3 cm.
The hemisphere has radius R cm.
The **total** surface area of the cone is equal to the **total** surface area of the hemisphere.

C H

Calculate the value of R.

$C = \text{area}_{\text{curve}} + \text{area}_{\text{base}}$

$= \pi \times 2.4 \times 6.3 + \pi \times 2.4^2 = 20.88\pi$

$H = \text{area}_{\text{curve}} + \text{area}_{\text{top}}$

$= \frac{1}{2} \times 4\pi R^2 + \pi R^2 = 3\pi R^2$

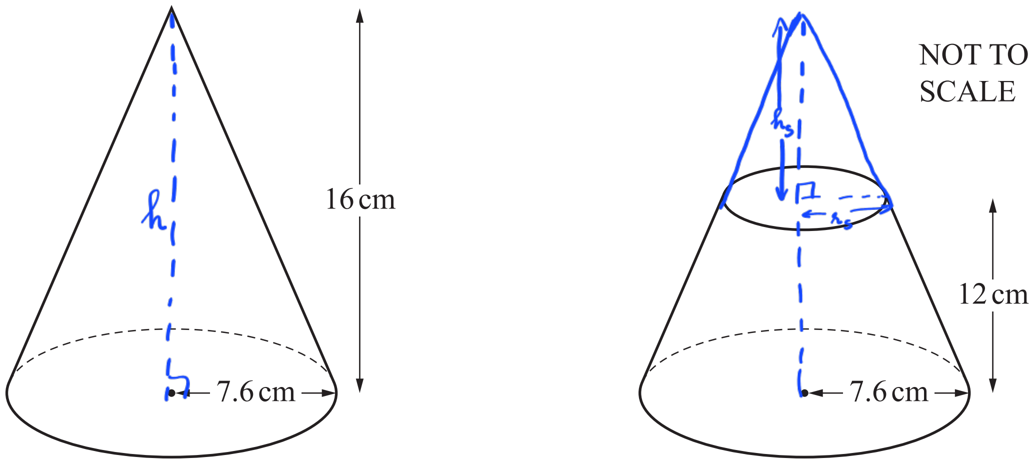
$\Rightarrow 20.88\pi = 3\pi R^2$

$\Rightarrow 6.96 = R^2$

$\Rightarrow R \approx 2.64$

R = ...2.64...... [4]

(b)



The diagram shows a solid cone with radius 7.6 cm and height 16 cm.
A cut is made parallel to the base of the cone and the top section is removed.
The remaining solid has height 12 cm, as shown in the diagram.

Calculate the volume of the remaining solid.

$$h_s = 16 - 12 = 4$$

$$\frac{h_s}{h} = \frac{r_s}{7.6} = \frac{4}{16} = \frac{1}{4} \Rightarrow r_s = 1.9$$

$$V_{\text{large solid}} = \frac{1}{3}\pi \times 7.6^2 \times 16 = \frac{23104}{75}\pi$$

$$V_{\text{small solid}} = \frac{1}{3}\pi \times 1.9^2 \times 4 = \frac{361}{75}\pi$$

$$\Rightarrow V_{\text{remaining solid}} = \frac{23104}{75}\pi - \frac{361}{75}\pi \approx 953$$

.........953......... cm^3 [4]

4 **(a)** The exchange rate is 1 euro = \$1.142 .

(i) Johann changes \$500 into euros.

Calculate the number of euros Johann receives.
Give your answer correct to the nearest euro.

$$\frac{500 \times 1}{1.142} \approx 437.83 \approx 438$$

.........438......... euros [2]

(ii) Johann buys a computer for \$329.
The same computer costs 275 euros.

Calculate the difference in cost in dollars.

275 euros = \$(275 × 1.142) = \$ 314.05

difference = 329 − 314.05 = 14.95

\$ ....14.95......... [2]

(b) Lucy spends $\frac{3}{8}$ of the money she has saved this month on a book that costs \$5.25 .

Calculate how much money Lucy has saved this month.

$\underbrace{\text{money Lucy has saved this month}}_{L}$

$$\frac{3}{8} L = 5.25$$

$$L = 5.24 : \frac{3}{8} = 14$$

\$ ....14......... [2]

(c) Kamal invests \$6130 at a rate of r% per year compound interest.
The value of his investment at the end of 5 years is \$6669.

Calculate the value of r.

$$6669 = 6130\left(1 + \frac{r}{100}\right)^5$$

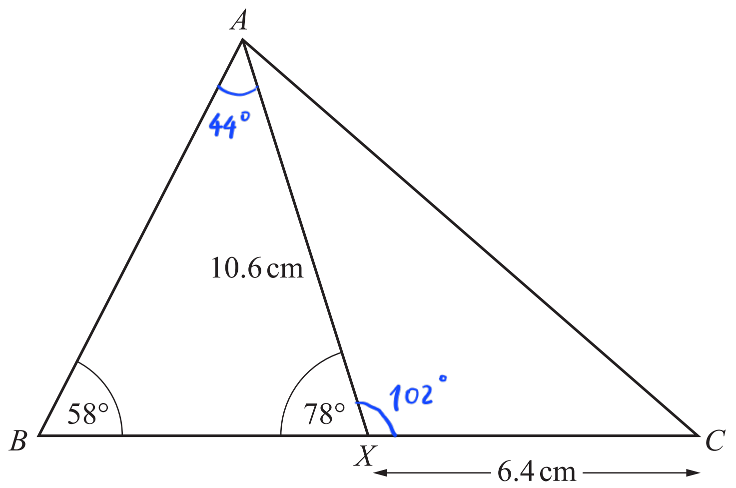
$$\frac{6669}{6130} = \left(1 + \frac{r}{100}\right)^5$$

$$1 + \frac{r}{100} = \sqrt[5]{\frac{6669}{6130}} \approx 1.017$$

$$\frac{r}{100} = 0.017$$

r = ...1.7......... [3]

5



NOT TO SCALE

The diagram shows triangle ABC.
X is a point on BC.
$AX = 10.6$ cm, $XC = 6.4$ cm, angle $ABC = 58°$ and angle $AXB = 78°$.

(a) Calculate AC.

$\widehat{AXC} = 180° - 78° = 102°$

$AC^2 = 6.4^2 + 10.6^2 - 2 \times 6.4 \times 10.6 \times \cos 102° = 181.53$

$AC \approx 13.5$

$AC =$ ....13.5.... cm [4]

(b) Calculate BX.

$\widehat{BAX} = 180° - 58° - 78° = 44°$

$\dfrac{BX}{\sin 44°} = \dfrac{10.6}{\sin 58°}$

$\Rightarrow BX = \dfrac{10.6 \times \sin 44°}{\sin 58°} \approx 8.6827$

$BX =$ ....8.68.... cm [4]

(c) Calculate the area of triangle ABC.

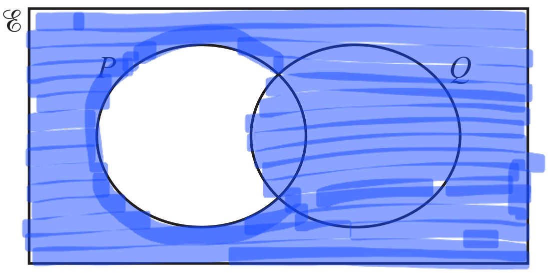
$A_{\Delta ABC} = A_{\Delta ABX} + A_{\Delta AXC}$

$= \frac{1}{2} \times 8.6827 \times 10.6 \times \sin 78° + \frac{1}{2} \times 10.6 \times 6.4 \times \sin 102°$

≈ 78.2

....78.2.... cm^2 [3]

6 **(a)** In the Venn diagram, shade the region $P' \cup Q$.

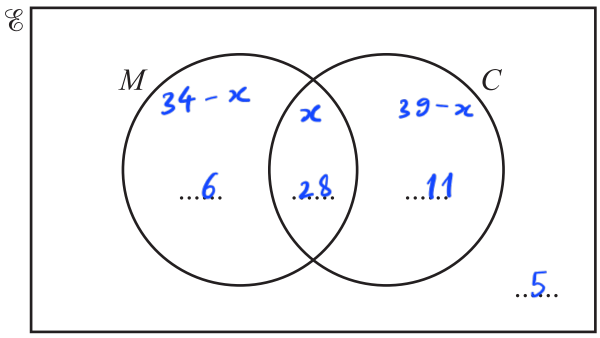


[1]

(b) There are 50 students in a group.

34 have a mobile phone (M).
39 have a computer (C).
5 have no mobile phone and no computer.

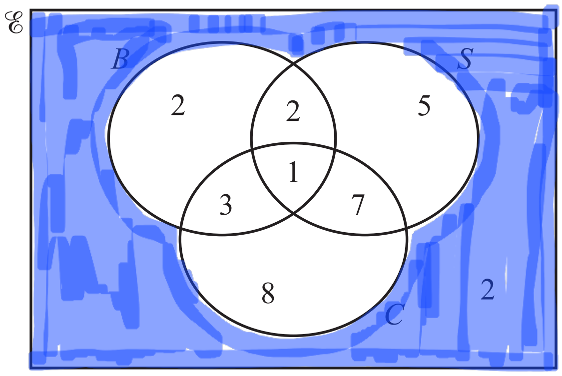
Complete the Venn diagram to show this information.



$(34-x)+x+(39-x)$
$+5=50$
$\Rightarrow 78-x=50$
$x=28$

[2]

(c) The Venn diagram shows the number of students in a group of 30 who have brothers (B), sisters (S) or cousins (C).



(i) Write down the number of students who have brothers.

2 + 2 + 1 + 3

........8........ [1]

(ii) Write down the number of students who have cousins but do not have sisters.

3 + 8

........11........ [1]

(iii) Find $n(B \cup S \cup C)'$.

........2........ [1]

(iv) Use set notation to describe the set of students who have both cousins and sisters but do not have brothers.

........$C \cap S \cap B'$........ [1]

(v) One student is picked at random from the 30 students.

Find the probability that this student has cousins.

$\frac{3+8+1+7}{30}$

........$\frac{19}{30}$........ [1]

(vi) Two students are picked at random from the students who have cousins.

Calculate the probability that both these students have brothers.

$\frac{4}{19} \times \frac{3}{18}$

........$\frac{2}{57}$........ [3]

(vii) One student is picked at random from the 30 students.

Event A This student has sisters.
Event B This student has cousins but does not have brothers.

Explain why event A and event B are equally likely.

$P(A) = \frac{2+5+1+7}{30} = \frac{1}{2}$ $P(C) = \frac{7+8}{30} = \frac{1}{2}$

.... [1]

7 **(a)** Simplify.

$$\frac{x^2-25}{x^2-x-20}$$

$$\frac{(x-5)(x+5)}{(x-5)(x+4)}$$

$\frac{x+5}{x+4}$ .... [3]

(b) Write as a single fraction in its simplest form.

$$\frac{x+5}{x}+\frac{x+8}{x-1}$$

$$\frac{(x+5)(x-1)+(x+8)x}{x(x-1)}$$

$$\frac{x^2+5x-x-5+x^2+8x}{x(x-1)}$$

$\frac{2x^2+12x-5}{x(x-1)}$ .... [3]

(c) A curve has equation $y = 2x^3 - 4x^2 + 6$.

(i) Find $\frac{dy}{dx}$, the derived function of y.

$$\frac{dy}{dx} = 6x^2 - 8x$$

.... $6x^2 - 8x$ .... [2]

(ii) Calculate the gradient of the curve $y = 2x^3 - 4x^2 + 6$ at $x = 4$.

$$6 \times 4^2 - 8 \times 4$$

.... 64 .... [2]

(iii) Find the coordinates of the two stationary points on the curve.

$$6x^2 - 8x = 0$$

$$2x(3x-4) = 0$$

$$x = 0 \text{ or } x = \frac{4}{3}$$

When $x = 0$, $y = 2 \times 0^3 - 4 \times 0^2 + 6 = 6$

When $x = \frac{4}{3}$, $y = 2 \times \left(\frac{4}{3}\right)^3 - 4 \times \left(\frac{4}{3}\right)^2 + 6 = \frac{98}{27}$

(...... 0 ...... , ... 6 ......) and (...... $\frac{4}{3}$ ...... , ... $\frac{98}{27}$ ......) [4]

8 **(a)** The table shows information about the mass, in kilograms, of each of 50 children.

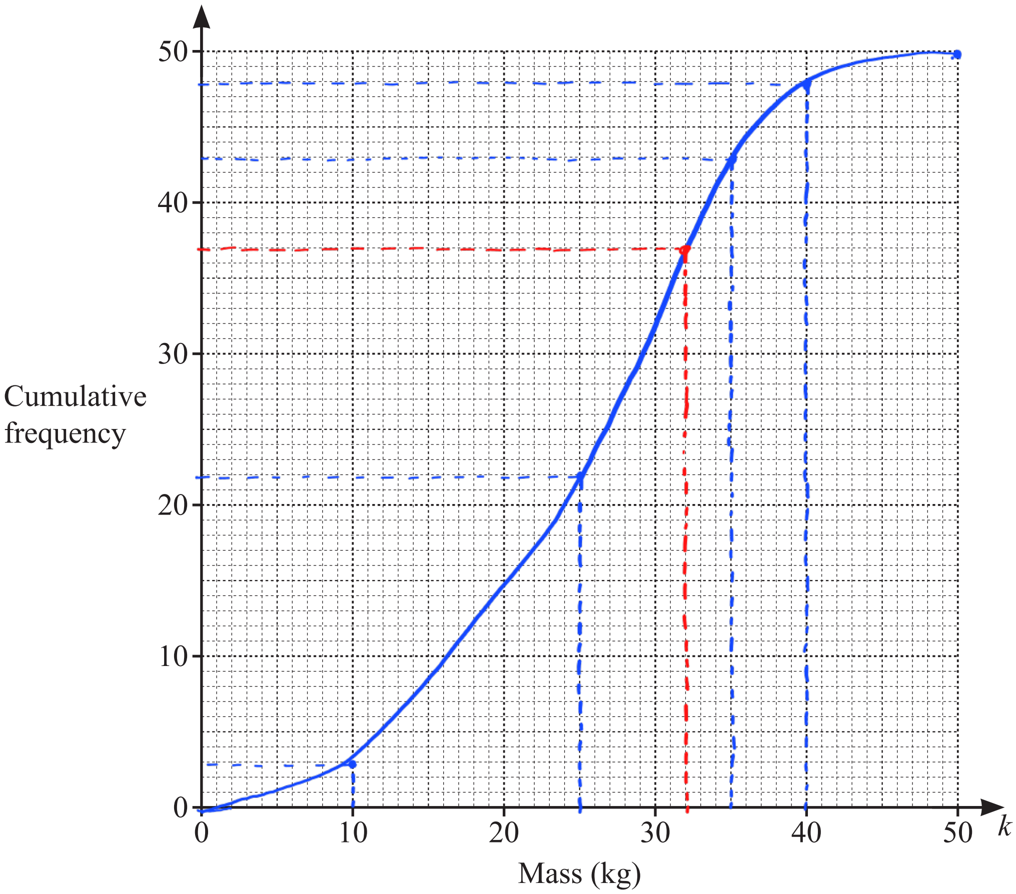
Mass (k kg)	$0 < k \leqslant 10$	$10 < k \leqslant 25$	$25 < k \leqslant 35$	$35 < k \leqslant 40$	$40 < k \leqslant 50$
Frequency	3	19	21	5	2

(i) Complete the cumulative frequency table.

Mass (k kg)	$k \leqslant 10$	$k \leqslant 25$	$k \leqslant 35$	$k \leqslant 40$	$k \leqslant 50$
Cumulative frequency	3	22	43	48	50

[2]

(ii) On the grid, draw a cumulative frequency diagram to show this information.

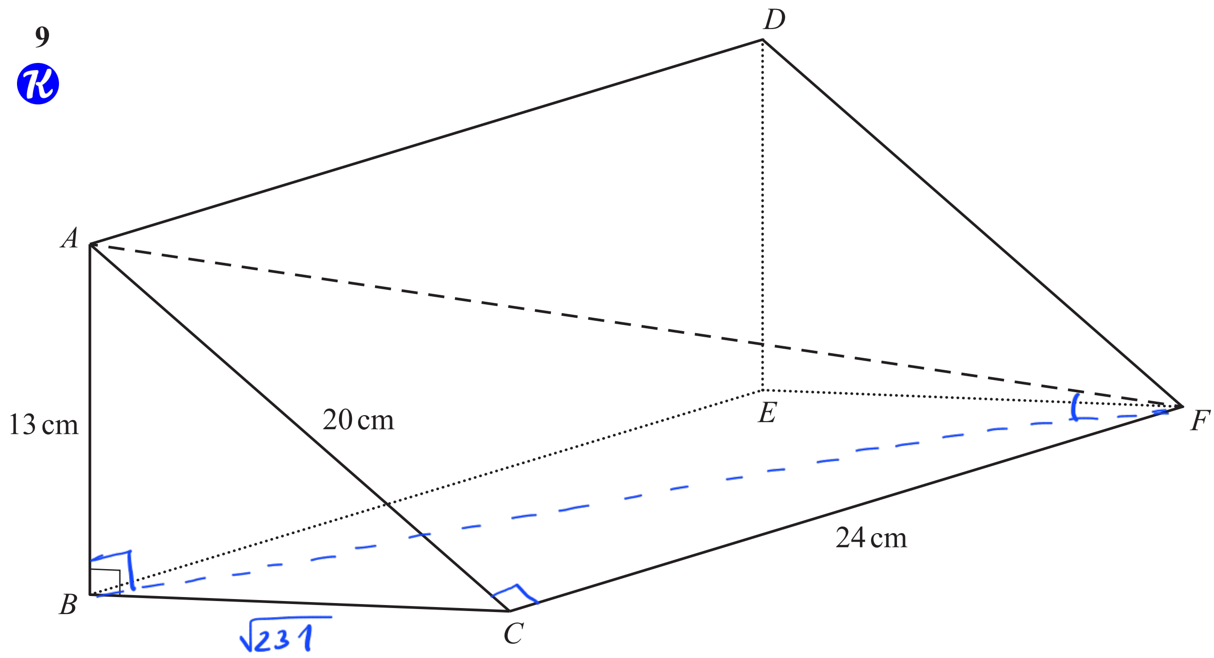


[3]

(iii) Use your diagram to find an estimate of the number of children with a mass of 32 kg or less.

.................37................. [1]

9



NOT TO SCALE

The diagram shows a prism, *ABCDEF*.
AB = 13 cm, *AC* = 20 cm, *CF* = 24 cm and angle *ABC* = 90°.

(a) Calculate the total surface area of the prism.

$$BC = \sqrt{20^2 - 13^2} = \sqrt{231}$$

$$2 \times \left(\frac{1}{2} \times 13 \times \sqrt{231}\right) + 20 \times 24 + 13 \times 24 + 24\sqrt{231}$$

$$\approx 1350$$

.......1350....... cm^2 [6]

(b) Calculate the volume of the prism.

$$V = \frac{1}{2} \times 13\sqrt{231} \times 24 \approx 2370$$

.......2370....... cm^3 [1]

(c) Calculate the angle that *AF* makes with the base *BCFE*.

$$BF = \sqrt{231 + 24^2} = \sqrt{807}$$

$$\tan \widehat{AFB} = \frac{13}{\sqrt{807}}$$

$$\widehat{AFB} \approx 24.6^\circ$$

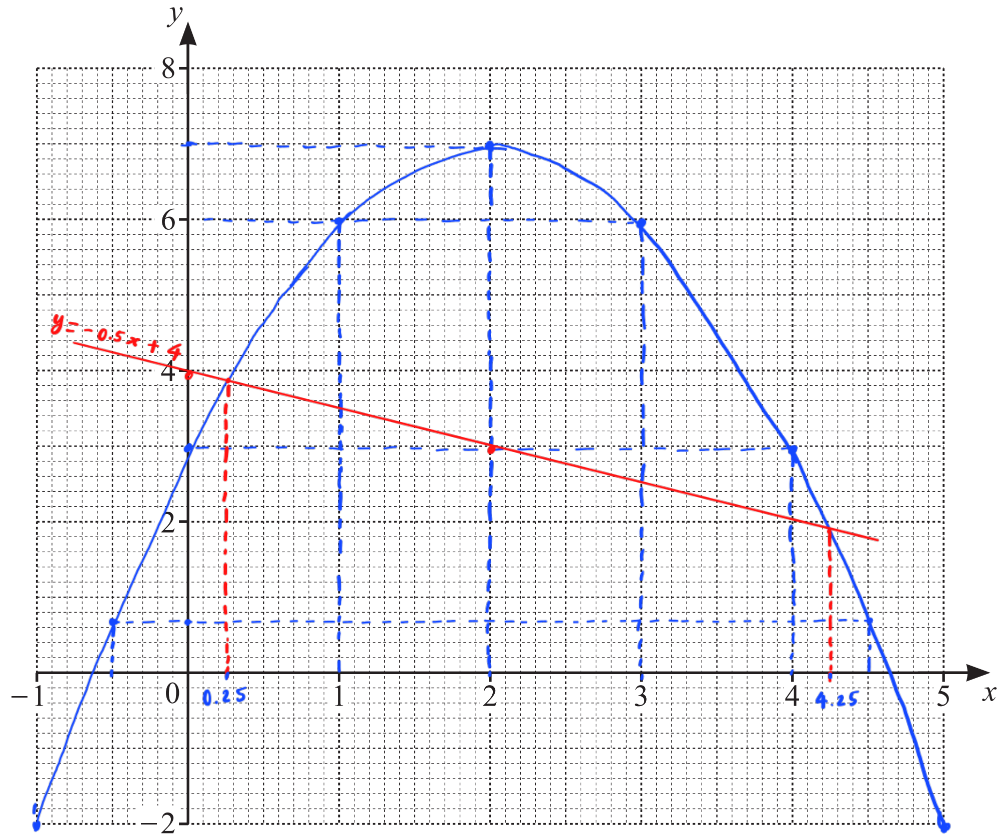
.......24.6°....... [4]

10 The table shows some values of $y = 3 + 4x - x^2$ for $-1 \leqslant x \leqslant 5$.

x	-1	-0.5	0	1	2	3	4	4.5	5
y	-2	0.75	3	6	7	6	3	0.75	-2

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 3 + 4x - x^2$ for $-1 \leqslant x \leqslant 5$.



[4]

(c) Write down an **integer** value of k for which the equation $3 + 4x - x^2 = k$ has no solutions.

.......8....... [1]

(d) By drawing a suitable straight line on the grid, solve the equation $-1 + \frac{9}{2}x - x^2 = 0$.

$(3 + 4x - x^2) - 4 + 0.5x = 0$

$3 + 4x - x^2 = -0.5x + 4$

$x =$ .....0.25..... or $x =$ .....4.25..... [4]

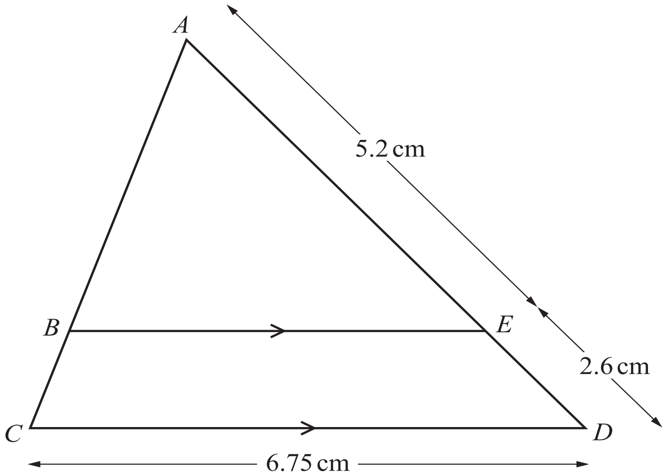
11 **(a)** Find the size of an exterior angle of a regular polygon with 18 sides.

interior angle $= \dfrac{(18-2)180^\circ}{18} = 160^\circ$

exterior angle $= 180^\circ - 160^\circ = 20^\circ$

.............20°............................. [2]

(b)



In triangle ACD, B lies on AC and E lies on AD such that BE is parallel to CD.
$AE = 5.2$ cm and $ED = 2.6$ cm.

Calculate BE.

$\Delta ABE \sim \Delta ACD$

$\dfrac{BE}{CD} = \dfrac{AE}{AD} \Rightarrow \dfrac{BE}{6.75} = \dfrac{5.2}{5.2+2.6}$

$BE = \dfrac{6.75 \times 5.2}{7.8}$

$BE =$ ....4.5............................ cm [2]

(c) Two solids are mathematically similar.
The smaller solid has height 2 cm and volume 32 cm^3.
The larger solid has volume 780 cm^3.

Calculate the height of the larger solid.

$\text{ratio}_{\text{volume}} = (\text{ratio}_{\text{height}})^3$

$\dfrac{780}{32} = \left(\dfrac{h}{2}\right)^3$

$\dfrac{h}{2} = \sqrt[3]{\dfrac{780}{32}} \approx 2.899$

$\Rightarrow h = 5.798$

..........5.80........................ cm [3]

12 $f(x) = 3 - 2x$ $g(x) = x^2 + 5$ $h(x) = x^3$

(a) Find f(−5).

$3 - 2(-5)$

.......13....... [1]

(b) Find ff(x).
Give your answer in its simplest form.

$3 - 2(3 - 2x)$

$= 3 - 6 + 4x$

.......$4x - 3$....... [2]

(c) Solve $g(x) = f(x) + 37$.

$x^2 + 5 = 3 - 2x + 37$

$x^2 + 2x - 35 = 0$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-35)}}{2 \times 1}$

$x =$ ....-7....... or $x =$ ....5....... [4]

(d) Find $f^{-1}(x)$.

$\times(-2) \longrightarrow +3$

$\div(-2) \longleftarrow -3$

$f^{-1}(x) =$ ....$\dfrac{x-3}{-2}$....... [2]

(e) Find $hf(x) + g(x)$.
Give your answer in its simplest form.

$(3 - 2x)^3 + (x^2 + 5)$

$(3 - 2x)(9 - 12x + 4x^2) + x^2 + 5$

$27 - 36x + 12x^2 - 18x + 24x^2 - 8x^3 + x^2 + 5$

.......$-8x^3 + 37x^2 - 54x + 32$....... [5]