1 **(a)** **(i)** Yasmin and Zak share an amount of money in the ratio 21 : 19.
Yasmin receives \$6 more than Zak.
Calculate the total amount of money shared by Yasmin and Zak.

$\frac{Y}{Z} = \frac{21}{19} \Rightarrow 19Y = 21Z$ (1)

$Y = Z + 6$ (2)

Sub (1) into (2): $19(Z+6) = 21Z$

$19Z + 114 = 21Z \Rightarrow Z = 57$

$Y = 57 + 6 = 63$

$Y + Z = 63 + 57 = 120$

\$ ....120........ [2]

(ii) In a sale, all prices are reduced by 15%.

(a) Yasmin buys a blouse with an original price of \$40.

Calculate the sale price of the blouse.

$40 - 40 \times 15\%$

\$ ....34........ [2]

(b) Zak buys a shirt with a sale price of \$29.75 .

Calculate the original price of the shirt.

$S - S \times 15\% = 29.75$

$0.85S = 29.75$

$S = 35$

\$ ....35........ [2]

(b) Xavier's salary increases by 2% each year.
In 2010, his salary was \$40 100.

(i) Calculate his salary in 2015.
Give your answer correct to the nearest dollar.

$$40\,100\left(1+\frac{2}{100}\right)^5 \approx 44\,273.64$$

\$ ......44 274...... [3]

(ii) In which year is Xavier's salary first greater than \$47 500?

$$40\,100\left(1+\frac{2}{100}\right)^t > 47\,500$$

use trial and error :

when $t = 8$: $40\,100\left(1+\frac{2}{100}\right)^8 = 46\,984 < 47\,500$

when $t = 9$: $40\,100\left(1+\frac{2}{100}\right)^9 = 47\,923 > 47\,500$

$\Rightarrow t = 9$

......2019...... [3]

(c) In January 2020, the population of a town was 5% **more** than its population in January 2018 (P_{18}).
In January 2021 (P_{21}), the population of this town was 2% **less** than its population in January 2020 (P_{20}).

Calculate the overall percentage increase in the population from January 2018 to January 2021.

$P_{20} = P_{18} + P_{18} \times 5\% = 1.05\,P_{18}$

$P_{21} = P_{20} - P_{20} \times 2\% = 0.98\,P_{20}$

$= 0.98\,(1.05\,P_{18}) = 1.029\,P_{18}$

$$\text{percentage increase} = \frac{P_{21} - P_{18}}{P_{18}} \times 100$$

$$= \frac{1.029\,P_{18} - P_{18}}{P_{18}} \times 100 = \frac{0.029\,P_{18}}{P_{18}} \times 100$$

......2.9...... % [2]

2 **(a)** $y = px^2 + t$

(i) Find the value of y when $p = 3$, $x = 2$ and $t = -13$.

$y = 3 \times 2^2 + (-13)$

$y =$ ......-1...... [2]

(ii) Rearrange the formula to write x in terms of p, t and y.

$y - t = px^2$

$\frac{y-t}{p} = x^2$

$x =$ ......$\pm\sqrt{\frac{y-t}{p}}$...... [3]

(b) **(i)** Factorise.

$15x^2 - 2x - 8$

$15x^2 - 12x + 10x - 8$

$3x(5x - 4) + 2(5x - 4)$

$(3x + 2)(5x - 4)$

......$(3x+2)(5x-4)$...... [2]

(ii) Solve the equation.

$15x^2 - 2x - 8 = 0$

$(3x + 2)(5x - 4) = 0$

$3x + 2 = 0$ or $5x - 4 = 0$

$x =$ ......$-\frac{2}{3}$...... or $x =$ ......$\frac{4}{5}$...... [1]

(c) Factorise completely.

$x^3 - 16xy^2$

$x(x^2 - 16y^2)$

$x[x^2 - (4y)^2]$

$x(x - 4y)(x + 4y)$

......$x(x-4y)(x+4y)$...... [3]

(d) Simplify. $\dfrac{2x-1-4ax+2a}{2x^2-x}$

$$\frac{2x-1-2a(2x-1)}{x(2x-1)}$$

$$=\frac{(2x-1)(1-2a)}{x(2x-1)}$$

$$=\frac{1-2a}{x}$$

$\dfrac{1-2a}{x}$ .... [4]

3 **(a)** Zoe's test scores last term were 6 7 7 7 8 9 9 10 10.

Find

(i) the range,

$10 - 6 = 4$

.......................4....................... [1]

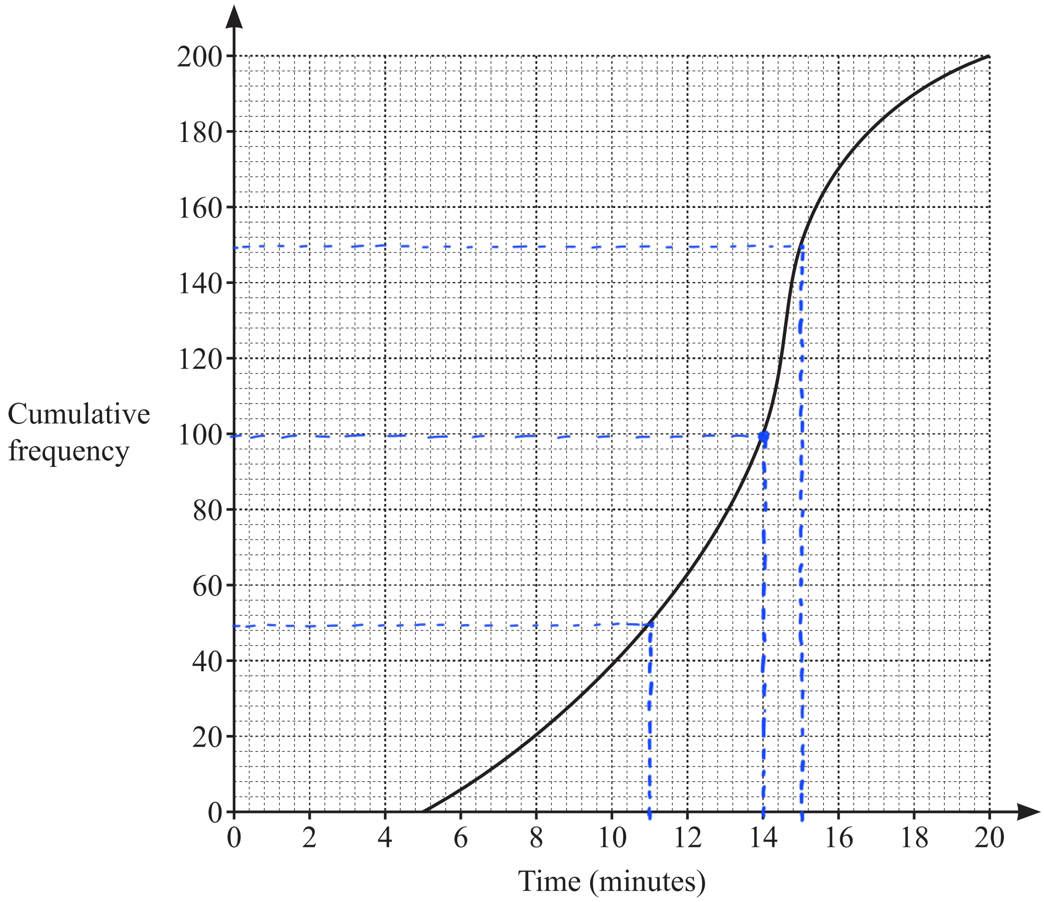
(ii) the mode,

.......................7....................... [1]

(iii) the median.

.......................8....................... [1]

(b) The cumulative frequency diagram shows information about the time taken by each of 200 students to solve a problem.



Use the diagram to find an estimate of

(i) the median,

.......................14....................... min [1]

(ii) the interquartile range.

$IQR = Q_3 - Q_1 = 15 - 11 = 4$

.......................4....................... min [2]

(c) The test scores of 200 students are shown in the table.

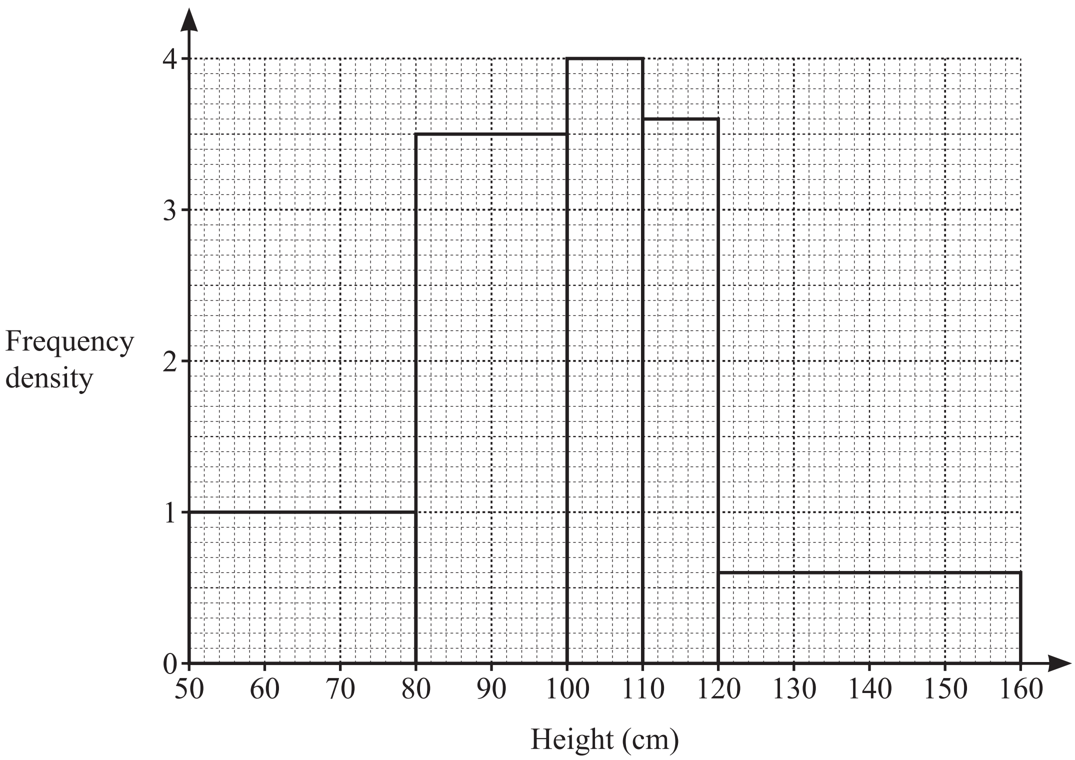
Score	5	6	7	8	9	10
Frequency	3	10	43	75	48	21

Calculate the mean.

$$\frac{(5 \times 3) + (6 \times 10) + (7 \times 43) + (8 \times 75) + (9 \times 48) + (10 \times 21)}{200}$$

............8.09............ [3]

(d) The height, in cm, of each of 200 plants is measured.
The histogram shows the results.



Calculate an estimate of the mean height.
You must show all your working.

Mid point	65	90	105	115	140
Class width	$50 \leqslant h < 80$	$80 \leqslant h < 100$	$100 \leqslant h < 110$	$110 \leqslant h < 120$	$120 \leqslant h < 160$
Frequency	30	70	40	36	24

$$\text{Mean} = \frac{(65 \times 30) + (90 \times 70) + (105 \times 40) + (115 \times 36) + (140 \times 24)}{200}$$

............99.75............ cm [6]

4 **(a)** A is the point (1, 5) and B is the point (3, 9).
M is the midpoint of AB.

(i) Find the coordinates of M.

$$M\left(\frac{1+3}{2}, \frac{5+9}{2}\right)$$

(.........2......... , ...7.........) [2]

ℓ

(ii) Find the equation of the line that is perpendicular to AB and passes through M.
Give your answer in the form $y = mx + c$.

$$m_{AB} = \frac{9-5}{3-1} = 2$$

$$m_\ell = -1 : 2 = -0.5$$

$\Rightarrow$ Equation of ℓ : $y - 7 = -0.5(x - 2)$

$y - 7 = -0.5x + 1$

$y =$ ......$-0.5x + 8$...... [4]

(b) The position vector of P is $\begin{pmatrix}-2\\3\end{pmatrix}$ and the position vector of Q is $\begin{pmatrix}-2\\5\end{pmatrix}$.

(i) Find the vector $\overrightarrow{PQ}$.

$$\begin{pmatrix}-2-(-2)\\5-3\end{pmatrix}$$

$\begin{pmatrix}0\\2\end{pmatrix}$ [2]

(ii) R is the point such that $\overrightarrow{PR} = 3\overrightarrow{PQ}$.

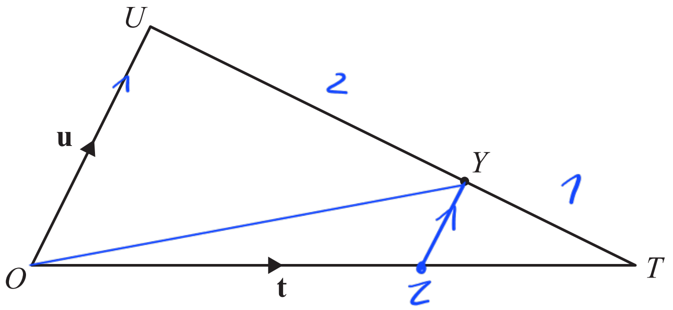
Find the position vector of R.

$$\overrightarrow{PR} = 3\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\6\end{pmatrix}$$

$$\begin{pmatrix}x_R-(-2)\\y_R-3\end{pmatrix} = \begin{pmatrix}0\\6\end{pmatrix}$$

$\begin{pmatrix}-2\\9\end{pmatrix}$ [2]

(c)



NOT TO SCALE

$\overrightarrow{OT} = \mathbf{t}$, $\overrightarrow{OU} = \mathbf{u}$ and $UY = 2YT$.

(i) Find $\overrightarrow{OY}$ in terms of **t** and **u**.
Give your answer in its simplest form.

$\overrightarrow{UT} = \overrightarrow{UO} + \overrightarrow{OT} = -u + t$

$\overrightarrow{UY} = \frac{2}{3}\overrightarrow{UT} = \frac{2}{3}(-u + t)$

$\overrightarrow{OY} = \overrightarrow{OU} + \overrightarrow{UY}$

$= u + \frac{2}{3}(-u + t)$

$\overrightarrow{OY} = \frac{1}{3}u + \frac{2}{3}t$ [2]

(ii) Z is on OT and YZ is parallel to UO.

Find $\overrightarrow{OZ}$ in terms of **t** and/or **u**.
Give your answer in its simplest form.

$\Delta TYZ \sim \Delta TUO$

$\frac{TZ}{TO} = \frac{TY}{TU} = \frac{1}{3}$

$\Rightarrow \frac{OZ}{OT} = \frac{2}{3}$

$\Rightarrow \overrightarrow{OZ} = \frac{2}{3}t$

$\overrightarrow{OZ} = \frac{2}{3}t$ [1]

5 Solve the simultaneous equations.

(a)

$$x + 2y = 13$$
$$-\quad x + 5y = 22$$

$2y - 5y = 13 - 22$

$-3y = -9$

$y = 3$

$\Rightarrow x + 2 \times 3 = 13$

$\Rightarrow x = 7$

$x =$ ....7....

$y =$ ....3.... [2]

(b)

$$y = 2 - x$$
$$y = x^2 + 2x + 2$$

$2 - x = x^2 + 2x + 2$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0$ or $x = -3$

When $x = 0$, $y = 2 - 0 = 2$

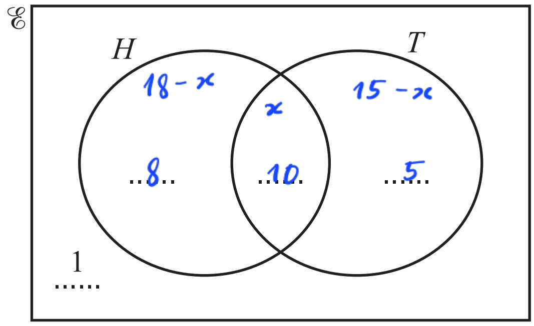
When $x = -3$, $y = 2 - (-3) = 5$

$x =$ ....0.... $y =$ ....2....

$x =$ ....-3.... $y =$ ....5.... [4]

6 In a class of 24 students, 18 students like homework (H), 15 students like tests (T) and 1 student does not like homework and does not like tests.

(a) Complete the Venn diagram to show this information.



$(18 - x) + x + (15 - x) + 1 = 24$

$34 - x = 24$

$x = 10$

[2]

(b) Write down the number of students who like both homework and tests.

.... 10 .... [1]

(c) Find $n(H' \cap T)$.

.... 5 .... [1]

(d) A student is picked at random from the class.

Write down the probability that this student likes tests but does not like homework.

.... $\frac{5}{24}$ .... [1]

(e) Two students are picked at random from the class.

Find the probability that both students do not like homework and do not like tests.

.... 0 .... [1]

(f) Two of the students who like homework are picked at random.

Find the probability that both students also like tests.

$\frac{10}{18} \times \frac{9}{17}$

.... $\frac{5}{17}$ .... [3]

7 (a)

Write down the inequality in x shown by the number line.

$-2 < x \leq 1$ .... [2]

(b) (i) Write $x^2 + 4x + 1$ in the form $(x + p)^2 + q$.

$(x^2 + 2x2 + 2^2) - 2^2 + 1$

$(x + 2)^2 - 3$

$(x+2)^2 - 3$ .... [2]

(ii) Use your answer to **part (b)(i)** to solve the equation $x^2 + 4x + 1 = 0$.

$(x + 2)^2 - 3 = 0$

$(x + 2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = \pm\sqrt{3} - 2$

$x \approx -3.73$ or $x \approx -0.268$

$x =$ ...... -3.73 ...... or $x =$ ...... -0.268 ...... [2]

(iii) Use your answer to **part (b)(i)** to write down the coordinates of the minimum point on the graph of $y = x^2 + 4x + 1$.

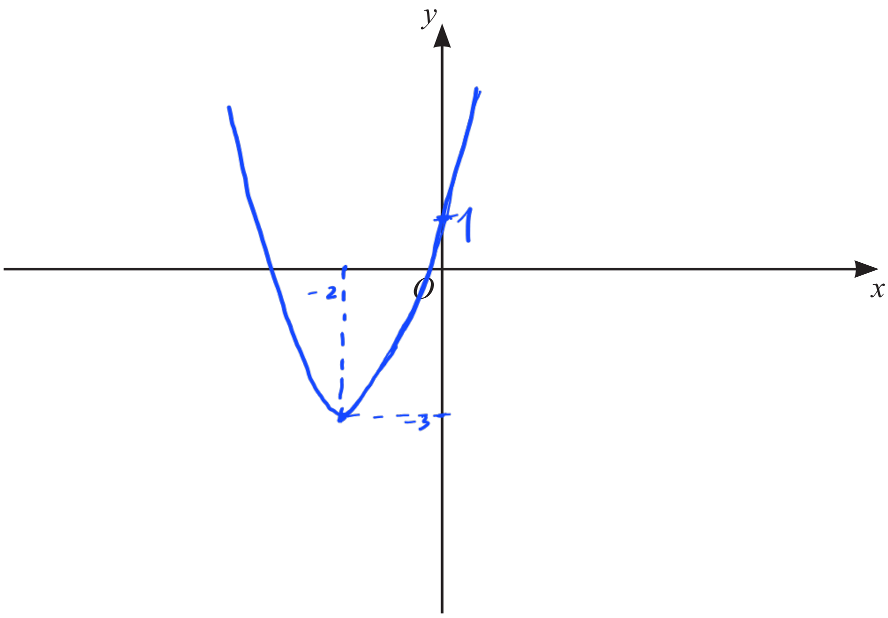
$(x+2)^2 - 3 \geqslant -3$

$\Rightarrow y_{min} = -3$ when $x + 2 = 0$

$x = -2$

(......-2...... , ...-3......) [2]

(iv) On the diagram, sketch the graph of $y = x^2 + 4x + 1$.



[2]

8 (a) A solid cuboid measures 20 cm by 12 cm by 5 cm.

(i) Calculate the volume of the cuboid.

$20 \times 12 \times 5$

.... 1200 .... cm^3 [1]

(ii) (a) Calculate the total surface area of the cuboid.

$2(20 \times 12) + 2(12 \times 5) + 2(20 \times 5)$

.... 800 .... cm^2 [3]

(b) The surface of the cuboid is painted.
The cost of the paint used is \$1.52 .

Find the cost to paint $1\,cm^2$ of the cuboid.
Give your answer in cents.

$800\ cm^2$ cost \$1.52

$\Rightarrow 1\ cm^2$ cost $\frac{1.52}{800} = \$0.0019$

.... 0.19 .... cents [1]

(b) A solid metal cylinder with radius x and height $\frac{9x}{2}$ is melted.
All the metal is used to make a sphere with radius r.

Find r in terms of x.

$V_{cylinder} = \pi x^2 \times \frac{9x}{2} = \frac{9\pi x^3}{2}$

$V_{sphere} = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{9\pi x^3}{2} = \frac{4}{3}\pi r^3$

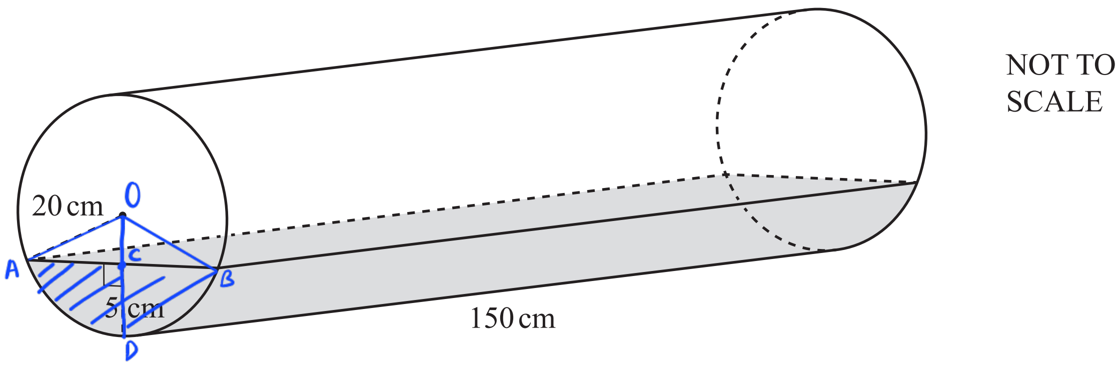
$x^3 = \frac{8}{27} r^3 = \left(\frac{2}{3} r\right)^3$

$\Rightarrow x = \frac{2}{3} r$

$r = \frac{3}{2} x$

$r =$ .... $\frac{3}{2}x$ .... [3]

(c)



The diagram shows a cylinder of length 150 cm on horizontal ground.
The cylinder has radius 20 cm.
The cylinder contains water to a depth of 5 cm, as shown in the diagram.

Calculate the volume of water in the cylinder.
Give your answer in litres.

$OC = OD - CD = 20 - 5 = 15$

$\cos \widehat{AOC} = \frac{OC}{OA} = \frac{15}{20}$

$\Rightarrow \widehat{AOC} \simeq 41.41^\circ$

$\Rightarrow \widehat{AOB} = 2\widehat{AOC} = 82.82^\circ = 1.4455$ radian

$A_{\text{sector } AOB} = \frac{1}{2} \times 20^2 \times 1.4455 = 289.1$

$A_{\Delta AOB} = \frac{1}{2} OA . OB . \sin \widehat{AOB}$

$= \frac{1}{2} \times 20 \times 20 \times \sin 82.82^\circ$

$= 198.43$

$A_{\text{shaded } ABD} = 289.1 - 198.43 = 90.67$

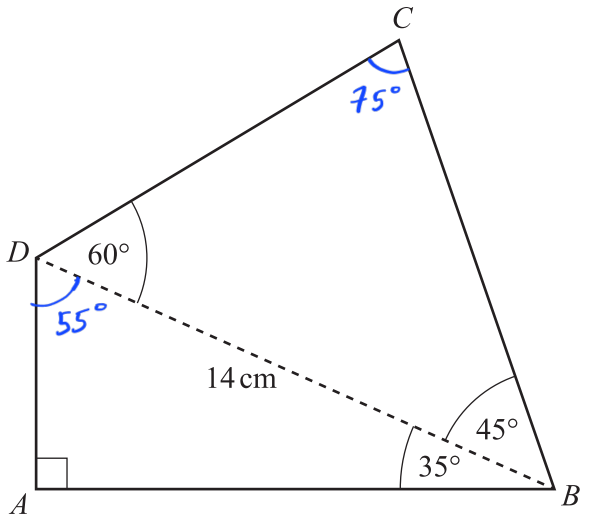
$V_{\text{water}} = 90.67 \times 150$

$= 13600.5 \text{ cm}^3$

$= 13.6 \ \ell$

..........13.6.......... litres [7]

9 (a)



NOT TO SCALE

Calculate the perimeter of the quadrilateral $ABCD$.

ΔBCD : $\dfrac{14}{\sin 75°} = \dfrac{BC}{\sin 60°} = \dfrac{CD}{\sin 45°}$

$\Rightarrow \begin{cases} BC = \dfrac{14 \sin 60°}{\sin 75°} \approx 12.552 \\ CD = \dfrac{14 \sin 45°}{\sin 75°} \approx 10.249 \end{cases}$

ΔABD : $\sin 35° = \dfrac{AD}{14} \Rightarrow AD = 14 \sin 35°$

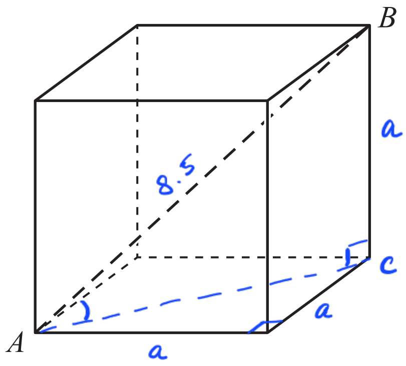
$\cos 35° = \dfrac{AB}{14} \Rightarrow AB = 14 \cos 35°$

$\text{Perimeter}_{ABCD} = 12.552 + 10.249 + 14 \sin 35° + 14 \cos 35°$

≈ 42.3

..........42.3.......... cm [7]

(b)



NOT TO SCALE

The diagram shows a cube.
The length of the diagonal *AB* is 8.5 cm.

(i) Calculate the length of an edge of the cube.

a

$AC^2 = a^2 + a^2 = 2a^2$

$AB^2 = AC^2 + BC^2$

$8.5^2 = 2a^2 + a^2$

$72.24 = 3a^2$

$a^2 = 24.08$

$a = \sqrt{24.08} \approx 4.91$

........4.91........ cm [3]

(ii) Calculate the angle between *AB* and the base of the cube.

$\sin \widehat{BAC} = \frac{\sqrt{24.08}}{8.5}$

$\Rightarrow \widehat{BAC} = \sin^{-1}\left(\frac{\sqrt{24.08}}{8.5}\right)$

$\Rightarrow \widehat{BAC} \approx 35.3°$

........35.3°........ [3]

10 $f(x) = 3x - 2$ $g(x) = 5x - 7$ $h(x) = x^2 + x$ $j(x) = 3^x$

(a) Find

(i) f(2),

$3 \times 2 - 2$

4 [1]

(ii) g(2),

$5 \times 2 - 7$

3 [1]

(iii) gf(2).

$gf(2) = g(4) = 5 \times 4 - 7$

13 [1]

(b) Find $f^{-1}(x)$.

$f: \times 3 \longrightarrow -2$

$\div 3 \longleftarrow +2 : f^{-1}$

$f^{-1}(x) = \dfrac{x+2}{3}$ [2]

(c) Find $hf(x)$, giving your answer in the form $ax^2 + bx + c$.

$(3x - 2)^2 + 3x - 2$

$= 9x^2 - 12x + 4 + 3x - 2$

$9x^2 - 9x + 2$ [3]

(d) Find the derivative of $h(x)$.

$h'(x) = 2x + 1$

$2x + 1$ [1]

(e) **(i)** Find x when $j^{-1}(x) = 4$.

$y = 3^x$

Swap: $x = 3^y = 3^{j^{-1}(x)} = 3^4$

$x =$ 81 [1]

(ii) Simplify $j^{-1}j(x)$.

x [1]

11 **(a)** These are the first four terms of a sequence.

$$11 \quad 7 \quad 3 \quad -1 \quad -5$$

$$-4 \quad -4 \quad -4 \quad -4$$

(i) Write down the next term.

.... -5 .... [1]

(ii) Write down the term to term rule for this sequence.

.... Subtract 4 .... [1]

(iii) Find the nth term of this sequence.

.... $15 - 4n$ .... [2]

(b) The nth term of a different sequence is $\frac{2n}{n+1}$.

(i) Find the difference between the 5th term and the 6th term of this sequence.
Give your answer as a fraction.

5^{th} term $= \frac{2 \times 5}{5+1} = \frac{5}{3}$

6^{th} term $= \frac{2 \times 6}{6+1} = \frac{12}{7}$

Difference $= \frac{12}{7} - \frac{5}{3} = \frac{1}{21}$

.... $\frac{1}{21}$ .... [2]

(ii) Is $\frac{3}{4}$ a term in this sequence?
Show how you decide.

$$\frac{2n}{n+1} = \frac{3}{4}$$

$$(2n) \times 4 = 3(n+1)$$

$$8n = 3n + 3$$

$$5n = 3$$

$n = \frac{3}{5}$ is not a natural number

$\Rightarrow \frac{3}{4}$ is not a term in this sequence

[3]