1

Painter	Plumber	Electrician
$35 per hour	Fixed charge $40 plus $26.50 per hour	$48 per hour for the first 2 hours then $32 per hour

These are the rates charged by a painter, a plumber and an electrician who do some work for Mr Sharma.

(a) The painter works for 7 hours.

Calculate the amount Mr Sharma pays the painter.

$7 \times 35 = 245$

$ ...245... [1]

(b) Mr Sharma pays the plumber $252.

Calculate how many hours the plumber works.

$40 + 26.50p = 252$

$26.50p = 212$

$p = 8$

.....8..... hours [2]

(c) Mr Sharma pays the electrician $224.

Calculate how many hours the electrician works.

$48 \times 2 + 32(e - 2) = 224$

$96 + 32e - 64 = 224$

$32e = 192$

$e = 6$

.....6..... hours [2]

(d) Write down the ratio of the amount Mr Sharma pays to the painter, the plumber and the electrician. Give your answer in its lowest terms.

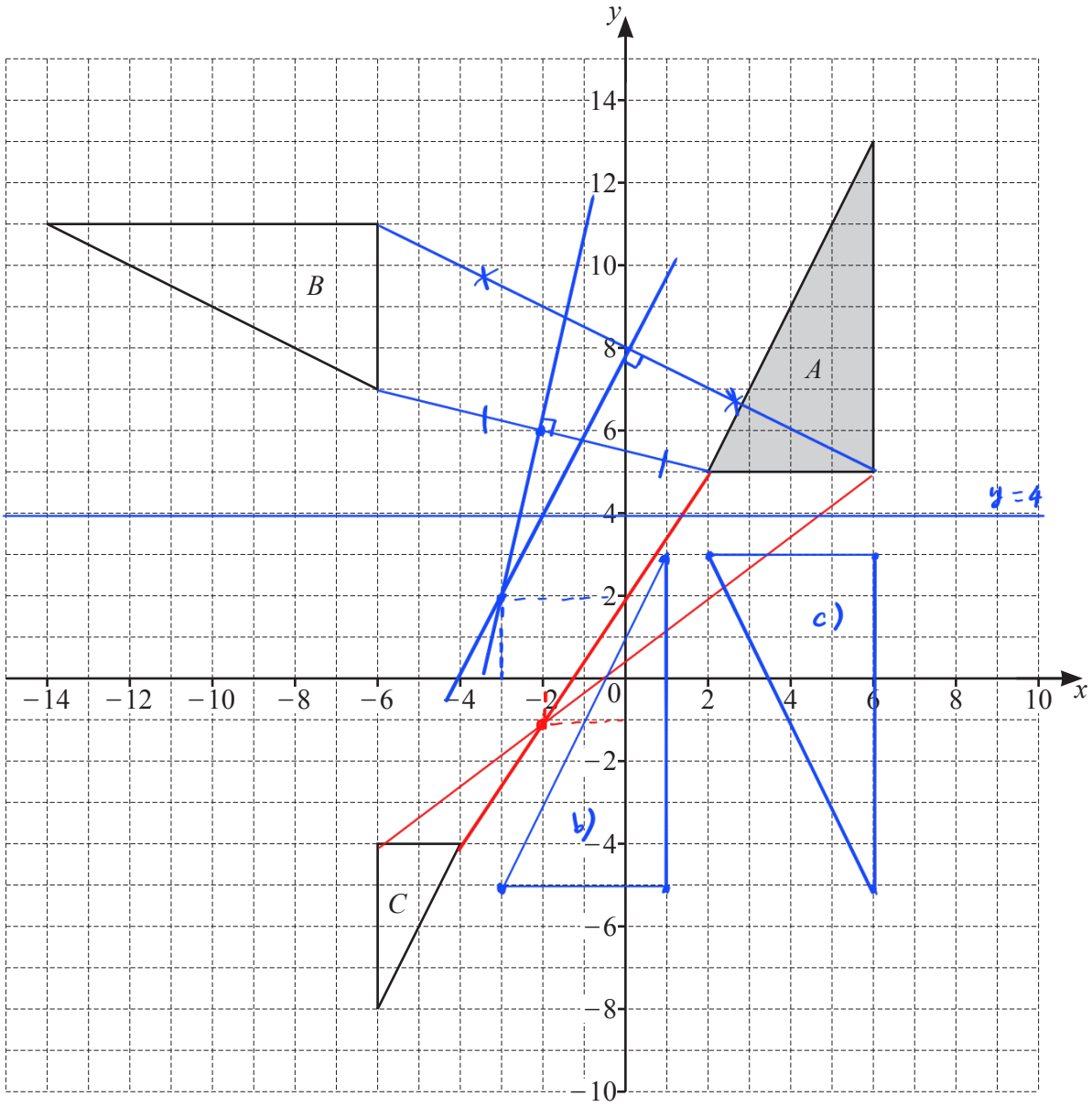
$245 : 252 : 224$

$HCF(245, 252, 224) = 7$

$\Rightarrow$ divide all by 7

painter : plumber : electrician = ...35..... : .....36..... : .....32..... [2]

2



(a) Describe fully the **single** transformation that maps

(i) triangle A onto triangle B,

Rotation, center (−3, 2), anticlockwise 90°

..... [3]

(ii) triangle A onto triangle C.

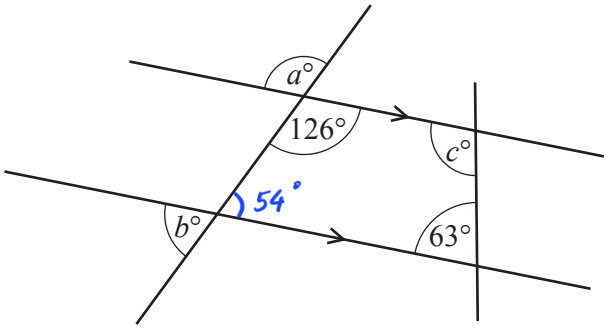
Enlargement, center (−2, −1), scale factor $-\frac{1}{2}$

..... [3]

(b) Draw the image of triangle A after a translation by the vector $\begin{pmatrix} -5 \\ -10 \end{pmatrix}$. [2]

(c) Draw the image of triangle A after a reflection in the line $y = 4$. [2]

3 **(a)**



NOT TO SCALE

The diagram shows two straight lines intersecting two parallel lines.

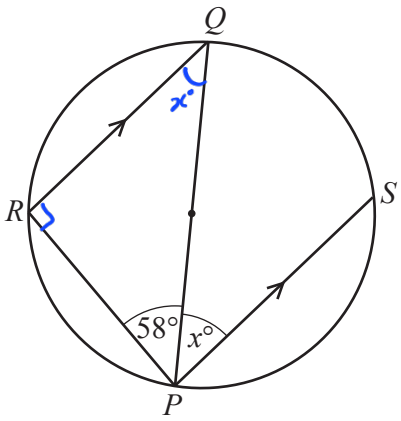
Find the values of a, b and c.

$a =$ 126°

$b =$ 54°

$c =$ 117° [3]

(b)



NOT TO SCALE

Points R and S lie on a circle with diameter PQ.
RQ is parallel to PS.
Angle $RPQ = 58°$.

Find the value of x, giving a geometrical reason for each stage of your working.

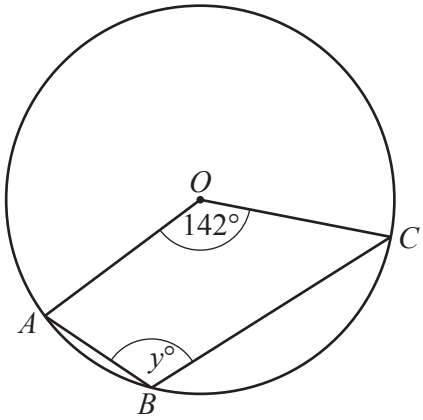
$\hat{PRQ} = 90°$ (angle in a semicircle)

$\hat{RQP} = 180° - 90° - 58° = 32°$ (sum of angles in triangle = 180°)

$x° = \hat{RQP} = 32°$ (alternate angles)

$x =$ 32 [3]

(c)



NOT TO SCALE

Points A, B and C lie on a circle, centre O.
Angle $AOC = 142°$.

Find the value of y.

$AOC_{reflex} = 360° - 142° = 218°$

$y° = \frac{1}{2} \times 218° = 109°$

$y =$ 109 [2]

4 **(a)** A shop gives each of 1000 people a voucher.
28 people use their voucher.
The shop now gives each of 16 500 people a voucher.

Calculate how many of these 16 500 people are expected to use their voucher.

1000 vouchers : 28 people use
16 500 vouchers : $\frac{16500 \times 28}{1000}$

.....462..... [1]

(b) In a class activity, all the 15 students wear hats.
7 students wear red hats, 6 students wear green hats and 2 students wear white hats.

(i) One of these students is picked at random.

Find the probability that this student wears a red hat.

.....$\frac{7}{15}$..... [1]

(ii) Two of the 15 students are picked at random.

Show that the probability that these two students wear hats of the same colour is $\frac{37}{105}$.

$P(R \cap R) + P(G \cap G) + P(W \cap W)$

$= \frac{7}{15} \times \frac{6}{14} + \frac{6}{15} \times \frac{5}{14} + \frac{2}{15} \times \frac{1}{14}$

$= \frac{37}{105}$

[3]

(iii) Three of the 15 students are picked at random.

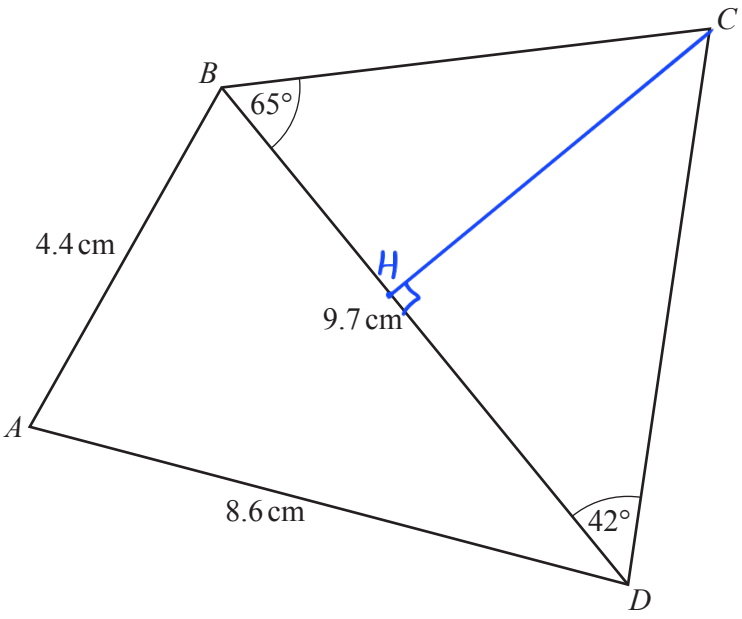
Find the probability that at least two of these three students wear red hats.

$3 \times P(R \cap R \cap G) + 3 \times P(R \cap R \cap W) + P(R \cap R \cap R)$

$3 \times \left(\frac{7}{15} \times \frac{6}{14} \times \frac{6}{13}\right) + 3 \times \left(\frac{7}{15} \times \frac{6}{14} \times \frac{2}{13}\right) + \left(\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13}\right)$

.....$\frac{29}{65}$..... [4]

5



NOT TO SCALE

(a) Calculate angle ADB.

$4.4^2 = 9.7^2 + 8.6^2 - 2 \times 9.7 \times 8.6 \cos \widehat{ADB}$

$-148.69 = -166.84 \cos \widehat{ADB}$

$\cos \widehat{ADB} \approx 0.8912$

Angle ADB = ... 27.0° ..... [3]

(b) Calculate DC.

$\widehat{BCD} = 180° - 42° - 65° = 73°$

$\frac{DC}{\sin 65°} = \frac{9.7}{\sin 73°}$

$\Rightarrow DC = \frac{9.7 \times \sin 65°}{\sin 73°} \approx 9.1929$

DC = ... 9.19 ..... cm [4]

(c) Calculate the shortest distance from C to BD.

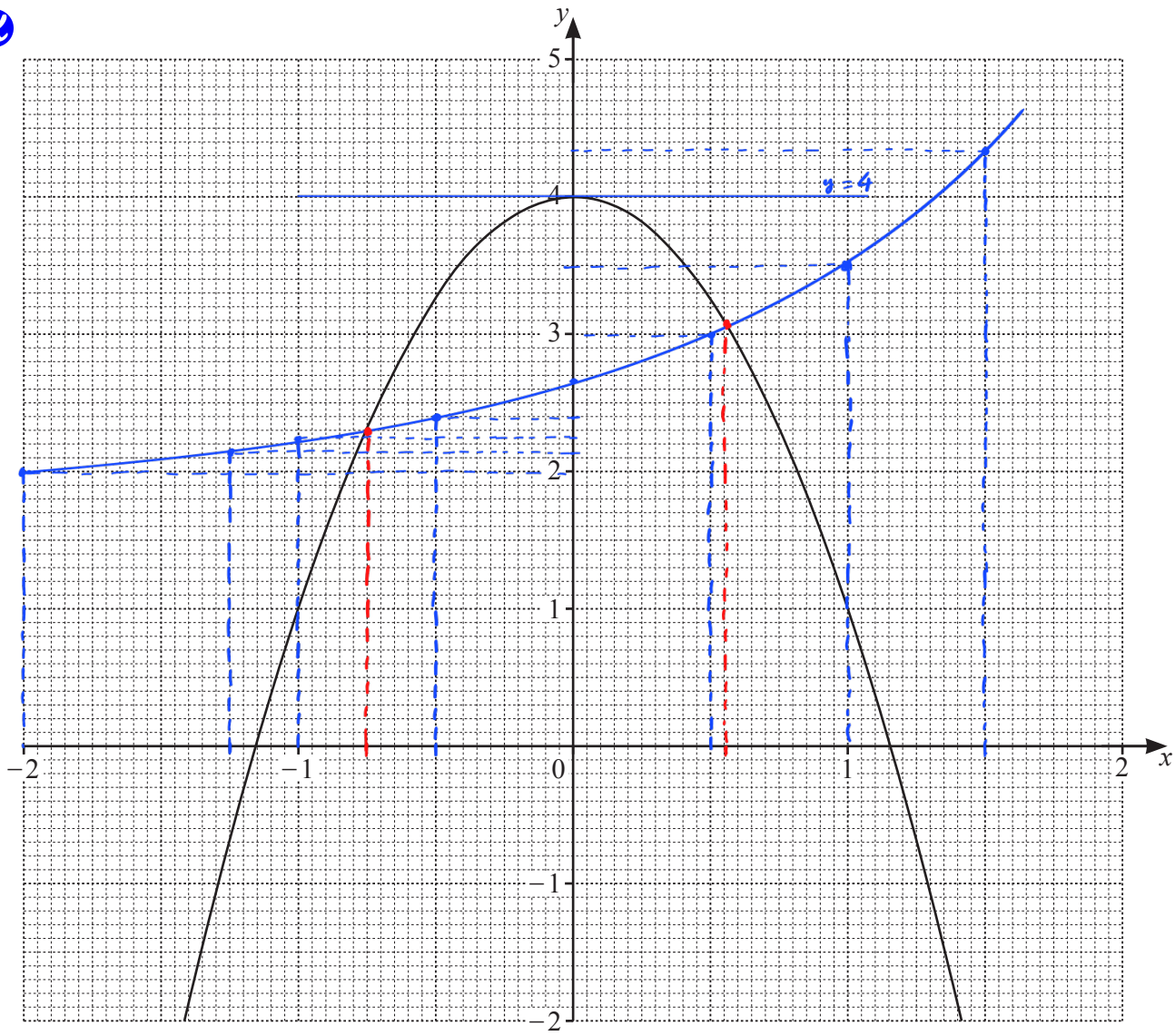
$\sin 42° = \frac{CH}{CD}$

$\Rightarrow CH = CD \sin 42°$

$CH \approx 9.1929 \sin 42°$

..... 6.15 ..... cm [3]

6



(a) The grid shows the graph of $y = a + bx^2$.

The graph passes through the points with coordinates (0, 4) and (1, 1).

(i) Find the value of a and the value of b.

(4, 0) => $4 = a + b \times 0^2$ => $a = 4$

(1, 1) => $1 = a + b \times 1^2$ => $b = 1 - 4 = -3$

$a =$ ....4....

$b =$ ....−3.... [2]

(ii) Write down the equation of the tangent to the graph at (0, 4).

.....$y = 4$..... [1]

(iii) The equation of the tangent to the graph at $x = -1$ is $y = 6x + 7$.

Find the equation of the tangent to the graph at $x = 1$.

2 tangents reflect over y-axis

.....$-6x + 7$..... [2]

(b) The table shows some values for $y = 1 + \frac{5}{3-x}$ for $-2 \leqslant x \leqslant 1.5$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	2	2.11	2.25	2.43	2.67	3	3.5	4.33

(i) Complete the table. [3]

(ii) On the grid, draw the graph of $y = 1 + \frac{5}{3-x}$ for $-2 \leqslant x \leqslant 1.5$. [4]

(c) (i) Write down the values of x where the two graphs intersect.

$x =$ .....-0.75..... or $x =$ .....0.55..... [2]

(ii) The answers to **part(c)(i)** are two solutions of a cubic equation in terms of x.

Find this equation in the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c and d are integers.

$$1 + \frac{5}{3-x} = 4 - 3x^2$$

$$\frac{5}{3-x} = 3 - 3x^2$$

$$5 = (3 - 3x^2)(3 - x)$$

$$5 = 9 - 9x^2 - 3x + 3x^3$$

.....$3x^3 - 9x^2 - 3x + 4 = 0$..... [4]

7 **(b)** The table shows information about the height, h cm, of each of 50 plants.

Mid value	10	25	32	37	50
Height (h cm)	$0 < h \leqslant 20$	$20 < h \leqslant 30$	$30 < h \leqslant 34$	$34 < h \leqslant 40$	$40 < h \leqslant 60$
Frequency	4	9	20	15	2

Calculate an estimate of the mean.

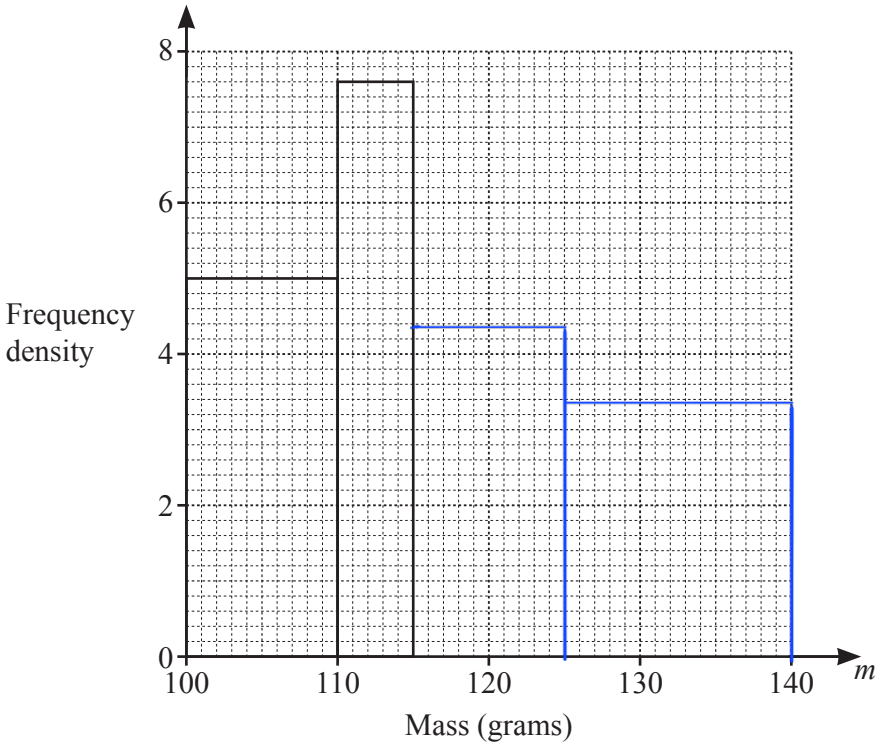
$$\frac{(10 \times 4) + (25 \times 9) + (32 \times 20) + (37 \times 15) + (50 \times 2)}{50}$$

..... 31.2 ..... cm [4]

(c) Some apples are weighed and the mass, m grams, of each apple is recorded.
The table shows the results.

Class width	10	5	10	15
Mass (m grams)	$100 < m \leqslant 110$	$110 < m \leqslant 115$	$115 < m \leqslant 125$	$125 < m \leqslant 140$
Frequency	50	x	44	51

The histogram shows some of the information from the table. 4.4 3.4



(i) Work out the value of x.

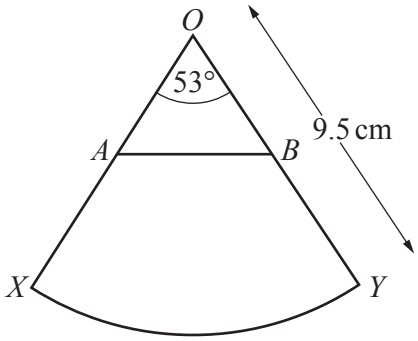
$$\frac{x}{5} = 7.6$$

$x =$ ..... 38 ..... [1]

(ii) Complete the histogram.

[2]

8 (a)



NOT TO SCALE

The diagram shows a sector OXY of a circle with centre O and radius 9.5 cm.
The sector angle is 53°.
A lies on OX, B lies on OY and $OA = OB$.

(i) Show that the area of the sector is 41.7 cm^2, correct to 1 decimal place.

$53° = 53 \times \frac{\pi}{180}$ radian

$A_{sector} = \frac{1}{2} \times 9.5^2 \times 53 \times \frac{\pi}{180} \approx 41.742 \approx 41.7$

[2]

(ii) The area of triangle OAB is $\frac{1}{3}$ of the area of sector OXY.

Calculate OA.

$A_{\Delta OAB} = \frac{1}{3} A_{sector\,OXY}$

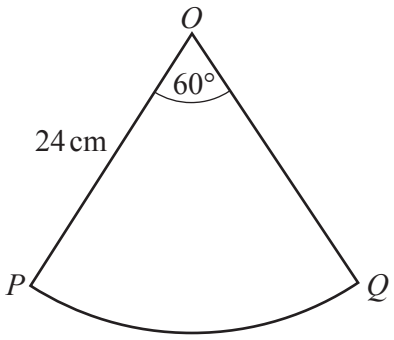
$\frac{1}{2} OA \times OB \sin 53° = \frac{1}{3} \times 41.742$

$OA^2 = 34.844$

$OA \approx 5.90$

$OA =$ 5.90 cm [4]

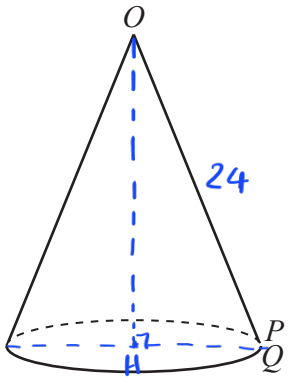
(b)



NOT TO SCALE

The diagram shows a sector OPQ of a circle with centre O and radius 24 cm. The sector angle is 60°.

A cone is made from this sector by joining OP to OQ.



NOT TO SCALE

Calculate the volume of the cone.

$60° = \frac{\pi}{3}$ radian

Arc length PQ = Perimeter of the cone's base

$24 \times \frac{\pi}{3} = 2\pi HQ$

$8 = 2 \times HQ$

$HQ = 4$

$OH = \sqrt{24^2 - 4^2} = 4\sqrt{35}$

$V_{cone} = \frac{1}{3} \pi \times 4^2 \times 4\sqrt{35} \approx 396$

.....396..... cm^3 [6]

9 **(a)** Factorise.

(i) $5am + 10ap - bm - 2bp$

$(5am + 10ap) - (bm + 2bp)$

$5a(m + 2p) - b(m + 2p)$

$(5a - b)(m + 2p)$ [2]

(ii) $15(k+g)^2 - 20(k+g)$

$5(k+g)[3(k+g) - 4]$

$5(k+g)(3k+3g-4)$ [2]

(iii) $4x^2 - y^4$

$(2x)^2 - (y^2)^2$

$(2x - y^2)(2x + y^2)$ [2]

(b) Expand and simplify.

$(x-3)(x+1)(3x-4)$

$(x^2 - 3x + x - 3)(3x - 4)$

$(x^2 - 2x - 3)(3x - 4)$

$3x^3 - 6x^2 - 9x - 4x^2 + 8x + 12$

$3x^3 - 10x^2 - x + 12$ [3]

(c) $(x+a)^2 = x^2 + 22x + b$

Find the value of a and the value of b.

$x^2 + 2ax + a^2 = x^2 + 22x + b$

$\Rightarrow \begin{cases} 2a = 22 \\ a^2 = b \end{cases}$

$\Rightarrow \begin{cases} a = 11 \\ 11^2 = b \end{cases}$

$a =$ 11

$b =$ 121 [2]

10 **(a)** A box is a cuboid with length 45 cm, width 30 cm and height 42 cm.
The box is completely filled with 90.72 kg of sand.

Calculate the density of this sand in kg/m^3.
[Density = mass ÷ volume]

$V_{box} = 45 \times 30 \times 42 = 56700\ cm^3 = 0.0567\ m^3$

$density = \frac{90.72}{0.0567} = 1600$

.....1600..... kg/m^3 [3]

(b) A bag contains 15000 cm^3 of sand.
Some of this sand is used to completely fill a hole in the shape of a cylinder.
The hole is 30 cm deep and has radius 10 cm.

Calculate the percentage of the sand from the bag that is used.

$V_{cylinder} = \pi \times 10^2 \times 30 = 3000\pi$

$\frac{3000\pi}{15000} \times 100 \approx 62.8\ \%$

.....62.8..... % [3]

(c) Sand costs $98.90 per tonne.
This cost includes a tax of 15%.

Calculate the amount of tax paid per tonne of sand.

Put s the cost of sand before tax

$s + 15\%\ s = 98.90$

$\Rightarrow 1.15\ s = 98.90$

$s = 86$

$\Rightarrow tax = 86 \times 15\% = 12.9$

$ .....12.9..... [3]

(d) Raj buys some sand for 3540 rupees.

Calculate the cost in dollars when the exchange rate is $1 = 70.8 rupees.

$\frac{3540 \times 1}{70.8} = 50$

$ ...50..... [2]

11 Gaya spends \$48 to buy books that cost \$$x$ each.

(a) Write down an expression, in terms of x, for the number of books Gaya buys.

$\frac{48}{x}$ [1]

(b) Myra spends \$60 to buy books that cost \$$(x+2)$ each.
Gaya buys 4 more books than Myra.

Show that $x^2+5x-24=0$.

The number of books Myra buys: $\frac{60}{x+2}$

$$\frac{48}{x} - \frac{60}{x+2} = 4$$

$$\frac{48(x+2) - 60x}{x(x+2)} = 4$$

$$48x + 96 - 60x = 4(x^2+2x)$$

$$96 - 12x = 4x^2 + 8x$$

$$4x^2 + 20x - 96 = 0$$

$$x^2 + 5x - 24 = 0$$

[4]

(c) Solve by factorisation.

$x^2+5x-24=0$

$$x^2 - 3x + 8x - 24 = 0$$

$$x(x-3) + 8(x-3) = 0$$

$$(x+8)(x-3) = 0$$

$$x+8 = 0 \quad \text{or} \quad x-3 = 0$$

$$x = -8 \quad \text{or} \quad x = 3$$

$x =$ ....−8.... or $x =$ ....3.... [3]

(d) Find the number of books Myra buys.

x cannot be negative $\Rightarrow x = 3$

$\Rightarrow$ Myra buys $\frac{60}{3+2} = 12$ books

....12.... [1]

12 **(a)** Find the gradient of the curve $y = 2x^3 - 7x + 4$ when $x = -2$.

$\frac{dy}{dx} = 6x^2 - 7$

gradient $= 6(-2)^2 - 7 = 17$

.....17..... [3]

(b) A is the point (7, 2) and B is the point (−5, 8).

(i) Calculate the length of AB.

$AB = \sqrt{(-5-7)^2 + (8-2)^2} = 6\sqrt{5} \approx 13.4$

.....13.4..... [3]

(ii) Find the equation of the line that is perpendicular to AB and that passes through the point (−1, 3).
Give your answer in the form $y = mx + c$.

$m_{AB} = \frac{8-2}{-5-7} = -0.5$

$m_\ell = -1 : (-0.5) = 2$

=> Equation of ℓ: $y - 3 = 2[x - (-1)]$

$y - 3 = 2(x + 1)$

$y - 3 = 2x + 2$

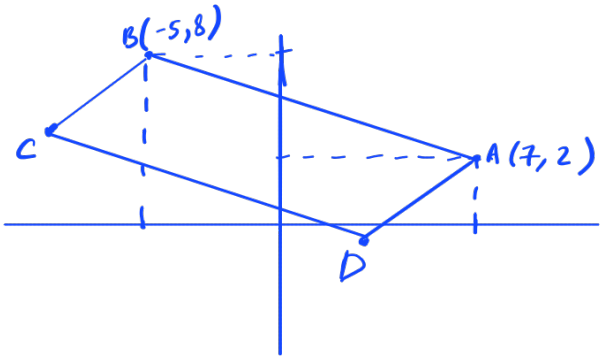
$y = 2x + 5$

$y =$ .....2x + 5..... [4]

(iii) AB is one side of the parallelogram $ABCD$ and

- $\overrightarrow{BC} = \begin{pmatrix} -a \\ -b \end{pmatrix}$ where $a > 0$ and $b > 0$
- the gradient of BC is 1
- $\left|\overrightarrow{BC}\right| = \sqrt{8}$.

Find the coordinates of D.



$x_C -$

$m_{BC} = 1 \Rightarrow \frac{y-8}{x-(-5)} = 1 \Rightarrow y - 8 = x + 5 \Rightarrow y = x + 13$

$\left|\overrightarrow{BC}\right| = \sqrt{8} \Rightarrow \sqrt{[x-(-5)]^2 + (y-8)^2} = \sqrt{8}$

$\Rightarrow (x+5)^2 + (y-8)^2 = 8$

$(x+5)^2 + (x+13-8)^2 = 8$

$(x+5)^2 = 4$

$x + 5 = 2$ or $x + 5 = -2$

$x_C = -3$ or $x_C = -7$

$\overrightarrow{BC} = \begin{pmatrix} x_C - x_B \\ y_C - y_B \end{pmatrix} = \begin{pmatrix} \text{negative} \\ \text{negative} \end{pmatrix}$

$\Rightarrow x_C < x_B$

$\Rightarrow x_C = -7$

$\Rightarrow y_C = -7 + 13 = 6$

ABCD is parallelogram

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$

$\Rightarrow \begin{pmatrix} -5-7 \\ 8-2 \end{pmatrix} = \begin{pmatrix} -7-x_D \\ 6-y_D \end{pmatrix}$

$\Rightarrow \begin{cases} -12 = -7 - x_D \\ 6 = 6 - y_D \end{cases} \Rightarrow \begin{cases} x_D = 5 \\ y_D = 0 \end{cases}$

(..... 5 ..... , ... 0 .....) [4]