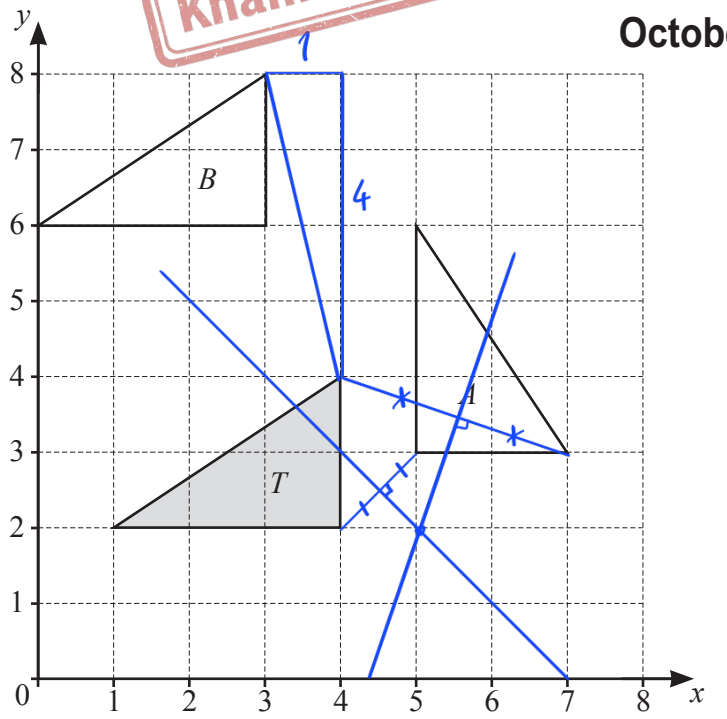
1 The diagram shows three triangles, T, A, and B, drawn on a 1 cm^2 grid.



(a) Describe fully the **single** transformation that maps triangle T onto triangle A.

Rotation, center (5, 2), clockwise 90°

..... [3]

(b) **(i)** Describe fully the **single** transformation that maps triangle T onto triangle B.

Translation by vector $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

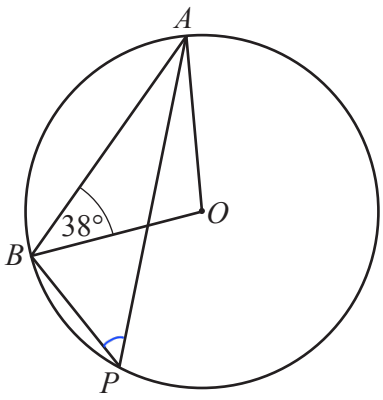
..... [2]

(ii) Calculate the distance that each point of triangle T moves when it is mapped onto triangle B.

$\sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12$

4.12 cm [2]

2 **(a)**



NOT TO SCALE

A, B and P are points on a circle, centre O and angle $OBA = 38°$.

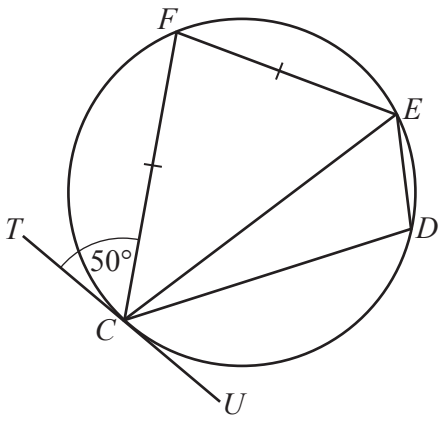
Find angle APB.

$\overset{\frown}{AOB} = 180° - 2 \times 38° = 104°$

$\overset{\frown}{APB} = \frac{1}{2} \overset{\frown}{AOB} = \frac{1}{2} \times 104°$

Angle APB = ...52°.... [3]

(b)



NOT TO SCALE

$CDEF$ is a cyclic quadrilateral and $FC = FE$.
TU is a tangent to the circle at C and angle $TCF = 50°$.

Find

(i) angle EFC,

$\overset{\frown}{FEC} = \overset{\frown}{FCT} = 50°$

$\overset{\frown}{EFC} = 180° - 2 \times 50° = 80°$

Angle EFC = ...80.... [2]

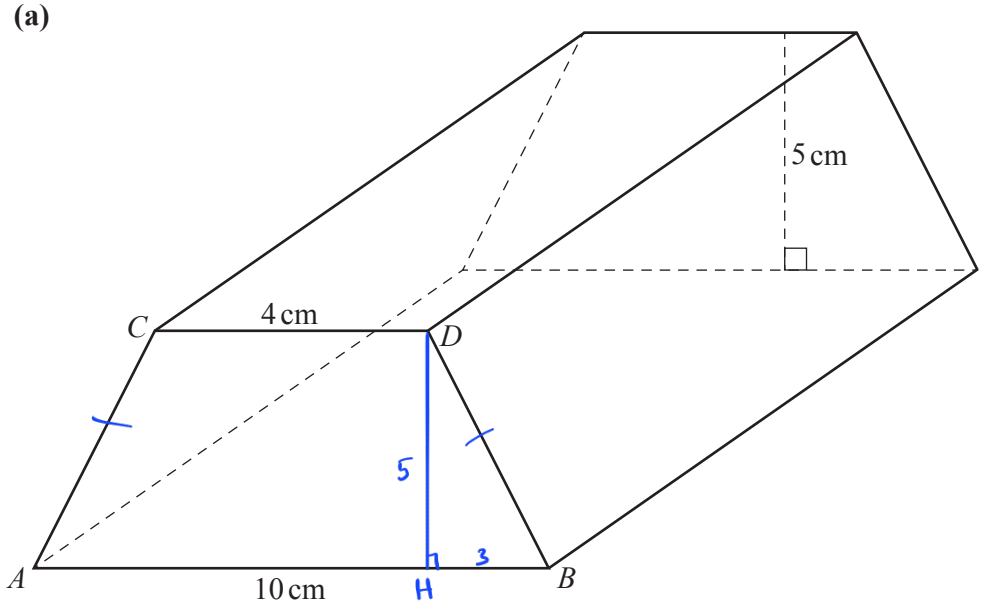
(ii) angle CDE.

$\overset{\frown}{CDE} = 180° - \overset{\frown}{EFC}$

$= 180° - 80°$

Angle CDE = ...100.... [1]

3 (a)



NOT TO SCALE

The diagram shows a prism.
The cross-section of the prism is a trapezium with CD parallel to AB and $AC = BD$.

$AB = 10$ cm, $CD = 4$ cm and the height of the trapezium is 5 cm.
The volume of the prism is 525 cm^3.

(i) The prism is made of iron.
1 cm^3 of iron has a mass of 7.8 g.

Calculate the mass of the prism.
Give your answer in kilograms.

$525 \times 7.8 = 4095$ (g)
$= 4.095$ kg

..... 4.095 ..... kg [2]

(ii) Calculate the length of the prism.

$V_{prism} = A_{ABCD} \times$ length

$525 = \frac{1}{2}(4 + 10) \times 5 \times$ length

length $= 15$

..... 15 ..... cm [3]

(iii) Calculate the total surface area of the prism.

$BH = \frac{10-4}{2} = 3$

$BD = \sqrt{3^2+5^2} = \sqrt{34}$

Total surface area $= 2(A_{front} + A_{right}) + A_{top} + A_{bottom}$

$= 2\left(\frac{1}{2}(4+10)5 + 15\sqrt{34}\right) + 4\times 15 + 10\times 15$

≈ 455

.....455..... cm^2 [6]

(iv) In a mathematically similar prism, the height of the trapezium is 10 cm.

Calculate the volume of this prism.

$\frac{V_{small}}{V_{large}} = \left(\frac{h_{small}}{h_{large}}\right)^3 = \left(\frac{5}{10}\right)^3 = \frac{1}{8} \Rightarrow V_{large} = 8 \times 525$

.....4200..... cm^3 [3]

(b) A cuboid measures 10 cm by 4 cm by 6 cm.
Each side is measured correct to the nearest centimetre. 1

Complete the inequality for the volume, V, of this cuboid.

$V_{min} = (10-\frac{1}{2})(4-\frac{1}{2})(6-\frac{1}{2})$

$V_{max} = (10+\frac{1}{2})(4+\frac{1}{2})(6+\frac{1}{2})$

.....182.875..... cm^3 $\leqslant V <$ .....307.125..... cm^3 [3]

4 **(a)** Solve the simultaneous equations.
You must show all your working.

$2p - q = 7$
$3p + 2q = 7$

$2 \times (2p - q = 7)$

$4p - 2q = 14$
$+ \quad 3p + 2q = 7$

$7p = 14 + 7 = 21$

$p = 3$

$2 \times 3 - q = 7$

$\Rightarrow \quad q = 6 - 7 = -1$

$p =$ 3

$q =$ −1 [3]

(b) Solve the equation.

$$\frac{x}{4} + \frac{2x}{3} = 1$$

$$\frac{3x + (2x) \times 4}{12} = 1$$

$11x = 12$

$x = \frac{12}{11}$

$x =$ $\frac{12}{11}$ [2]

(c) $-8 < 3x - 2 \leqslant 7$

(i) Solve the inequality.

$-8 < 3x - 2$ and $3x - 2 \leqslant 7$

$-6 < 3x$ and $3x \leqslant 9$

$-2 < x$ and $x \leqslant 3$

$-2 < x \leqslant 3$ [3]

(ii) Find the integer values of x that satisfy the inequality.

−1, 0, 1, 2, 3 [1]

(d) Factorise completely.

$$16a - 4a^2$$

$4a(4-a)$ [2]

(e) Write each of the following as a single fraction, in its simplest form.

(i) $\frac{1}{2a} \div \frac{3}{4b}$

$\frac{1}{2a} \times \frac{4b}{3} = \frac{4b}{6a} = \frac{2b}{3a}$

$\frac{2b}{3a}$ [2]

(ii) $2 - \frac{x}{x-1}$

$\frac{2(x-1) - x}{x-1}$

$= \frac{2x - 2 - x}{x-1}$

$= \frac{x-2}{x-1}$

$\frac{x-2}{x-1}$ [2]

5 **(a)** $500 is invested at a rate of 3% per year.

Calculate the total interest earned at the end of 7 years when

(i) simple interest is paid,

$500 \times \frac{3}{100} \times 7 = 105$

$ ....105.... [2]

(ii) compound interest is paid.

$500\left(1 + \frac{3}{100}\right)^7 - 500$

≈ 115

$ ....115.... [3]

(b) The value of a car decreases exponentially by 10% each year.
The value now is $6269.40 .

Calculate the value of the car 3 years ago.

c

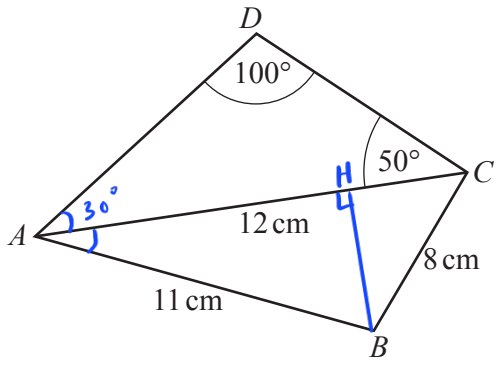
$c\left(1 - \frac{10}{100}\right)^3 = 6269.40$

$0.729c = 6269.40$

$c = 8600$

$ ....8600.... [3]

6



NOT TO SCALE

(a) Calculate AD.

$$\frac{AD}{\sin 50^\circ} = \frac{12}{\sin 100^\circ}$$

$$AD = \frac{12 \sin 50^\circ}{\sin 100^\circ} \approx 9.3343$$

$AD =$ 9.33 cm [3]

(b) Calculate angle BAC and show that it rounds to 40.42°, correct to 2 decimal places.

$$8^2 = 11^2 + 12^2 - 2 \times 11 \times 12 \cos \widehat{BAC}$$

$$264 \cos \widehat{BAC} = 201$$

$$\cos \widehat{BAC} = \frac{67}{88}$$

$$\widehat{BAC} \approx 40.415^\circ \approx 40.42^\circ$$

[4]

(c) Calculate the area of the quadrilateral $ABCD$.

$$A_{ABCD} = A_{\Delta ABC} + A_{\Delta ACD}$$

$$= \frac{1}{2} \times 11 \times 12 \times \sin 40.415^\circ + \frac{1}{2} \times 12 \times 9.3343 \sin 30^\circ$$

$$\approx 70.8$$

70.8 cm^2 [3]

(d) Calculate the shortest distance from B to AC.

$$\sin \widehat{BAC} = \frac{BH}{AB}$$

$$\Rightarrow BH = 11 \sin 40.415^\circ \approx 7.13$$

7.13 cm [3]

7 **(a)** Amir buys 3 cakes that cost c cents each and 2 loaves of bread that cost $(2c - 11)$ cents each.
He spends a total of \$5.87 .

Find the value of c.

\$ 5.87 = 587 cents

$3c + 2(2c - 11) = 587$

$3c + 4c - 22 = 587$

$7c = 60$

$c = 87$

$c =$ 87 [3]

(b) A bottle of water costs \$$w$.
A bottle of juice costs \$$(w + 1)$.

Alex spends \$22 on bottles of water and \$42 on bottles of juice.
The number of bottles of water is equal to the number of bottles of juice.

Find the value of w.

$\frac{22}{w} = \frac{42}{w+1}$

$\Rightarrow 22(w + 1) = 42w$

$22w + 22 = 42w$

$22 = 20w$

$w = 1.10$

$w =$ 1.10 [3]

(c) Alicia walks a distance of 9 km at a speed of x km/h.
She then runs a distance of 5 km at a speed of $(2x + 1)$ km/h.

The total time Alicia takes is 2.5 hours.

(i) Show that $10x^2 - 41x - 18 = 0$.

$\frac{9}{x} + \frac{5}{2x+1} = 2.5$

$\frac{9(2x+1) + 5x}{x(2x+1)} = 2.5$

$18x + 9 + 5x = 2.5(2x^2 + x)$

$23x + 9 = 5x^2 + 2.5x$

$5x^2 - 20.5x - 9 = 0$

multiply by 2: $10x^2 - 41x - 18 = 0$

[4]

(ii) Work out Alicia's running speed.
You must show all your working.

$10x^2 - 45x + 4x - 18 = 0$

$5x(2x - 9) + 2(2x - 9) = 0$

$(5x + 2)(2x - 9) = 0$

$5x + 2 = 0$ or $2x - 9 = 0$

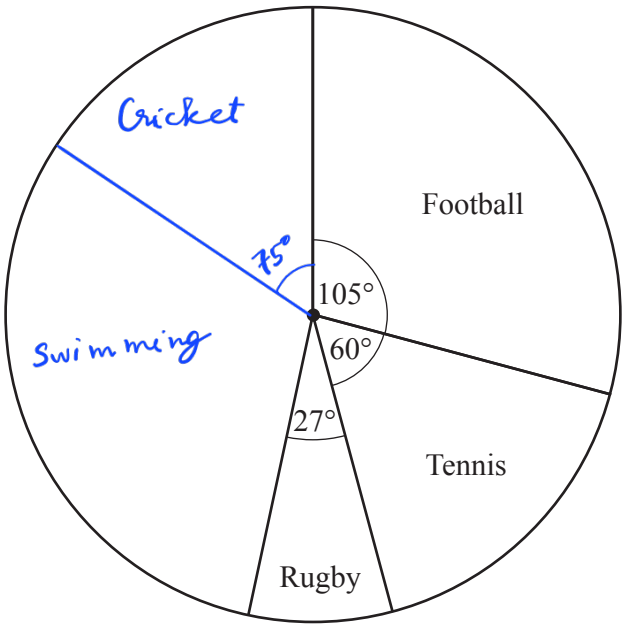
$x = -\frac{2}{5}$ or $x = \frac{9}{2}$

Because $x > 0$ so $x = \frac{9}{2}$

$\Rightarrow$ Alicia's running speed $= 2 \times \frac{9}{2} + 1 = 10$

.....10..... km/h [4]

8 **(a)** Jean asks 600 people to choose their favourite sport.
The pie chart shows some of this information.



(i) Show that 100 people choose tennis.

$\frac{600}{360^\circ} \times 60^\circ = 100$

[1]

(ii) Work out how many people choose rugby.

$\frac{600}{360^\circ} \times 27^\circ = 45$

45 [2]

(iii) 125 people choose cricket and the rest choose swimming.

Complete the pie chart to show this information.

600 : 360°
125

[2]

Cricket: $\frac{125 \times 360^\circ}{600} = 75^\circ$

(c) A dice is rolled 100 times.
The frequency table shows the results.

Score	1	2	3	4	5	6
Frequency	16	25	17	19	8	15
Cumulative freq	16	41	58	77	85	100

Find

(i) the range,

$6 - 1 = 5$

.....5..... [1]

(ii) the mode,

.....2..... [1]

(iii) the median.

.....3..... [1]

(d) 50 students answer a mathematics question.
The table shows the time, t seconds, taken by each student to answer the question.

Mid value	15	22.5	27.5	40	65
Time (t seconds)	$10 < t \leqslant 20$	$20 < t \leqslant 25$	$25 < t \leqslant 30$	$30 < t \leqslant 50$	$50 < t \leqslant 80$
Frequency	2	8	12	16	12

Calculate an estimate of the mean.

$$\frac{(15 \times 2) + (22.5 \times 8) + (27.5 \times 12) + (40 \times 16) + (65 \times 12)}{50}$$

.....39.2..... s [4]

9 $f(x) = x(x-1)(x-2)$

(a) Find the coordinates of the points where the graph of $y = f(x)$ crosses the x-axis.

cross x-axis $\Rightarrow$ $y = 0$

$\Rightarrow$ $x(x-1)(x-2) = 0$

$x = 0$ or $x = 1$ or $x = 2$

(0 , 0)

(1 , 0)

(2 , 0) [2]

(b) Show that $f(x) = x^3 - 3x^2 + 2x$.

$f(x) = (x^2 - x)(x - 2)$

$= x^3 - x^2 - 2x^2 + 2x$

$= x^3 - 3x^2 + 2x$

[2]

(c) Find the coordinates of the turning points of the graph of $y = f(x)$.
Show all your working and give your answers correct to 1 decimal place.

$\frac{dy}{dx} = 3x^2 - 6x + 2 = 0$

$\Rightarrow x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3}$

$x = \frac{3 - \sqrt{3}}{3} \approx 0.4$ or $x = \frac{3 + \sqrt{3}}{3} \approx 1.6$

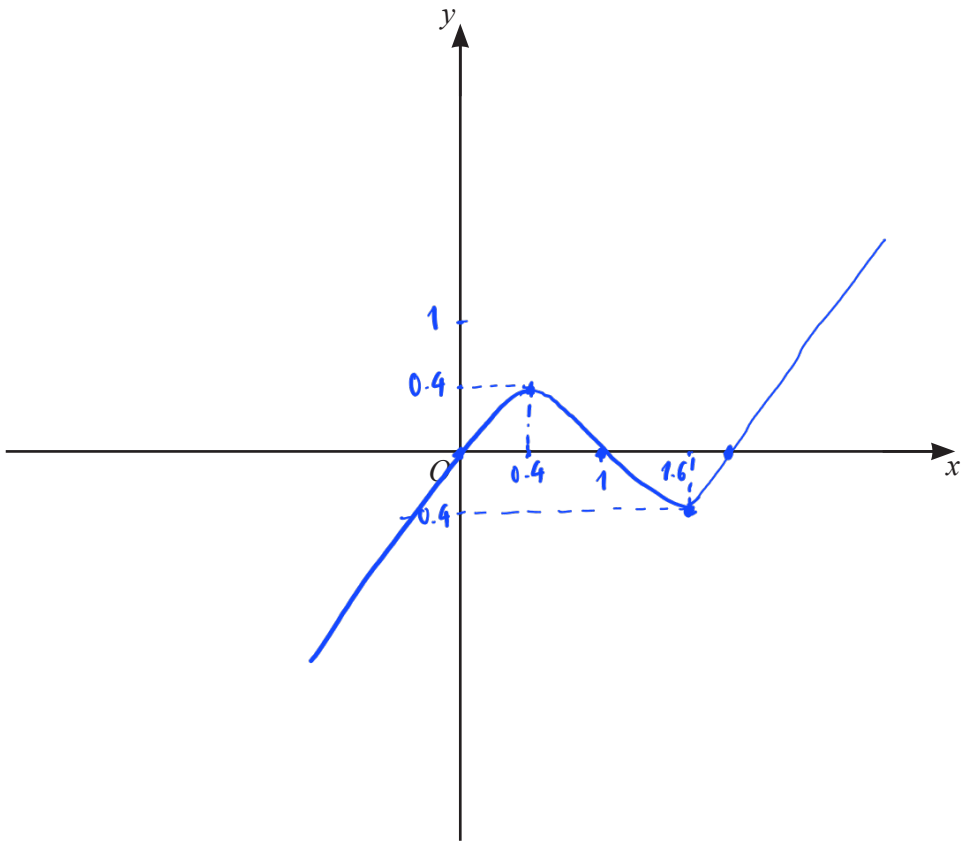
When $x = \frac{3 - \sqrt{3}}{3}$: $y = \left(\frac{3 - \sqrt{3}}{3}\right)^3 - 3\left(\frac{3 - \sqrt{3}}{3}\right)^2 + 2 \times \frac{3 - \sqrt{3}}{3} \approx 0.4$

When $x = \frac{3 + \sqrt{3}}{3}$: $y = \left(\frac{3 + \sqrt{3}}{3}\right)^3 - 3\left(\frac{3 + \sqrt{3}}{3}\right)^2 + 2 \times \frac{3 + \sqrt{3}}{3} \approx -0.4$

(0.4 , 0.4)

(1.6 , −0.4) [8]

(d) Sketch the graph of $y = f(x)$.



[2]

10 **(a)** Sarah spins a fair four-sided spinner numbered 0, 1, 1 and 3.

(i) What number is the spinner most likely to land on?

1 [1]

(ii) Sarah spins the spinner twice.

Find the probability that it lands on the number 1 both times.

$\frac{2}{4} \times \frac{2}{4} = \frac{1}{4}$

$\frac{1}{4}$ [2]

(iii) Sarah spins the spinner until it lands on the number 3.

The probability that this happens on the nth spin is $\frac{729}{16384}$.

Find the value of n.

It lands on number 3 on the n^{th} spin

$\Rightarrow$ It does not land on 3 on $(n-1)$ previous times

$$\left(\frac{3}{4}\right)^{n-1} \times \frac{1}{4} = \frac{729}{16384}$$

$$\left(\frac{3}{4}\right)^{n-1} = \frac{729}{4096}$$

Trial & error: $\left(\frac{3}{4}\right)^{6} = \frac{729}{4096} \Rightarrow n = 6 + 1 = 7$

$n =$ ....7.... [2]

(b) Scott takes an examination.
The examination is in two parts, a theory test and a practical test.
Both parts must be passed to pass the examination.

The probability that Scott passes the theory test is 0.9 .
The probability that Scott passes the practical test is 0.8 .

Find the probability that

(i) Scott passes the examination,

0.9×0.8

....0.72.... [2]

(ii) Scott passes the theory test or the practical test but not both.

$0.9 \times (1 - 0.8) + (1 - 0.9) \times 0.8 = 0.26$

....0.26.... [3]

11 $f(x) = 2x - 1$ $g(x) = x^2 + 2x$ $h(x) = 4^x$ $j(x) = 2^x$

(a) Find the value of

(i) h(3),

$4^3 = 64$

64 [1]

(ii) fh(3).

$f(64) = 2 \times 64 - 1$

127 [1]

(b) Solve the equation $gf(x) = 0$.

$gf(x) = (2x-1)^2 + 2(2x-1) = 0$

$(2x-1)(2x-1+2) = 0$

$(2x-1)(2x+1) = 0$

$2x - 1 = 0$ or $2x + 1 = 0$

$x = \frac{1}{2}$ or $x = -\frac{1}{2}$

$x =$ $\frac{1}{2}$ or $x =$ $-\frac{1}{2}$ [4]

(c) $p^{-1}(x) = f(x)$

Find $p(x)$.

$p^{-1}(x) = 2x - 1$

$\times 2 \rightarrow -1$

$\div 2 \leftarrow +1$

$\frac{x+1}{2}$ [2]

(d) $h(x)j(x) = \frac{1}{\sqrt{2}}$

Find the value of x.

$4^x \times 2^x = \frac{1}{\sqrt{2}}$

$(4 \times 2)^x = \frac{1}{2^{0.5}}$

$8^x = 2^{-0.5}$

$2^{3x} = 2^{-0.5}$

$3x = -0.5$

$x =$ $-\frac{1}{6}$ [3]