1 **(a)** Malena has 450 fruit trees.
The fruit trees are in the ratio apple : pear : plum = 8 : 7 : 3.

(i) Show that Malena has 200 apple trees.

$$\frac{450}{8+7+3} \times 8 = 200$$

[2]

(ii) Find the number of plum trees.

$$\frac{450}{8+7+3} \times 3$$

..........75.......... [1]

(iii) Malena wants to increase the number of pear trees by 32%.

Calculate the number of extra pear trees she needs.

$$\text{pear}_{\text{now}} = \frac{450}{8+7+3} \times 7 = 175$$

$$\text{pear}_{\text{after - increase}} = 175 + 175 \times 32\% = 231$$

Extra pear = 231 − 175

..........56.......... [2]

(iv) Each apple tree produces 48.5 kg of apples.
The apples have an average mass of 165 g each. 0.165 kg

Calculate the total number of apples produced by the 200 trees.
Give your answer correct to the nearest 1000 apples.

200 trees produce 200 × 48.5 = 9700 kg apples

$$\text{Number of apples: } \frac{9700}{0.165} \approx 58788 \approx 59000$$

..........59000.......... [3]

(b) Malena's land is valued at three million and seventy-five thousand dollars.

(i) Write this number in figures.

3 075 000 .... [1]

(ii) Write your answer to **part (b)(i)** in standard form.

3.075×10^6 .... [1]

(c) In 2020, each plum tree produced 37.7 kg of plums.
This was 16% more than in 2019.

Calculate the mass of plums produced by each plum tree in 2019. m_{19}

$m_{19} + m_{19} \times 16\% = 37.7$

$1.16\, m_{19} = 37.7$

$m_{19} = 32.5$

32.5 .... kg [2]

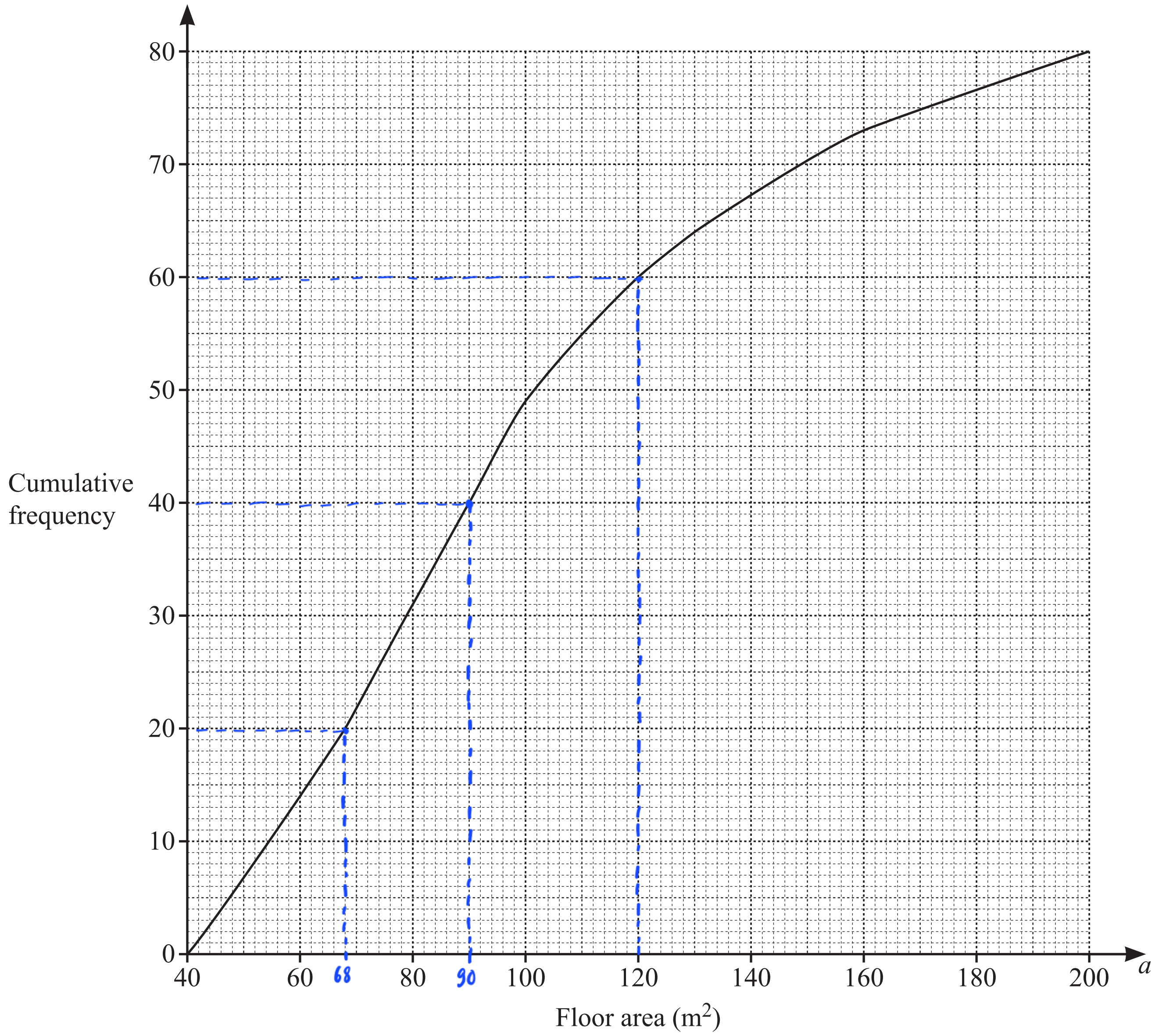
(d) Malena invests \$1800 at a rate of 2.1% per year compound interest.

Calculate the value of her investment at the end of 15 years.

$1800\left(1+\frac{2.1}{100}\right)^{15} \approx 2460$

\$ 2460 .... [2]

2 **(a)** The cumulative frequency diagram shows information about the floor area, $a\,\text{m}^2$, of each of 80 houses.



Use the diagram to find an estimate of

(i) the median,

........90........ m^2 [1]

(ii) the lower quartile,

........6.8........ m^2 [1]

(iii) the interquartile range,

120 − 68

........52........ m^2 [1]

(iv) the number of houses with a floor area greater than $120\,\text{m}^2$.

80 − 60

........20........ [2]

(b) The information about the 80 floor areas is shown in this frequency table.

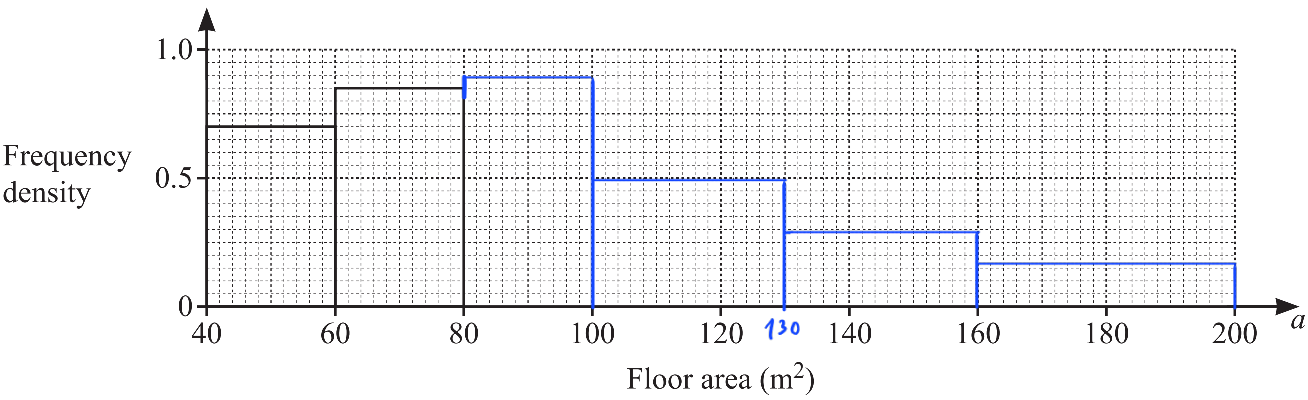
Mid value	50	70	90	115	145	180
Floor area ($a\,\text{m}^2$)	$40 < a \leqslant 60$	$60 < a \leqslant 80$	$80 < a \leqslant 100$	$100 < a \leqslant 130$	$130 < a \leqslant 160$	$160 < a \leqslant 200$
Frequency	14	17	18	15	9	7
Freq. density	0.7	0.85	0.9	0.5	0.3	0.175

(i) Calculate an estimate of the mean floor area.

$$\frac{(50\times 14)+(70\times 17)+(90\times 18)+(115\times 15)+(145\times 9)+180\times 7}{80}$$

........97.5........ m^2 [4]

(ii) Complete the histogram to show the information in the frequency table.



[4]

(iii) Two of the houses are picked at random.

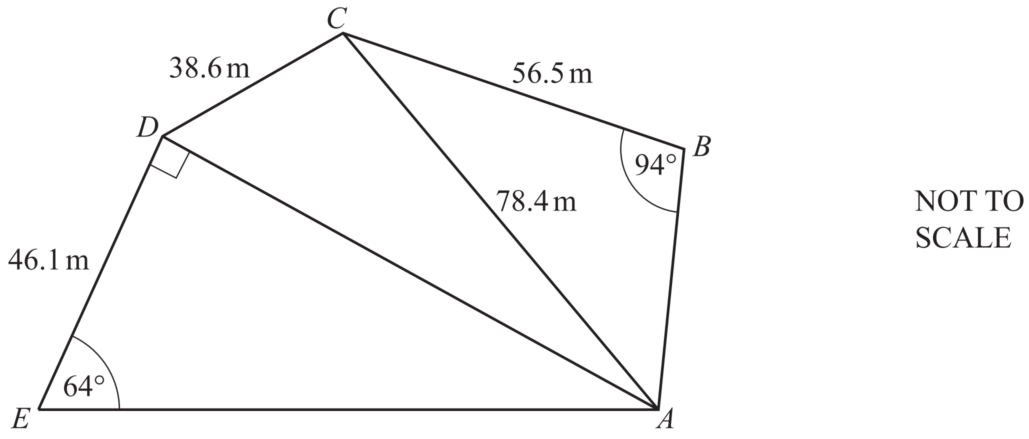
Find the probability that one of the houses has a floor area greater than $130\,\text{m}^2$ and the other has a floor area $60\,\text{m}^2$ or less.

$P(>130) \times P(\leq 60) \quad \times 2$

$\frac{9+7}{80} \times \frac{14}{79} \times 2$

........$\frac{28}{395}$........ [3]

3 (a)



ABCDE is a pentagon.

(i) Calculate *AD* and show that it rounds to 94.5 m, correct to 1 decimal place.

$\tan 64° = \dfrac{AD}{46.1}$

$\Rightarrow AD = 46.1 \tan 64° \approx 94.519 \approx 94.5$

[2]

(ii) Calculate angle *BAC*.

$\dfrac{78.4}{\sin 94°} = \dfrac{56.5}{\sin \widehat{BAC}}$

$\Rightarrow \sin \widehat{BAC} = \dfrac{56.5 \times \sin 94°}{78.4} \approx 0.7189$

$\Rightarrow \widehat{BAC} \approx 46.0°$

Angle *BAC* = ...46.0°.................................... [3]

(iii) Calculate the largest angle in triangle *CAD*.

$\widehat{ACD}$ is opposite to the largest side (AD)

$\Rightarrow \widehat{ACD}$ is the largest angle in ΔCAD

$AD^2 = CD^2 + CA^2 - 2 \times CD \times CA \cos \widehat{ACD}$

$94.519^2 = 38.6^2 + 78.4^2 - 2 \times 38.6 \times 78.4 \cos \widehat{ACD}$

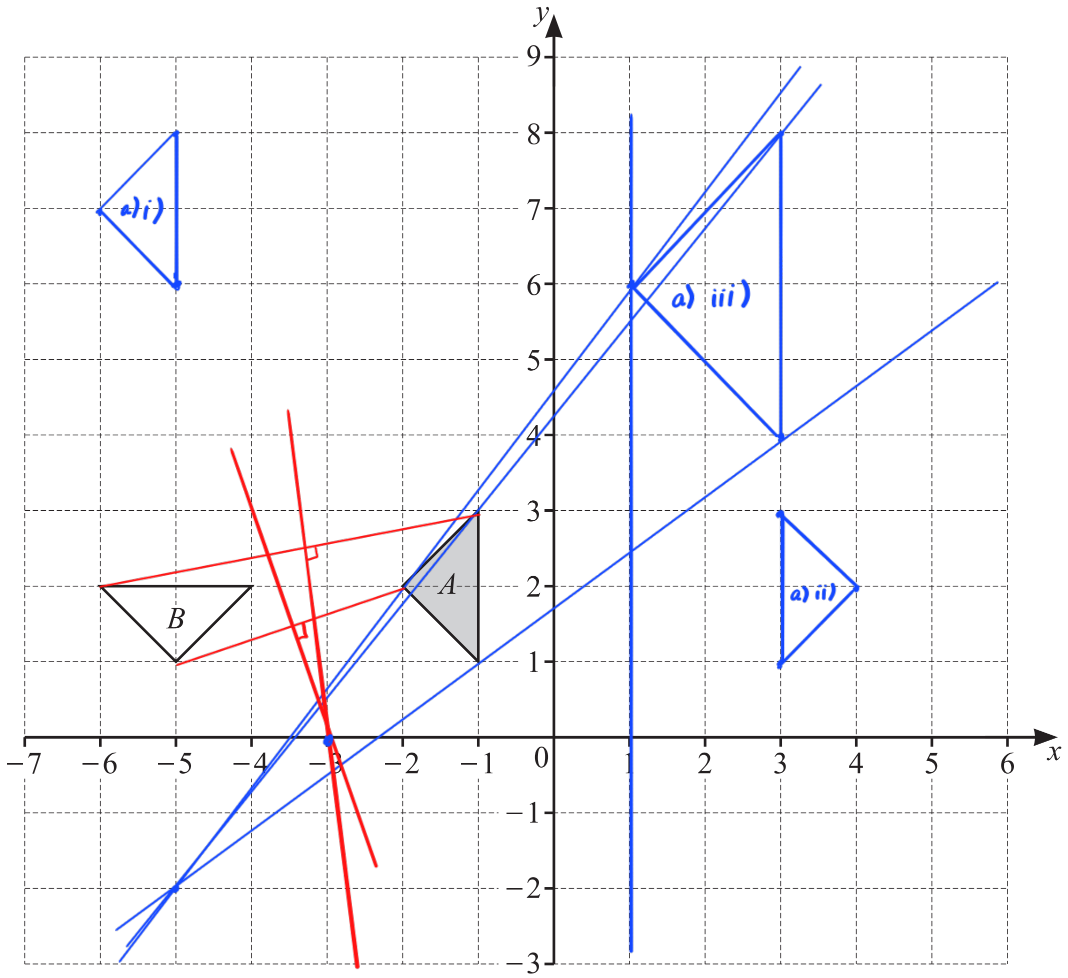
$1297.32 = -6052.48 \cos \widehat{ACD}$

$\Rightarrow \cos \widehat{ACD} = -0.21435$

$\widehat{ACD} = 102.4°$

....102.4°.................................... [4]

4



(a) On the grid, draw the image of triangle A after

(i) a translation by the vector $\begin{pmatrix}-4\\5\end{pmatrix}$, [2]

(ii) a reflection in the line $x = 1$, [2]

(iii) an enlargement, scale factor 2 and centre $(-5, -2)$. [2]

(b) Describe fully the **single** transformation that maps triangle A onto triangle B.

Rotation , center (-3,0) , anticlockwise 90°

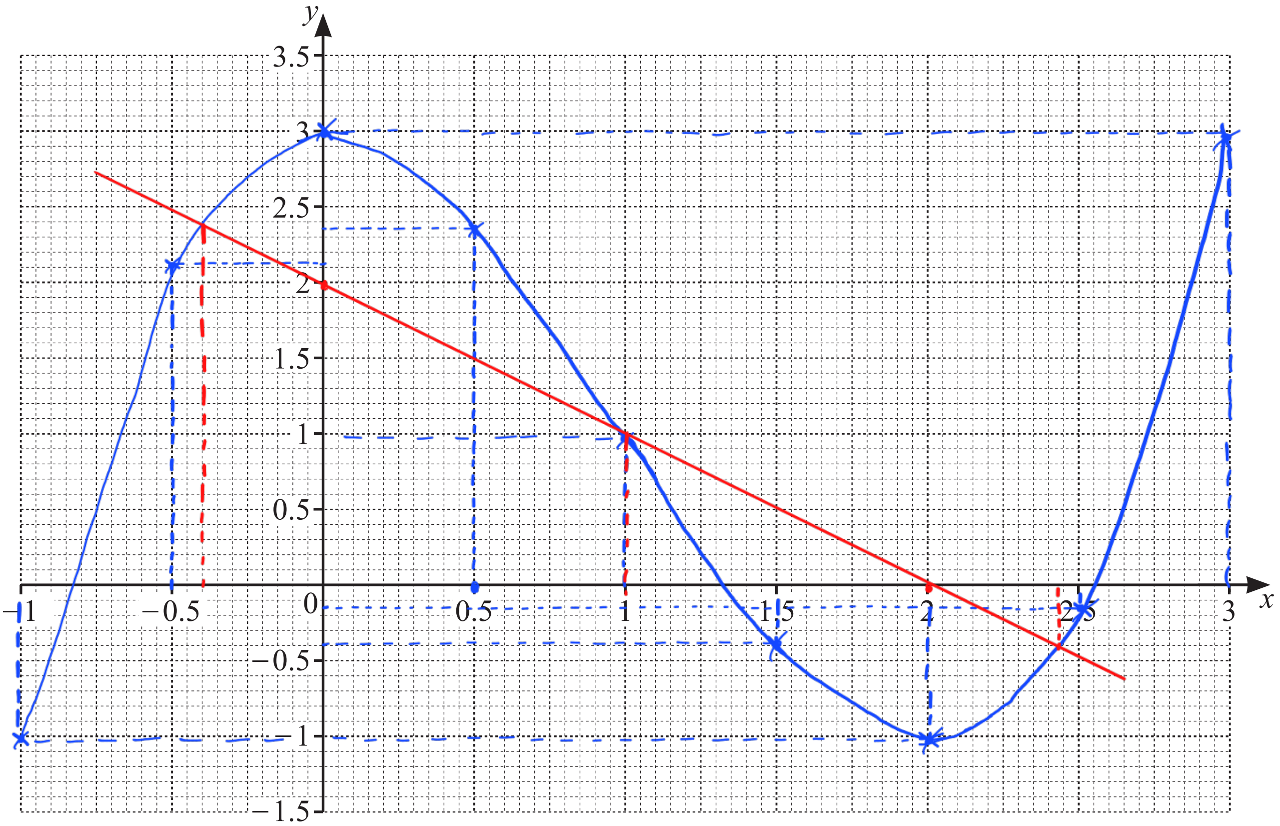
.... [3]

5 The table shows some values for $y = x^3 - 3x^2 + 3$.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	-1	2.125	3	2.375	1	-0.375	-1	-0.125	3

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 - 3x^2 + 3$ for $-1 \leqslant x \leqslant 3$.



[4]

(c) By drawing a suitable straight line on the grid, solve the equation $x^3 - 3x^2 + x + 1 = 0$.

$(x^3 - 3x^2 + 3) + x - 2 = 0$

$x^3 - 3x^2 + 3 = -x + 2$

Draw line $y = -x + 2$

$x =$ ....-0.4..... or $x =$ ....1.......... or $x =$ ...2.375.... [4]

6 **(a)** Solve.

(i) $4(2x-3)=24$

$8x - 12 = 24$

$8x = 36$

$x =$ ....4.5.................... [3]

(ii) $6x+14>6$

$6x > -8$

$x > \frac{-8}{6}$

....$x > \frac{-8}{6}$.................... [2]

(b) Rearrange the formula $V=2x^3-3y^3$ to make y the subject.

$3y^3 = 2x^3 - V$

$y^3 = \frac{2x^3 - V}{3}$

$y =$ ....$\sqrt[3]{\frac{2x^3-V}{3}}$.................... [3]

(c) Show that $(2n-5)^2-13$ is a multiple of 4 for all integer values of n.

$4n^2 - 20n + 25 - 13$

$4n^2 - 20n + 12$

$4(n^2 - 5n + 3) \vdots 4$ for $n \in \mathbb{Z}$

[3]

(e) The energy of a moving object is directly proportional to the square of its speed.
The speed of the object is increased by 30%.

Calculate the percentage increase in the energy of the object.

$e \propto s^2 \Rightarrow e = k s^2$

$s_{new} = s + s \times 30\% = 1.3s$

$\Rightarrow e_{new} = k s^2_{new} = k(1.3s)^2 = 1.69ks^2 = 1.69e$

$\Rightarrow$ percentage increase $= \frac{1.69e - e}{e} \times 100 = 69$

....69.................... % [2]

(d) The expression $5+12x-2x^2$ can be written in the form $q-2(x+p)^2$.

(i) Find the value of p and the value of q.

$-2x^2+12x+5$

$= -2(x^2-6x)+5$

$= -2(x^2-2x3+3^2-3^2)+5$

$= -2[(x-3)^2-9)+5$

$= -2(x-3)^2+18+5$

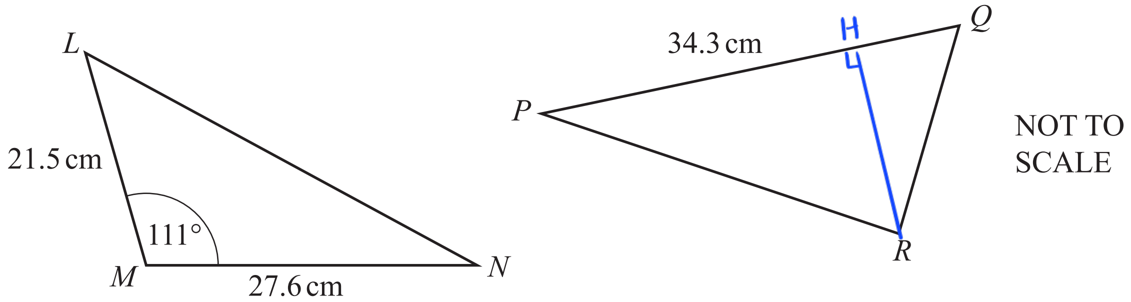
$= 23-2(x-3)^2$

$p =$ ....-3.... , $q =$ ....23.... [3]

(ii) Write down the coordinates of the maximum point of the curve $y=5+12x-2x^2$.

(....3.... , ...23....) [1]

(b)



Triangle *PQR* has the same area as triangle *LMN*.
Calculate the shortest distance from *R* to the line *PQ*.

$A_{\Delta LMN} = A_{\Delta PRQ}$

$\frac{1}{2} \times 21.5 \times 27.6 \times \sin 111° = \frac{1}{2} RH \times 34.3$

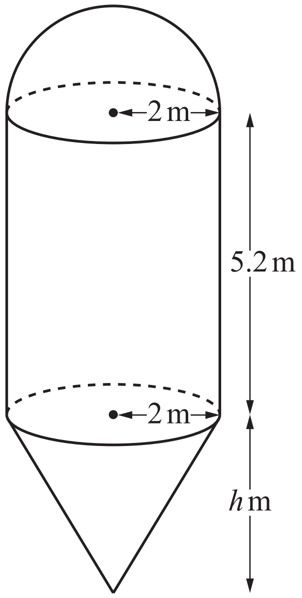
$593.4 \sin 111° = 34.3\, RH$

$RH = 16.1512$

....16.2.... cm [3]

7 **(a)** The diagram shows a container for storing grain.

The container is made from a hemisphere, a cylinder and a cone, each with radius 2 m. The height of the cylinder is 5.2 m and the height of the cone is h m.



(i) Calculate the volume of the hemisphere. Give your answer as a multiple of π.

$$V_{Sphere} = \frac{4}{3}\pi 2^3 = \frac{32}{3}\pi$$

$$V_{hemisphere} = \frac{1}{2} \times \frac{32}{3}\pi = \frac{16\pi}{3}$$

.......$\frac{16\pi}{3}$....... m^3 [2]

(ii) The total volume of the container is $\frac{88\pi}{3}$ m^3.

Calculate the value of h.

$$V_{total} = V_{hemisphere} + V_{cylinder} + V_{cone}$$

$$\frac{88\pi}{3} = \frac{16\pi}{3} + \pi 2^2 \times 5.2 + \frac{1}{3}\pi 2^2 \times h$$

$$\frac{88\pi}{3} = \frac{16\pi}{3} + 20.8\pi + \frac{4}{3}\pi h$$

$$\frac{16\pi}{5} = \frac{4}{3}\pi h$$

$$h = 2.40$$

h = ...2.40....... [4]

(iii) The container is full of grain.
Grain is removed from the container at a rate of 35 000 kg per hour.
1 m^3 of grain has a mass of 620 kg.

Calculate the time taken to empty the container.
Give your answer in hours and minutes.

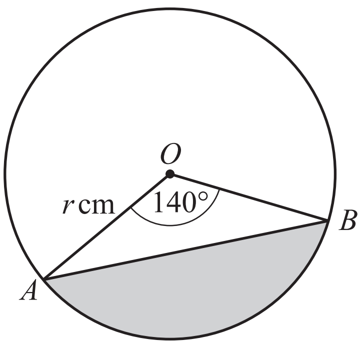
$V_{container} = \frac{88\pi}{3} m^3$

Total mass of grain $= \frac{88\pi}{3} \times 620 = \frac{54560\pi}{3}$ kg

Time taken $= \frac{54560\pi}{3} \div 35000 = 1.6324$ h
$= 1$ h 38 min

..........1.......... h ....38.......... min [3]

(b)



NOT TO SCALE

A and B are points on a circle, centre O, radius r cm.
The area of the shaded segment is 65 cm^2.

Calculate the value of r.

$140° = \frac{140\pi}{180}$ radian $= \frac{7\pi}{9}$ rad

$A_{shaded\ segment} = A_{sector} - A_{\Delta AOB}$

$65 = \frac{1}{2} r^2 \times \frac{7\pi}{9} - \frac{1}{2} r^2 \sin 140°$

$65 = 0.9003 r^2$

$72.198 = r^2$

$r \approx 8.50$

$r =$ ....8.50.......... [4]

8 **(a)** Kaito runs along a 12 km path at an average speed of x km/h.

(i) Write down an expression, in terms of x, for the number of hours he takes.

$\frac{12}{x}$ hours [1]

(ii) Yuki takes 1.5 hours longer to walk along the same path as Kaito.
She walks at an average speed of $(x-4)$ km/h.

Write down an equation, in terms of x, and show that it simplifies to $x^2-4x-32=0$.

$$\frac{12}{x-4}-\frac{12}{x}=1.5$$

$$\frac{12x-12(x-4)}{(x-4)x}=1.5$$

$$12x-12x+48=1.5x(x-4)$$

$$48=1.5x^2-6x$$

$$1.5x^2-6x-48=0$$

$$1.5(x^2-4x-32)=0$$

$$x^2-4x-32=0$$

[4]

(iii) Solve by factorisation.

$$x^2-4x-32=0$$

$$x^2+4x-8x-32=0$$

$$x(x+4)-8(x+4)=0$$

$$(x-8)(x+4)=0$$

$x-8=0$ or $x+4=0$

$x =$ 8 or $x =$ -4 [3]

(iv) Find the number of hours it takes Yuki to walk along the 12 km path.

Because $x>0$ so $x=8$

$$\frac{12}{8-4}=3$$

3 hours [2]

(b) A bus travels 440 km, correct to the nearest 10 km.
The time taken to complete the journey is 6 hours, correct to the nearest half hour.
0.5 h

Calculate the lower bound of the speed of the bus.

$$\text{speed}_{min} = \frac{\text{distance}_{min}}{\text{time}_{max}}$$

$$= \frac{440 - \frac{10}{2}}{6 + \frac{0.5}{2}} = 69.6$$

..........69.6.......... km/h [3]

9 **(a)** F is the point $(5, -2)$ and $\overrightarrow{FG} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

Find

(i) the coordinates of point G,

$$\overrightarrow{FG} = \begin{pmatrix} x_G - 5 \\ y_G - (-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

(.......3....... , ...1.......) [1]

(ii) $5\overrightarrow{FG}$,

$$\begin{pmatrix} 5 \times (-2) \\ 5 \times 3 \end{pmatrix}$$

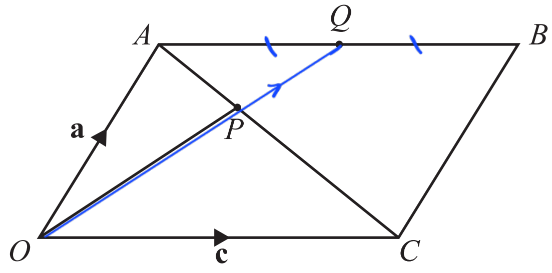
$\begin{pmatrix} -10 \\ 15 \end{pmatrix}$ [1]

(iii) $|\overrightarrow{FG}|$.

$$\sqrt{(-2)^2 + 3^2} = \sqrt{13}$$

..........3.61.......... [2]

(b)



NOT TO SCALE

$OABC$ is a parallelogram.
P is a point on AC and Q is the midpoint of AB.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

(i) Find, in terms of **a** and/or **c**

(a) $\overrightarrow{AQ}$,

$\overrightarrow{AQ} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}c$

$\overrightarrow{AQ} =$ ...... $\frac{1}{2}c$ ...... [1]

(b) $\overrightarrow{OQ}$.

$OQ = \overrightarrow{OA} + \overrightarrow{AQ}$
$= a + \frac{1}{2}c$

$\overrightarrow{OQ} =$ ...... $a + \frac{1}{2}c$ ...... [1]

(ii) $\overrightarrow{OP} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$

(a) Show that O, P and Q lie on a straight line.

$\overrightarrow{OP} = \frac{2}{3}\left(a + \frac{1}{2}c\right) = \frac{2}{3}\overrightarrow{OQ}$

$\Rightarrow$ O, P, Q are collinear

[2]

(b) Write down the ratio $OP : OQ$.
Give your answer in the form $1 : n$.

$\frac{OP}{OQ} = \frac{2}{3} = \frac{1}{1.5}$

1 : ...... 1.5 ...... [1]

10 **(a)** Find the coordinates of the turning points of the graph of $y = x^3 - 12x + 6$.
You must show all your working.

$$\frac{dy}{dx} = 3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

when $x = -2$, $y = (-2)^3 - 12(-2) + 6 = 22$

when $x = 2$, $y = 2^3 - 12(2) + 6 = -10$

(..-2.. , ..22..) and (..2.. , ..-10..) [5]

(b) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

$$\frac{d^2y}{dx^2} = 6x$$

when $x = -2$, $6(-2) = -12 < 0$

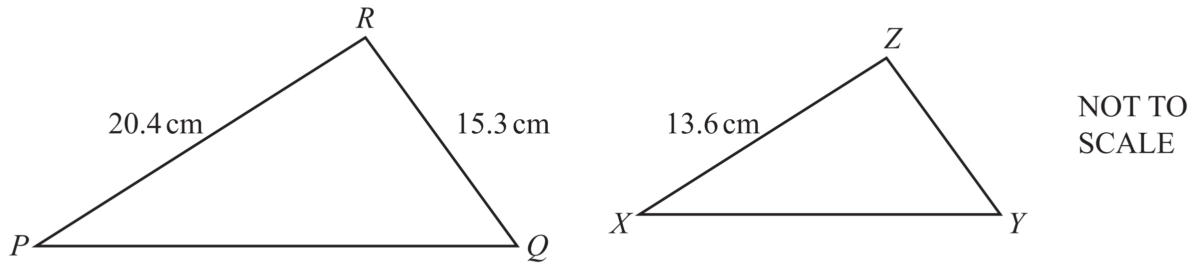
$\Rightarrow (-2, 22)$ is maximum

when $x = 2$, $6(2) = 12 > 0$

$\Rightarrow (2, -10)$ is minimum

[3]

11 (a)



Triangle *PQR* is mathematically similar to triangle *XYZ*.

(i) Find *YZ*.

$$\frac{QR}{YZ} = \frac{PR}{XZ} \Rightarrow YZ = \frac{15.3 \times 13.6}{20.4}$$

YZ = ...10.2.......................... cm [2]

(ii) The area of triangle *XYZ* is 63.6 cm^2.

Calculate the area of triangle *PQR*.

$$\text{Ratio}_{\text{area}} = (\text{ratio}_{\text{side}})^2$$

$$\frac{A_{\Delta XYZ}}{A_{\Delta PQR}} = \left(\frac{XZ}{PR}\right)^2 = \left(\frac{13.6}{20.4}\right)^2 = \frac{4}{9}$$

$$\Rightarrow A_{\Delta PQR} = 63.6 : \frac{4}{9}$$

..........143.1.................. cm^2 [3]

(b) Two containers are mathematically similar.
The larger container has a capacity of 64.8 litres and a surface area of 0.792 m^2.
The smaller container has a capacity of 37.5 litres.

Calculate the surface area of the smaller container.

$$\text{Ratio}_{\text{volume}} = (\text{ratio}_{\text{side}})^3$$

$$\text{Ratio}_{\text{area}} = (\text{ratio}_{\text{side}})^2 \Rightarrow \text{ratio}_{\text{side}} = \sqrt{\text{ratio}_{\text{area}}}$$

$$\Rightarrow \text{Ratio}_{\text{volume}} = \left(\sqrt{\text{ratio}_{\text{area}}}\right)^3$$

$$\Rightarrow \frac{64.8}{37.5} = \left(\sqrt{\frac{0.792}{A_{\text{small}}}}\right)^3$$

$$\Rightarrow \frac{0.792}{A_{\text{small}}} = \frac{36}{25}$$

$$\Rightarrow A_{\text{small}} = \frac{25 \times 0.792}{36}$$

..........0.55.................. m^2 [3]