1 A company employed 300 workers when it started and now employs 852 workers.

(a) Calculate the percentage increase in the number of workers.

$$\frac{852-300}{300} \times 100$$

184 % [2]

(b) Of the 852 workers, the ratio part-time workers : full-time workers = 5 : 7.

Calculate the number of full-time workers.

$$\frac{852}{5+7} \times 7$$

497 [2]

(c) The company makes 40 600 headphones in one year.

Write this number

(i) in words,

forty thousand six hundred [1]

(ii) in standard form.

4.06×10^4 [1]

(d) In one month, the company sells 3 000 headphones.

Of these, 48% are exported, $\frac{3}{8}$ are sold to shops and the rest are sold online.

Calculate the number of headphones that are sold online.

$$3000 - (48\% \times 3000) - \left(\frac{3}{8} \times 3000\right) = 435$$

435 [3]

(e) One year, sales increased by 15%.
The following year sales increased by 18%.
Calculate the overall percentage increase in sales.

$$x \longrightarrow x + 15\%\ x = 1.15x \longrightarrow 1.15x + 18\% \times 1.15x = 1.357x$$

$$\text{overall percentage increase} = \frac{1.357x - x}{x} \times 100$$

$$= 35.7$$

35.7 % [3]

2 The table shows some values for $y = x^2 - \frac{1}{3x}$, $x \neq 0$.
The y-values are rounded to 1 decimal place.

x	−2	−1.5	−1	−0.75	−0.5	−0.25	−0.1
y	4.2	2.5	1.3	1	0.9	1.4	3.3

(a) Complete the table. [2]

(b) On the grid, draw the graph of $y = x^2 - \frac{1}{3x}$ for $-2 \leqslant x \leqslant -0.1$.

The graph of $y = x^2 - \frac{1}{3x}$ for $x > 0$ has been drawn for you.

[4]

(c) By drawing a suitable line on the grid, solve the equation $x^2 - \frac{1}{3x} + 1 = 0$.

$x^2 - \frac{1}{3x} = -1$

$\Rightarrow$ Draw line $y = -1$

$x =$ ......0.31...... [2]

3 $\qquad f(x) = 1 + 4x \qquad g(x) = x^2$

(a) Find

(i) gf(3),

$f(3) = 1 + 4 \times 3 = 13$

$g(13) = 13^2 = 169$

.......169....... [2]

(ii) fg(x),

$1 + 4x^2$

.......$1 + 4x^2$....... [1]

(iii) $f^{-1}f(x)$.

.......x....... [1]

(b) Find the value of x when $f(x) = 15$.

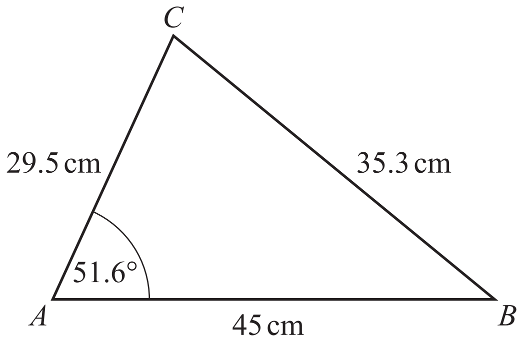
$1 + 4x = 15$

$4x = 14$

$x = \frac{14}{4}$

$x =$ ....$\frac{14}{4}$....... [2]

4 **(a)**



NOT TO SCALE

In triangle ABC, $AB = 45$ cm, $AC = 29.5$ cm, $BC = 35.3$ cm and angle $CAB = 51.6°$.

(i) Calculate angle ABC.

$$\frac{35.3}{\sin 51.6°} = \frac{29.5}{\sin \widehat{ABC}}$$

$$\Rightarrow \sin \widehat{ABC} = \frac{29.5 \times \sin 51.6°}{35.3} \approx 0.65493$$

$$\widehat{ABC} \approx 40.9°$$

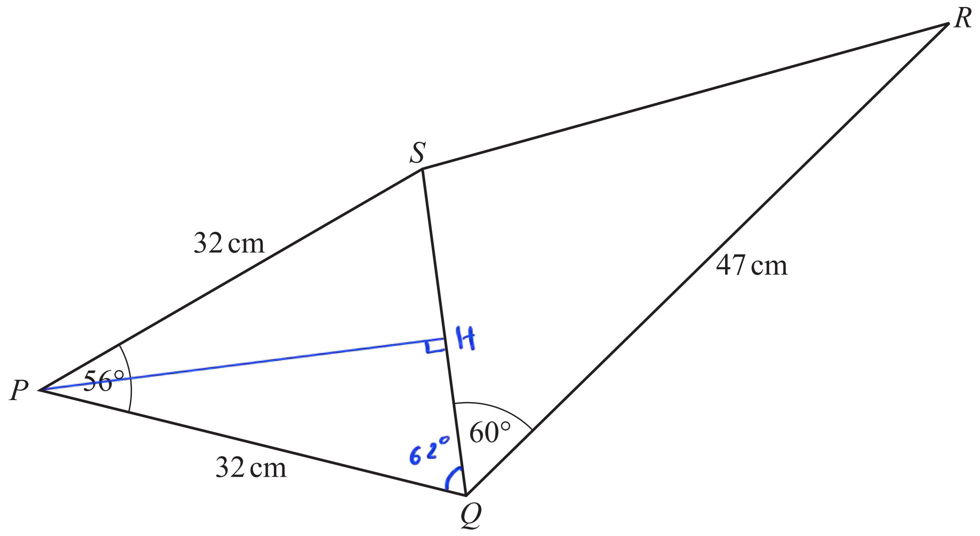
Angle ABC = ...40.9°........ [3]

(ii) Calculate the area of triangle ABC.

$$\frac{1}{2} \times 29.5 \times 45 \sin 51.6° \approx 520$$

........520........ cm^2 [2]

(b)



The diagram shows a quadrilateral *PQRS* formed from two triangles, *PQS* and *QRS*.
Triangle *PQS* is isosceles, with $PQ = PS = 32$ cm and angle $SPQ = 56°$.
$QR = 47$ cm and angle $SQR = 60°$.

(i) Calculate *SR*.

ΔSPQ : $SQ^2 = 32^2 + 32^2 - 2 \times 32 \times 32 \times \cos 56° \approx 902.773$

ΔSQR : $SR^2 = 902.773 + 47^2 - 2 \times \sqrt{902.773} \times 47 \cos 60°$

$= 1699.602$

$SR \approx 41.2$

SR = ...41.2...... cm [4]

(ii) Calculate the shortest distance from *P* to *SQ*.

$\widehat{PQS} = \dfrac{180° - 56°}{2} = 62°$

$\sin 62° = \dfrac{PH}{PQ} = \dfrac{PH}{32}$

$PH = 32 \sin 62° \approx 28.3$

......28.3...... cm [3]

5 The table shows information about the mass, m grams, of each of 120 letters.

Mid value	25	75	150	350
Mass (m grams)	$0 < m \leqslant 50$	$50 < m \leqslant 100$	$100 < m \leqslant 200$	$200 < m \leqslant 500$
Frequency	43	31	25	21
Freq density	0.86	0.62	0.25	0.07

(a) Calculate an estimate of the mean mass.

$$\frac{(25 \times 43) + (75 \times 31) + (150 \times 25) + (350 \times 21)}{120}$$

.... $\frac{725}{6}$ .... g [4]

(b) Iraj draws a histogram to show this information.
He makes the height of the first bar 17.2 cm.

Calculate the height of each of the remaining bars.

Freq density	0.86	0.62	0.25	0.07
height (cm)	17.2	12.4	5	1.4

height of bar for $50 < m \leqslant 100$ .... 12.4 .... cm

height of bar for $100 < m \leqslant 200$ .... 5 .... cm

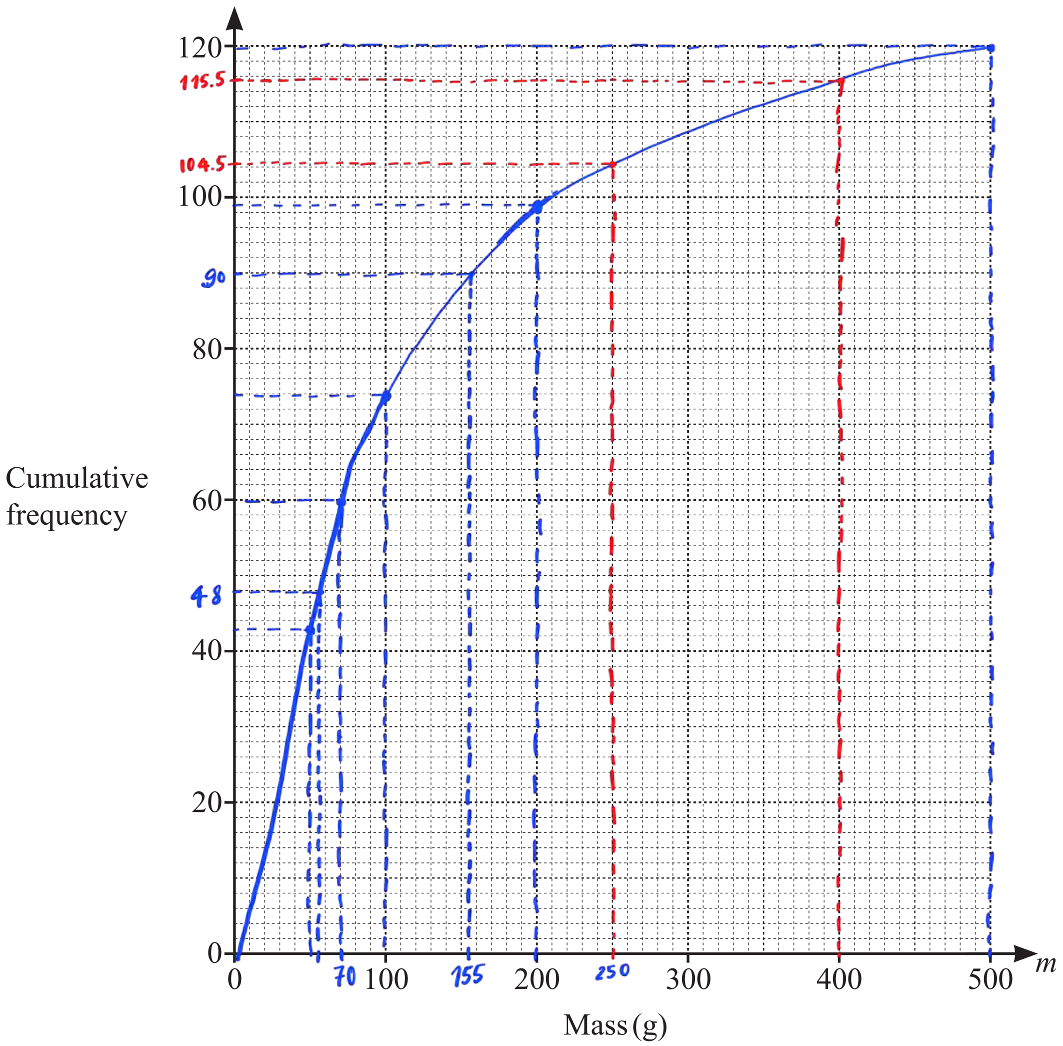
height of bar for $200 < m \leqslant 500$ .... 1.4 .... cm [3]

(c) Complete the cumulative frequency table.

Mass (m grams)	$m \leqslant 50$	$m \leqslant 100$	$m \leqslant 200$	$m \leqslant 500$
Cumulative frequency	43	74	99	120

[2]

(d) Draw a cumulative frequency diagram.



[3]

(e) Use the cumulative frequency diagram to find an estimate for

(i) the median,

120 × 50% → 60th → 70

.............70............. g [1]

(ii) the upper quartile,

120 × 75% → 90th → 155

.............155............. g [1]

(iii) the 40th percentile,

120 × 40% → 48th → 55

.............55............. g [2]

(iv) the number of letters with a mass m where $250 < m \leqslant 400$.

115.5 − 104.5

.............11............. [2]

6 **(a)** The interior angle of a regular polygon is 156°.

Calculate the number of sides of this polygon.

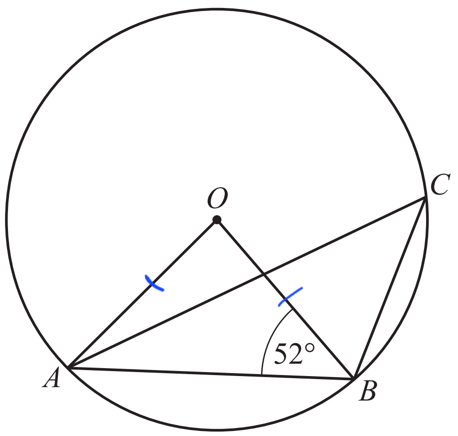
$$\frac{(n-2)180}{n} = 156$$

$$\Rightarrow 180n - 360 = 156n$$

$$24n = 360$$

.................15................. [2]

(b)



NOT TO SCALE

A, *B* and *C* lie on a circle, centre *O*.
Angle *OBA* = 52°.

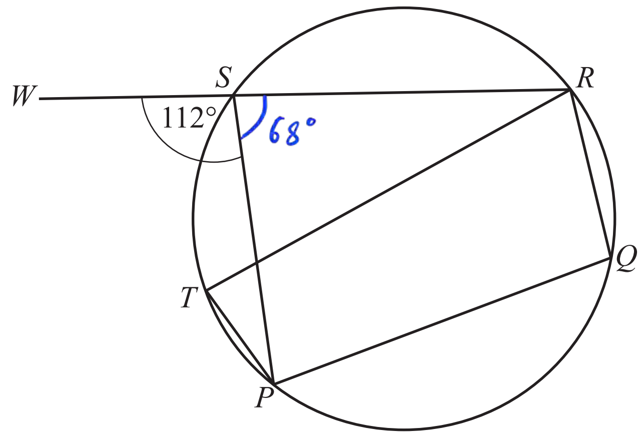
Calculate angle *ACB*.

$\widehat{AOB} = 180° - 52° \times 2 = 76°$

$\widehat{ACB} = \frac{1}{2}\widehat{AOB} = \frac{1}{2} \times 76° = 38°$

Angle *ACB* = ...38°............. [2]

(c)



NOT TO SCALE

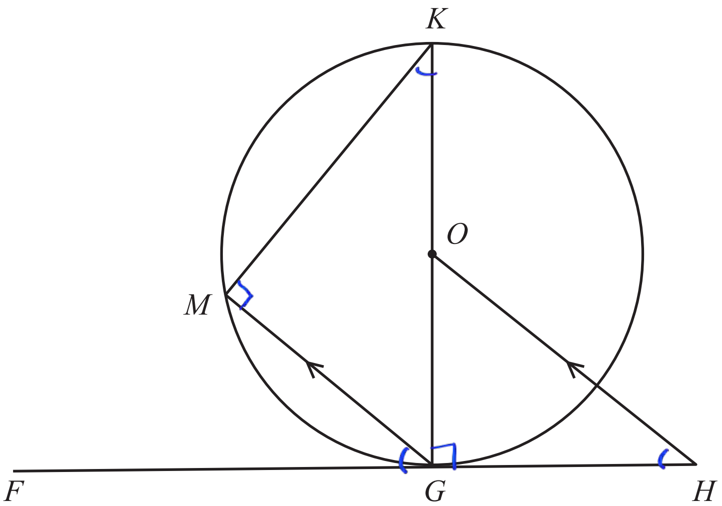
P, Q, R, S and T lie on a circle.
WSR is a straight line and angle $WSP = 112°$.

Calculate angle PTR.

$\widehat{PSR} = 180° - 112° = 68°$
$\widehat{PQR} = 180° - 68° = 112°$
$\widehat{PTR} = 180° - 112° = 68°$

Angle PTR = ...68°............ [2]

(d)



NOT TO SCALE

G, K and M lie on a circle, centre O.
FGH is a tangent to the circle at G and MG is parallel to OH.

Show that triangle GKM is mathematically similar to triangle OHG.
Give a geometrical reason for each statement you make.

$\widehat{OGH} = 90°$ (tangent ⊥ radius)
$\widehat{KMG} = 90°$ (angle at semicircle)
$\widehat{MKG} = \widehat{MGF}$ (alternate segment theorem)
$\widehat{MGF} = \widehat{OHG}$ (corresponding angles)
2 pairs of angles are equal => similar [4]

$$\frac{A_{small}}{A_{large}} = \left(\frac{w_{small}}{w_{large}}\right)^2$$

7 Two rectangular picture frames are mathematically similar.

(a) The areas of the frames are 350 cm^2 and 1134 cm^2.
The width of the smaller frame is 17.5 cm.

Calculate the width of the larger frame. w

$$\Rightarrow \quad \frac{350}{1134} = \left(\frac{17.5}{w_{large}}\right)^2$$

$$\Rightarrow \quad w_{large} = 17.5 : \sqrt{\frac{350}{1134}}$$

..........31.5.......... cm [3]

(b) A picture in the smaller frame has length 15 cm and width 10.5 cm, both correct to the nearest 5 mm. $= 0.5\text{ cm}$

Calculate the upper bound for the area of this picture.

$$Area_{max} = length_{max} \times width_{max}$$

$$= \left(15 + \frac{0.5}{2}\right)\left(10.5 + \frac{0.5}{2}\right)$$

..........163.9375.......... cm^2 [2]

(c) In a sale, the price of a large frame is reduced by 18%. x
Parthi pays \$166.05 for 5 large frames in the sale.

Calculate the original price of one large frame.

The cost of 1 large frame after reduction: $\frac{166.05}{5} = 33.21$

$$x - 18\% \; x = 33.21$$

$$0.82x = 33.21$$

\$ ....40.5.......... [2]

(d) Parthi advertises a large frame for a price of \$57 or 48.20 euros.
The exchange rate is \$1= 0.88 euros.

Calculate the difference between these prices, in dollars and cents, correct to the nearest cent.

$$48.20 \text{ euros} = \$\,\frac{48.20}{0.88} = \$\,\frac{1205}{22}$$

$$\text{difference} = 57 - \frac{1205}{22} = \$\,2\,\frac{5}{22}$$

$\approx$ 2 dollars 22.723 cents

$\approx$ 2 dollars 23 cents

\$ ....2.23.......... [3]

8 Darpan runs a distance of 12 km and then cycles a distance of 26 km.
His running speed is x km/h and his cycling speed is 10 km/h faster than his running speed.
He takes a total time of 2 hours 48 minutes. $2.8h$

(a) An expression for the time, in hours, Darpan takes to run the 12 km is $\frac{12}{x}$.

Write an equation, in terms of x, for the total time he takes in hours.

cycling speed $= x + 10$

Time$_{cycle}$ $= \frac{26}{x+10}$

Total time $= \frac{26}{x+10} + \frac{12}{x}$

$\frac{26}{x+10} + \frac{12}{x} = 2.8$ [3]

(b) Show that this equation simplifies to $7x^2 - 25x - 300 = 0$.

$\frac{26x + 12(x+10)}{(x+10)x} = 2.8$

$26x + 12x + 120 = 2.8(x^2 + 10x)$

$38x + 120 = 2.8x^2 + 28x$

$2.8x^2 - 10x - 120 = 0$

Multiply by 2.5 : $7x^2 - 25x - 300 = 0$ ✓

[4]

(c) Use the quadratic formula to solve $7x^2 - 25x - 300 = 0$.
You must show all your working.

$$x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4 \times 7 \times (-300)}}{2 \times 7}$$

$x =$ ....-5.... or $x =$ ....$\frac{60}{7}$.... [4]

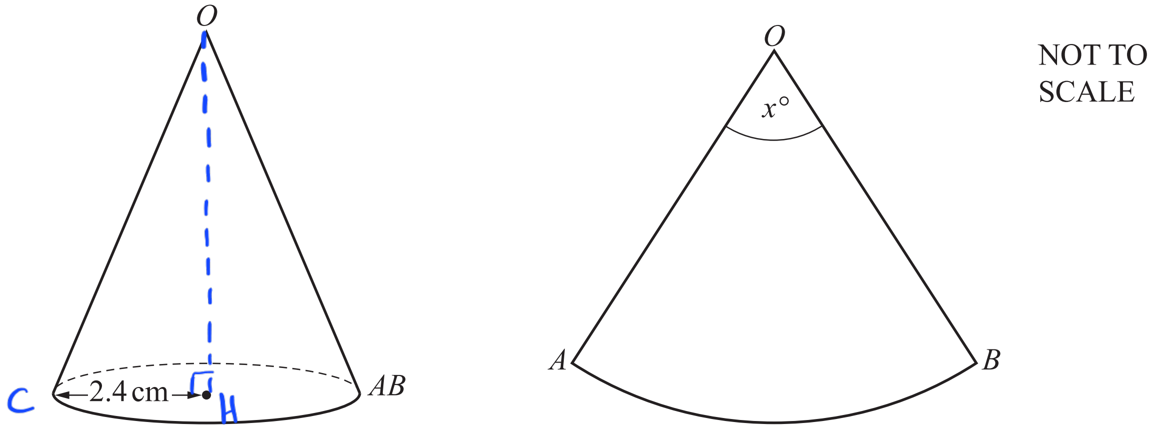
(d) Calculate the number of minutes Darpan takes to run the 12 km.

Because $x > 0$ so $x = \frac{60}{7}$

Time$_{run}$ $= \frac{12}{\frac{60}{7}} = 1.4\,h = 84$ minutes

....84.... min [2]

9 **(a)**



The volume of a paper cone of radius 2.4 cm is 95.4 cm^3.
The paper is cut along the slant height from O to AB.
The cone is opened to form a sector OAB of a circle with centre O.

Calculate the sector angle $x°$.

$$V = 95.4 \Rightarrow \frac{1}{3}\pi \times 2.4^2\, OH = 95.4$$

$$\Rightarrow OH = \frac{795}{16\pi}$$

$$OA = OB = OC = \sqrt{2.4^2 + \left(\frac{795}{16\pi}\right)^2} \approx 15.997$$

$$\text{Arc length}_{AB} = \text{perimeter}_{\text{base}} = 2\pi \times 2.4 = 4.8\pi$$

$$\Rightarrow OA\, x = 4.8\pi$$

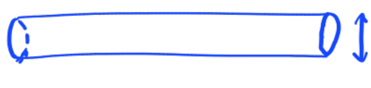
$$x = 4.8\pi : 15.997 = \frac{4800}{15997}\pi \text{ radian}$$

$$x = \frac{4800}{15997}\pi \times \frac{180}{\pi} \approx 54.0°$$

........54.0........ [6]

(b) An empty fuel tank is filled using a cylindrical pipe with diameter 8 cm. 0.08 m
Fuel flows along this pipe at a rate of 2 metres per second.
It takes 24 minutes to fill the tank.
1440 s

Calculate the capacity of the tank.
Give your answer in litres.



The fuel flows a length of $2 \times 1440 = 2880$ m

$$V_{\text{tank}} = \pi \times \left(\frac{0.08}{2}\right)^2 \times 2880 \approx 14.4765 \text{ m}^3$$

$$\approx 14476.5 \ \ell$$

........14500........ litres [4]

10 **(a)** Expand and simplify.

$$(x+1)(x-2)(x+3)$$

$$(x^2 + x - 2x - 2)(x+3)$$
$$(x^2 - x - 2)(x+3)$$
$$x^3 - x^2 - 2x + 3x^2 - 3x - 6$$

$x^3 + 2x^2 - 5x - 6$ .... [3]

(b) Make g the subject of the formula.

$$M = \frac{2fg}{g-c}$$

$$M(g-c) = 2fg$$
$$Mg - Mc = 2fg$$
$$Mg - 2fg = Mc$$
$$(M-2f)g = Mc$$

$g = \dfrac{Mc}{M-2f}$ .... [4]

(c) Simplify.

$$\frac{4x^2 - 16x}{x^2 - 16}$$

$$\frac{4x(x-4)}{x^2 - 4^2}$$

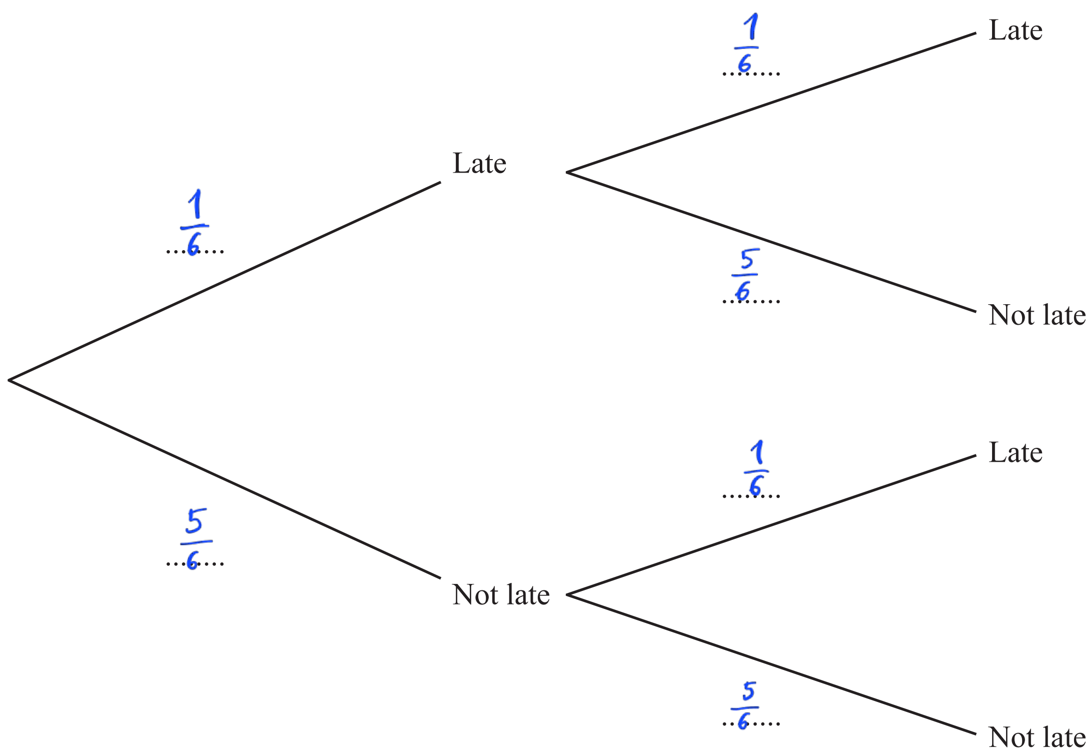
$$\frac{4x(x-4)}{(x-4)(x+4)}$$

$\dfrac{4x}{x+4}$ .... [3]

11 **(a)** The probability that Shalini is late for school on any day is $\frac{1}{6}$.

(i) Complete the tree diagram for Monday and Tuesday.

Monday Tuesday



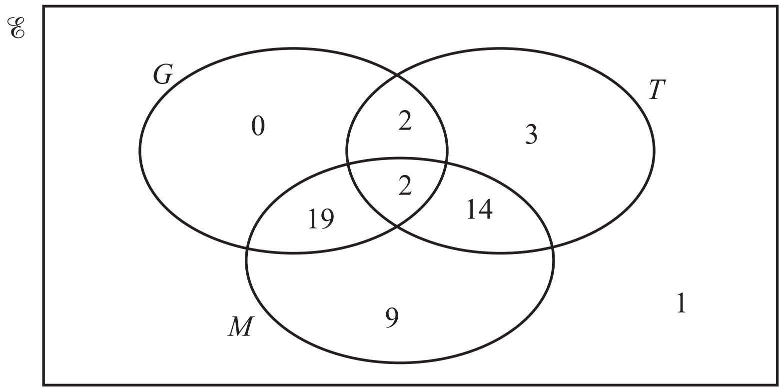
[2]

(ii) Calculate the probability that Shalini is late on Monday but is not late on Tuesday.

$\frac{1}{6} \times \frac{5}{6}$

$\frac{5}{36}$ [2]

(b) The Venn diagram shows the number of students in a group of 50 students who wear glasses (G), who wear trainers (T) and who have a mobile phone (M).



(i) Use set notation to describe the region that contains only one student.

$(G \cup T \cup M)'$ .... [1]

(ii) Find $n(T' \cap (G \cup M))$.

$0 + 19 + 9$

.... 28 .... [1]

(iii) One student is picked at random from the 50 students.

Find the probability that this student wears trainers but does not wear glasses.

$\frac{3 + 14}{50}$

.... $\frac{17}{50}$ .... [1]

(iv) Two students are picked at random from those wearing trainers.

Find the probability that both students have mobile phones.

$\frac{16}{21} \times \frac{15}{20}$

.... $\frac{4}{7}$ .... [3]

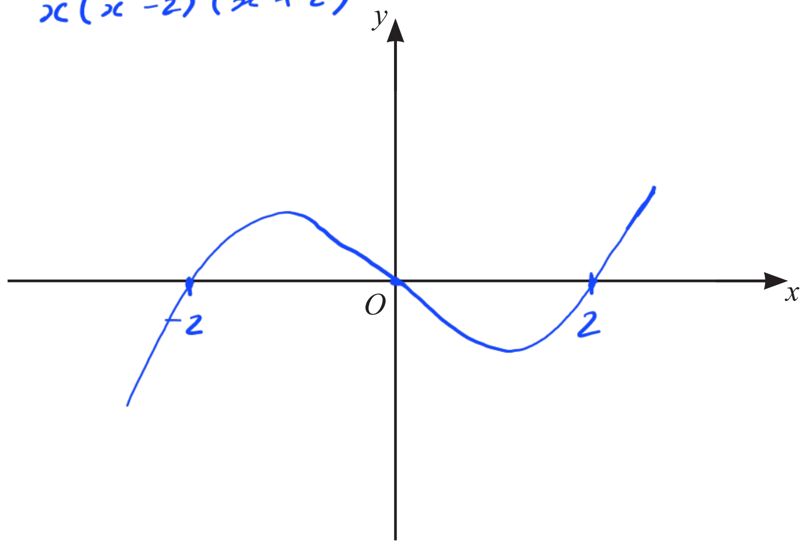
12 **(a)** Solve the equation $\tan x = 11.43$ for $0° \leqslant x \leqslant 360°$.

$x = 85.0°$ or $x = 85.0° + 180° = 265°$

$x =$ ....85.0°.... or $x =$ ....265.0°.... [2]

(b) Sketch the curve $y = x^3 - 4x$.

$x(x-2)(x+2)$



[3]

(c) A curve has equation $y = x^3 + ax + b$.
The stationary points of the curve have coordinates $(2, k)$ and $(-2, 10 - k)$.
Work out the value of a, the value of b and the value of k.

$\frac{dy}{dx} = 3x^2 + a = 0$

$\Rightarrow x^2 = \frac{-a}{3}$

$x = \pm\sqrt{\frac{-a}{3}}$

$\Rightarrow \sqrt{\frac{-a}{3}} = 2 \Rightarrow \frac{-a}{3} = 4 \Rightarrow a = -12$

Sub 2 and -2 into y : $\begin{cases} k = 2^3 - 12 \times 2 + b \\ 10 - k = (-2)^3 - 12(-2) + b \end{cases}$

$\Rightarrow \begin{cases} b - k = 16 \\ b + k = -6 \end{cases}$

$a =$ ....-12.... , $b =$ ....5.... , $k =$ ....-11.... [6]