1 **(a)** The list shows 15 midday temperatures, in degrees Celsius, in Suntown.

17 21 21 18 23 22 25 19

21 17 19 18 21 24 23

(i) Complete the stem-and-leaf diagram to show this information.

1	7 7 8 8 9 9
2	1 1 1 1 2 3 3 4 5

Key: 1|7 represents 17 °C

[2]

(ii) Find the median.

.....21..... °C [1]

(iii) Find the upper quartile.

.....23..... °C [1]

(iv) Rahul draws a pie chart to show this information.

Calculate the sector angle for the number of days the temperature is 18 °C.

$\frac{2}{15} \times 360^\circ = 48^\circ$

.....48°..... [2]

(c) **(i)** The time, t minutes, spent on homework in one week by each of 200 students is recorded. The table shows the results.

Mid value	50	70	85	95	125
Time (t minutes)	$40 < t \leqslant 60$	$60 < t \leqslant 80$	$80 < t \leqslant 90$	$90 < t \leqslant 100$	$100 < t \leqslant 150$
Frequency	6	10	70	84	30

Calculate an estimate of the mean.

$$\frac{(50 \times 6) + (70 \times 10) + (85 \times 70) + (95 \times 84) + (125 \times 30)}{200}$$

.....93.4..... min [4]

(ii) A new table with different class intervals is completed.

Time (t minutes)	$40 < t \leqslant 90$	$90 < t \leqslant 150$
Frequency	86	114
Freq density	1.72	1.9

On a histogram the height of the bar for the $40 < t \leqslant 90$ interval is 17.2 cm.

Calculate the height of the bar for the $90 < t \leqslant 150$ interval.

$\frac{17.2 \times 1.9}{1.72} = 19$

19 cm [2]

2 **(a)** Alex, Bobbie and Chris share strawberries in the ratio Alex : Bobbie : Chris = 3 : 2 : 2.
Chris receives 12 strawberries.
Calculate the total number of strawberries shared.

$\frac{12}{2} \times (3 + 2 + 2)$

42 [2]

(b) In a sale, a shop reduces all prices by 12%.

(i) Dina buys a book which has an original price of $6.50 .

Calculate how much Dina pays for the book.

$6.50 - 6.50 \times 12\%$

$ 5.72 [2]

(ii) Elu pays $11 for a toy.

Calculate the original price of the toy.

$t - 12\% t = 11$

$0.88 t = 11$

$t = \frac{11}{0.88}$

$ 12.5 [2]

(c) Feri invests some money. x
The rate of interest for the first year is 2.5%.
At the end of the second year the overall percentage increase of Feri's investment is 6.6%.
Find the rate of interest for the second year.

Invest x

End of 1st year: $x + 2.5\% x = 1.025x$

End of 2nd year: $x + 6.6\% x = 1.066x$

$\Rightarrow \text{Rate}_{2nd} = \frac{1.066x - 1.025x}{1.025x} \times 100 = 4$

4 % [2]

(d) A radioactive substance decays at an exponential rate of 2% per day.
The initial mass is 80 g.

(i) Find the mass at the end of 5 days.

$80\left(1-\frac{2}{100}\right)^5 \approx 72.3$

.....72.3..... g [2]

(ii) Find how many **more** whole days, after day 5, it takes for the mass to reduce to less than 67 g.

$80\left(1-\frac{2}{100}\right)^n < 67$

use trial & error method:

$80\left(1-\frac{2}{100}\right)^8 = 68.1 > 67$

$80\left(1-\frac{2}{100}\right)^9 = 66.7 < 67$ $\Rightarrow n = 9$

$9 - 5 = 4$

.....4..... [3]

3 **(a)** Geeta buys x apples, $(x+7)$ oranges and $(2x-1)$ bananas.
The total number of pieces of fruit Geeta buys is 30.

(i) Find the number of apples Geeta buys.

$x + (x+7) + (2x-1) = 30$

$4x + 6 = 30$

$4x = 24$

$x = 6$

.....6..... [3]

(ii) The cost of one apple is 15 cents.
The cost of one orange is 18 cents.
The total cost of all the fruit is \$5.55 . = 555 cents

Find the cost, in cents, of one banana. b

$(15 \times 6) + 18(6+7) + b(2\times 6 - 1) = 555$

$324 + 11b = 555$

$11b = 231$

$b = 21$

.....21..... cents [3]

(b) **(i)** Solve.

$$\frac{3w}{16} - 1 = \frac{1}{2}$$

$\frac{3w}{16} = \frac{1}{2} + 1 = 1.5$

$3w = 1.5 \times 16 = 24$

$w =$ .....8..... [2]

(ii) $$\frac{3(2^{-y})}{16} - 1 = \frac{1}{2}$$

Find the value of y.

$\frac{3}{16 \times 2^y} = \frac{1}{2} + 1 = \frac{3}{2}$

$3 \times 2 = 3 \times 16 \times 2^y$

$6 = 48 \times 2^y$

$2^y = \frac{1}{8} = 2^{-3}$

$y =$ .....-3..... [2]

(c) **(i)** Solve the simultaneous equations.

$$2p+q=2$$
$$+\quad p-q=-\frac{1}{2}$$

$2p+p = 2-\frac{1}{2}$

$3p = 1.5$

$p = 0.5$

$\Rightarrow 2\times 0.5 + q = 2$

$\Rightarrow 1 + q = 2$

$p =$ ....0.5....

$q =$ ....1.... [2]

(ii) Hence, for $0° \leqslant u \leqslant 360°$ and $0° \leqslant v \leqslant 360°$, solve the simultaneous equations.

$$2\sin u+\cos v=2$$
$$\sin u-\cos v=-\frac{1}{2}$$

$\sin u = 0.5$ and $\cos v = 1$

$u = 30°$ or $u = 180° - 30°$ and $v = 0°$ or $v = 360°$

$= 150°$

$u =$ ....30.0°.... or $u =$ ....150°....

$v =$ ....0°.... or $v =$ ....360°.... [4]

4 $f(x) = 2x - 1$ $g(x) = 3x - 2$ $h(x) = \frac{1}{x}$, $x \neq 0$ $j(x) = 5^x$

(a) Find

(i) $f(2)$,

$2 \times 2 - 1$

.....3..... [1]

(ii) $gf(2)$.

$g(3) = 3 \times 3 - 2$

.....25..... [1]

(b) Find $g^{-1}(x)$.

g: $\times 3 \longrightarrow -2$

$\div 3 \longleftarrow +2$ g^{-1}

$g^{-1}(x) =$ ..... $\frac{x+2}{3}$ ..... [2]

(c) Find x when $h(x) = j(-2)$.

$\frac{1}{x} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

$x =$ .....25..... [2]

(d) Write $f(x) - h(x)$ as a single fraction.

$(2x - 1) - \frac{1}{x}$

$= \frac{(2x - 1)x - 1}{x}$

$= \frac{2x^2 - x - 1}{x}$

..... $\frac{2x^2 - x - 1}{x}$ ..... [2]

(e) Find the value of $jj(2)$.

$j(2) = 5^2 = 25$

$j(25) = 5^{25}$

$\approx 2.98 \times 10^{17}$

.....2.98×10^{17}..... [1]

(f) Find x when $j^{-1}(x) = 4$.

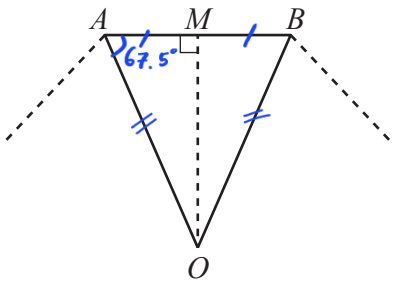
$y = 5^x$

Swap: $x = 5^y = 5^{j^{-1}(x)}$

When $j^{-1}(x) = 4$: $x = 5^4$

$x =$ .....625..... [2]

5 **(a)** *ABCDEFGH* is a regular octagon with sides of length 6 cm.
The diagram shows part of the octagon.
O is the centre of the octagon and *M* is the midpoint of *AB*.



NOT TO SCALE

(i) **(a)** Show that angle *OAM* is 67.5°.

$\widehat{AOB} = \frac{360°}{8} = 45°$

$\widehat{OAB} = \frac{180° - 45°}{2} = 67.5°$ ✓

[2]

(b) Calculate the area of the octagon.

$AM = \frac{1}{2} AB = 3$

$\tan 67.5° = \frac{OM}{AM} = \frac{OM}{3}$

$\Rightarrow OM = 3 \tan 67.5° \approx 7.24264$

$A_{\Delta OAB} = \frac{1}{2} \times 7.24264 \times 6 = 21.72792$

$A_{octagon} = 21.72792 \times 8 \approx 174$

.....174..... cm^2 [4]

(ii) Find the area of the circle that passes through the vertices of the octagon.

The radius of this circle is OA

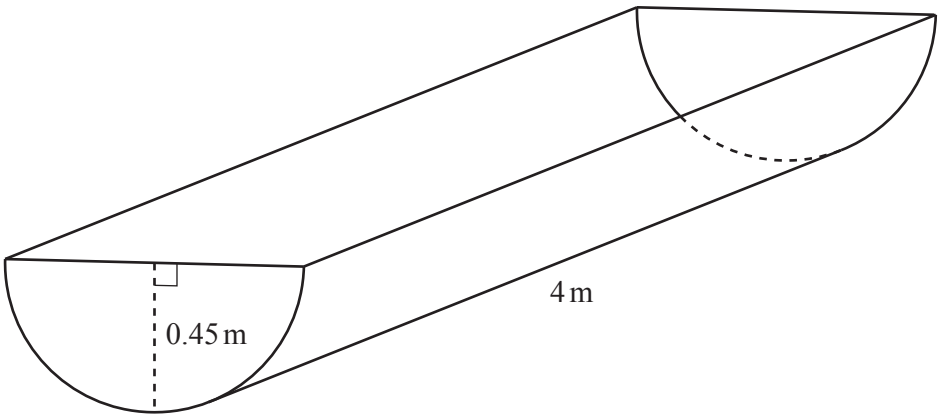
$\cos 67.5° = \frac{AM}{OA} = \frac{3}{OA}$

$\Rightarrow OA = \frac{3}{\cos 67.5°}$

$\Rightarrow$ Area $_{circle} = \pi \times \left(\frac{3}{\cos 67.5°}\right)^2 \approx 193$

.....193..... cm^2 [3]

(b)



NOT TO SCALE

The diagram shows a horizontal container for water with a uniform cross-section.
The cross-section is a semicircle.
The radius of the semicircle is 0.45 m and the length of the container is 4 m.

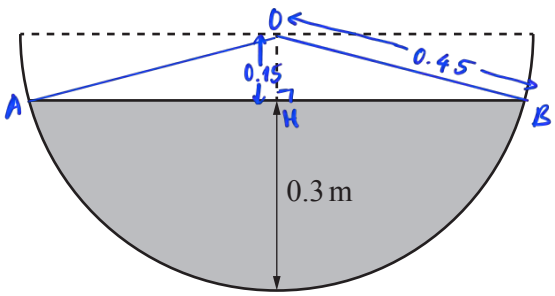
(i) Calculate the volume of the container.

Area of cross-section $= \frac{\pi \times 0.45^2}{2} =$

$V_{container} = \frac{\pi \times 0.45^2}{2} \times 4 \approx 1.27$

.....1.27..... m^3 [2]

(ii)



NOT TO SCALE

The greatest depth of the water in the container is 0.3 m.
The diagram shows the cross-section.

Calculate the number of litres of water in the container.
Give your answer correct to the nearest integer.

$\cos \widehat{BOH} = \frac{0.15}{0.45} = \frac{1}{3}$

$\widehat{BOH} = 70.529°$

$\Rightarrow \widehat{AOB} = 2 \times 70.529° = 141.058° \approx 2.4619$ radian

$A_{sector\ OAB} = \frac{1}{2} \times 0.45^2 \times 2.4619 = 0.24927$

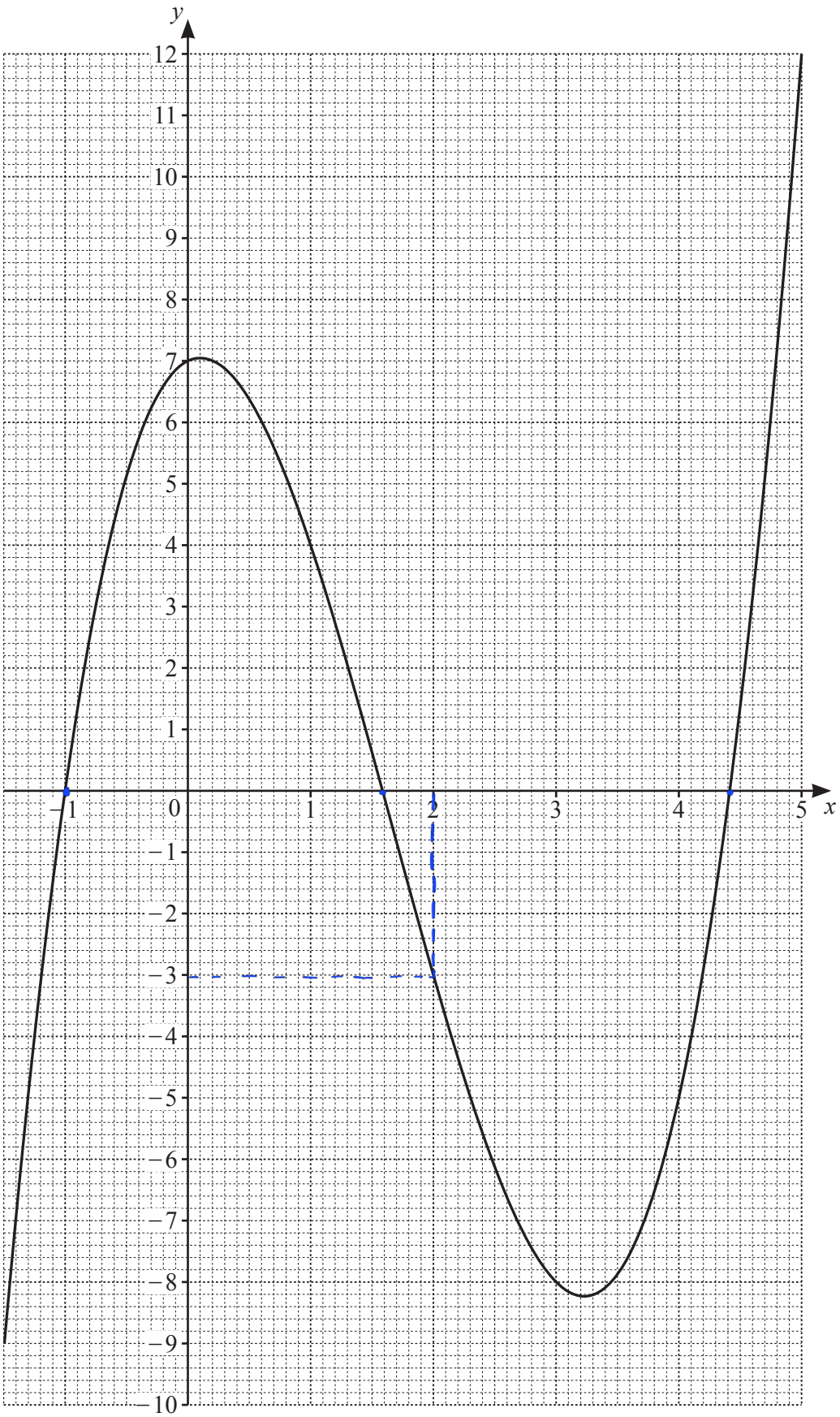
$A_{\Delta OAB} = \frac{1}{2} \times 0.45 \times 0.45 \sin 141.058° = 0.063639$

$A_{shaded\ cross-section} = 0.24927 - 0.063639 = 0.185631$

$\Rightarrow V_{water} = 0.185631 \times 4 = 0.742524\ m^3 \approx 743\ l$

.....743..... litres [6]

6 (a)



The diagram shows the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 5$.

(i) Find $f(2)$.

-3 [1]

(ii) Solve the equation $f(x) = 0$ for $-1.5 \leqslant x \leqslant 5$.

$x = -1$ or $x = 1.6$ or $x = 4.4$ [3]

(iii) $f(x) = k$ has three solutions for $-1.5 \leqslant x \leqslant 5$ where k is an integer.

Find the smallest possible value of k.

$k = -8$ [1]

(iv) On the grid, draw a line $y = mx$ so that $f(x) = mx$ has exactly one solution for $-1.5 \leqslant x \leqslant 5$. [2]

(b) $y = 3x^2 - 12x + 7$

(i) Find the value of $\frac{dy}{dx}$ when $x = 5$.

$\frac{dy}{dx} = 6x - 12$

$\frac{dy}{dx}\Big|_{x=5} = 6 \times 5 - 12$

18 [3]

(ii) Find the coordinates of the point on the graph of $y = 3x^2 - 12x + 7$ where the gradient is 0.

$6x - 12 = 0$

$\Rightarrow x = 2$

$y = 3 \times 2^2 - 12 \times 2 + 7 = -5$

(2 , -5) [2]

(c) When $y = 2x^p + qx^2$, $\frac{dy}{dx} = 14x^6 + 6x$.

Find the value of p and the value of q.

$\frac{dy}{dx} = 2px^{p-1} + 2qx$

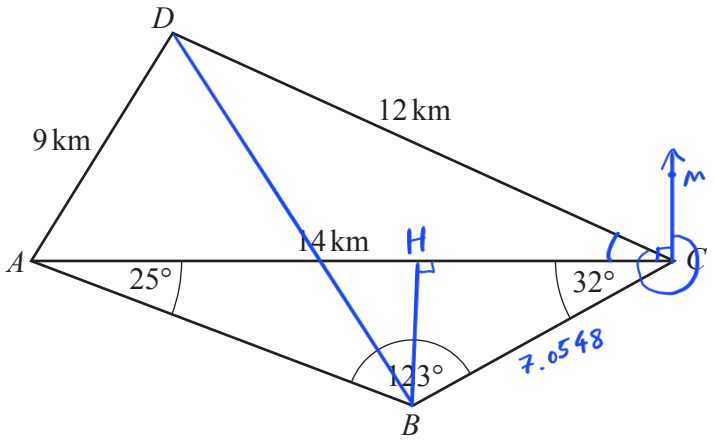
$= 14x^6 + 6x$

$\Rightarrow \begin{cases} 2q = 6 \\ p - 1 = 6 \\ 2p = 14 \end{cases}$

$p = 7$

$q = 3$ [2]

7



NOT TO SCALE

(a) Calculate angle ACD.

$9^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \times \cos \widehat{ACD}$

$-259 = -336 \cos \widehat{ACD}$

$\cos \widehat{ACD} = \frac{37}{48}$

$\widehat{ACD} \approx 39.571°$

Angle ACD = 39.6° [4]

(b) Show that $BC = 7.05\,\text{km}$, correct to 2 decimal places.

$\frac{BC}{\sin 25°} = \frac{14}{\sin 123°}$

$BC = \frac{14 \sin 25°}{\sin 123°} \approx 7.0548 \approx 7.05$

[3]

(c) Calculate the shortest distance from B to AC.

ΔBHC ; $\sin 32^\circ = \frac{BH}{BC} = \frac{BH}{7.0548}$

$\Rightarrow BH = 7.0548 \sin 32^\circ$

$BH \approx 3.74$

3.74 ..... km [3]

(d) Calculate the length of the straight line BD.

$\overset{\frown}{BCD} = 32^\circ + 39.571^\circ = 71.571^\circ$

$BD^2 = 12^2 + 7.0548^2 - 2 \times 12 \times 7.0548 \cos 71.571^\circ$

$BD^2 = 140.2447$

$BD \approx 11.8$

$BD =$ 11.8 ..... km [4]

(e) C is due east of A.

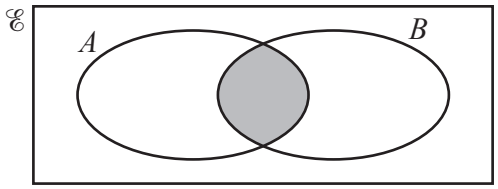
Find the bearing of D from C.

$\overset{\frown}{DCM} = 90^\circ - 39.571^\circ = 50.429^\circ$

$\text{Bearing}_{C \to D} = 360^\circ - 50.429^\circ \approx 309.6^\circ$

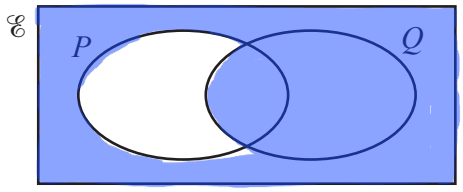
309.6° ..... [2]

8 **(a)** **(i)** Use set notation to describe the shaded region in the Venn diagram.

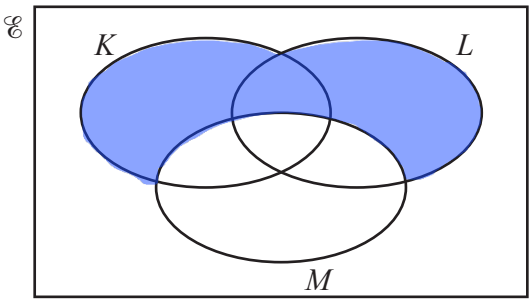


.....$A \cap B$..... [1]

(ii) Shade the correct region in each Venn diagram.



$Q \cup P'$



$(K \cup L) \cap M'$

[2]

(b)

V	E	N	N	D	I	A	G	R	A	M

The diagram shows 11 cards.

(i) One of these cards is chosen at random.

Write down the probability that the letter on the card is **not** A.

..... $\frac{9}{11}$ ..... [1]

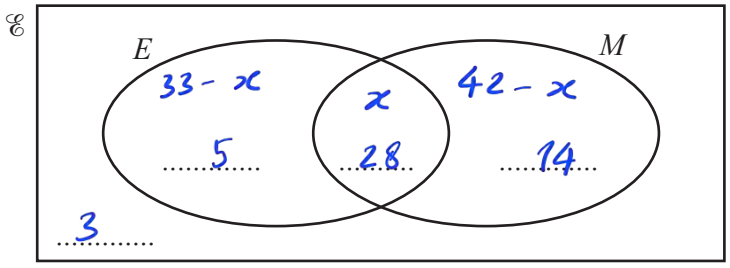
(ii) A card is chosen at random from these 11 cards and then replaced.
A second card is then chosen at random.

Find the probability that exactly one card has the letter N.

$\left(\frac{2}{11} \times \frac{9}{11}\right) \times 2$

..... $\frac{36}{121}$ ..... [3]

(c)



50 students are asked if they like English (E) and if they like mathematics (M).
3 say they do not like English and do not like mathematics.
33 say they like English.
42 say they like mathematics.

$(33-x)+x+(42-x)+3=50$

$78-x=50$

$x=28$

(i) Complete the Venn diagram. [2]

(ii) A student is chosen at random.

Find the probability that this student likes English and likes mathematics.

$\frac{28}{50}$ [1]

(iii) Two students are chosen at random.

Find the probability that they both like mathematics.

$\frac{42}{50} \times \frac{41}{49}$

$\frac{123}{175}$ [2]

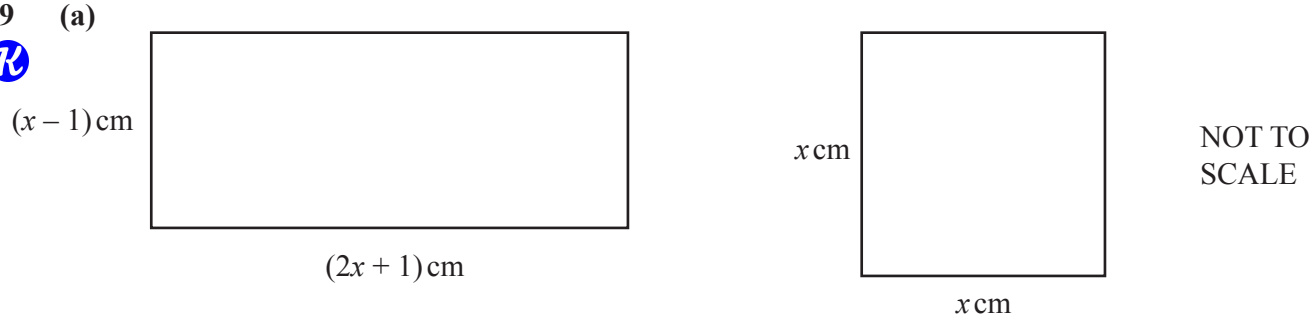
(iv) Two students who like English are chosen at random.

Find the probability that they both also like mathematics.

$\frac{28}{33} \times \frac{27}{32}$

$\frac{63}{88}$ [2]

9 (a)



The area of the rectangle is 29 cm^2 greater than the area of the square.
The difference between the perimeters of the two shapes is k cm.

Find the value of k.
You must show all your working.

$(x-1)(2x+1) - x^2 = 29$

$2x^2 - 2x + x - 1 - x^2 = 29$

$x^2 - x - 30 = 0$

$x^2 - 6x + 5x - 30 = 0$

$x(x-6) + 5(x-6) = 0$

$(x+5)(x-6) = 0$

$x = -5$ or $x = 6$

Because $x > 0$ so $x = 6$

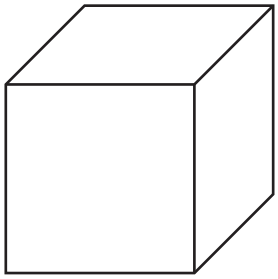
perimeter of rectangle $= 2(6 - 1 + 2 \times 6 + 1) = 36$

perimeter of square $= 4 \times 6 = 24$

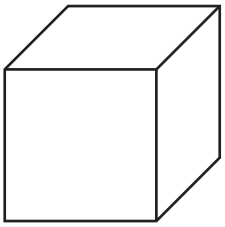
difference $= 36 - 24 = 12$

$k =$ ..... 12 ..... [6]

(b)



$(y + 1)$ cm



y cm

NOT TO SCALE

The volume of the larger cube is $5\,\text{cm}^3$ greater than the volume of the smaller cube.

(i) Show that $3y^2 + 3y - 4 = 0$.

$$(y+1)^3 - y^3 = 5$$
$$(y+1)(y+1)^2 - y^3 = 5$$
$$(y+1)(y^2 + 2y + 1) - y^3 = 5$$
$$(y^3 + 2y^2 + y) + (y^2 + 2y - 1) - y^3 = 5$$
$$3y^2 + 3y - 4 = 0$$

[4]

(ii) Find the volume of the smaller cube.
Show all your working and give your answer correct to 2 decimal places.

$$y = \frac{-3 \pm \sqrt{3^2 - 4 \times 3(-4)}}{2 \times 3}$$
$$y = \frac{-3 \pm \sqrt{57}}{6}$$

Because $y > 0$ so $y = \frac{-3 + \sqrt{57}}{6}$

$$\Rightarrow V_{cube} = \left(\frac{-3 + \sqrt{57}}{6}\right)^3 \approx 0.44$$

..... 0.44 ..... cm^3 [4]