0580/43

May/June 2022

1 Here is part of a bus timetable.

Abbots	06 50	08 25	09 20
Callet	07 12	08 47	09 42
North Moor	07 30	09 05	10 00
South Moor	07 37	09 12	10 07
Centre Point	08 00	09 35	10 30

(a) Rashid catches the 09 20 bus at Abbots.

Find the time the bus arrives at South Moor.

..........10.07.......... [1]

(b) Annisa leaves home at 8.27 am and takes 25 minutes to walk to the bus stop at Callet.
She catches the next bus to Centre Point.

Find the total time, in minutes, for her journey from leaving home to arriving at Centre Point.

Annisa arrives at Callet at 8:27 am + 25' = 8:52 am

She has to catch the 9:42 bus and arrives Central point at 10:30

$8:27 \xrightarrow{33'} 9:00 \xrightarrow{90'} 10:30$ 33' + 90'

..........123.......... min [2]

(c) The distance from Abbots to Centre Point is 29.4 km.
Each bus takes the same time for the journey.

Calculate the average speed of a bus for this journey.
Give your answer in kilometres per hour.

Time = 8:00 − 6:50 = 1h 10' = $1\frac{1}{6}$ h

Speed = $\dfrac{29.4}{1\frac{1}{6}}$

..........25.2.......... km/h [2]

(d) On one journey, all 56 seats on the bus are filled.
The ratio of adults to children on this journey is adults : children = 5 : 3.
a : c
The cost for an adult ticket is \$2.80 .
The cost for a child ticket is $\frac{3}{4}$ of the adult cost. $= \frac{3}{4} \times 2.8 = 2.1$

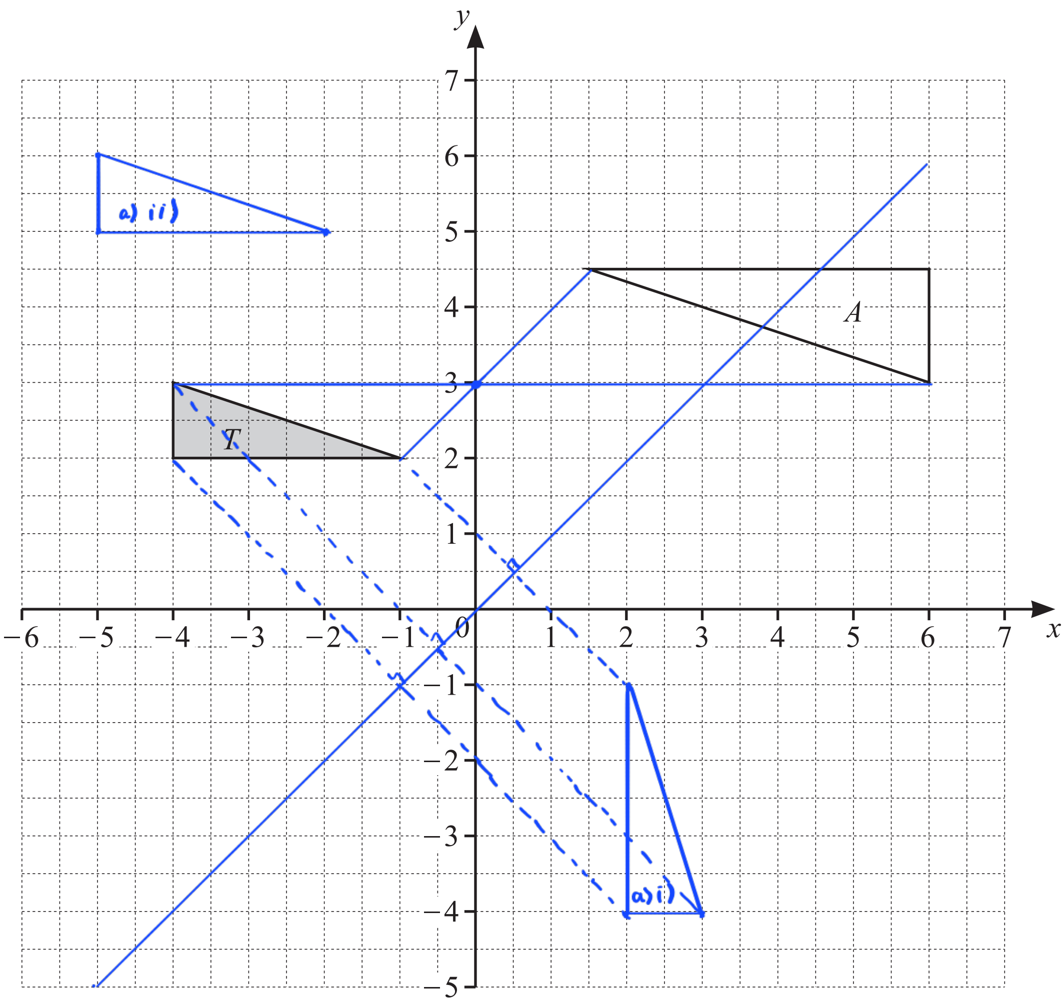
Work out the total cost of the tickets for this journey.

$a = \dfrac{56}{5+3} \times 5 = 35$ $c = 56 - 35 = 21$

Total cost = $35 \times 2.8 + 21 \times 2.1$

\$ ....142.1.......... [4]

2 (a)



(i) Draw the image of triangle T after a reflection in the line $y = x$. [2]

(ii) Draw the image of triangle T after a translation by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$. [2]

(iii) Describe fully the **single** transformation that maps triangle T onto triangle A.

Enlargement, center (0, 3), scale factor = −1.5

.... [3]

(b) A quadrilateral P is enlarged by a scale factor of 1.2 to give quadrilateral Q.
The area of quadrilateral P is 20 cm^2.

Calculate the area of quadrilateral Q.

20×1.2^2

28.8 cm^2 [2]

3 **(a)** The table shows the numbers of tigers reported to be living in the wild in the year 2014 in some countries.

Country	Number
India	2226
Indonesia	371
Nepal	198
Bangladesh	106

(i) Using the table,

(a) find the number of tigers in Nepal as a percentage of the number of tigers in Bangladesh,

$\frac{198}{106} \times 100 \approx 187$

.........187......... % [1]

(b) find the ratio tigers in Bangladesh : tigers in Indonesia : tigers in India, giving your answer in its simplest form.

$106 : 371 : 2226$ HCF(106, 371, 2226) = 53

Divide by 53: $2 : 7 : 42$

.....2..... : .....7..... : .....42..... [2]

(ii) Five years later, the number of tigers reported in India was 2967.

Find the percentage increase in the population of tigers in India.

$\frac{2967 - 2226}{2226} \times 100 \approx 33.3$

.........33.3......... % [2]

(iii) The number of tigers in India in the year 2014 is approximately 30.48% greater than in the year 2010.

Find the number of tigers in India in the year 2010.
Give your answer correct to the nearest integer.

$I_{2014} = I_{2010} + I_{2010} \times 30.48\%$

$2226 = 1.3048\ I_{2010}$

$\Rightarrow I_{2010} = \frac{2226}{1.3048} \approx 1706.009$

.........1706......... [3]

(b) At the start of June, a hive has a population of 2000 bees.
Three months after the start of June the hive has a population of 2662 bees.

The population of this hive can be calculated using the formula

$$P = ab^x,$$

where P is the population of the hive x months after the start of June.

By finding the value of a and the value of b, calculate the population of the hive 7 months after the start of June.
Give your answer correct to the nearest integer.

At the start of June, $x = 0$

$\Rightarrow 2000 = ab^0 = a$

3 moths later: $2662 = 2000\, b^3$

$\Rightarrow b = \sqrt[3]{\dfrac{2662}{2000}} = 1.1$

$\Rightarrow P = 2000 \times 1.1^x$

7 months later: $P = 2000 \times 1.1^7 \approx 3897.4$

.........3897......... [5]

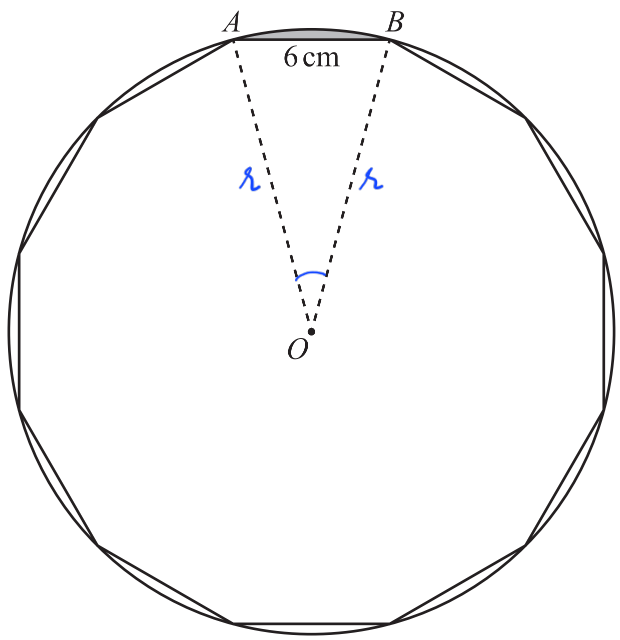
4 A regular 12-sided polygon has side length 6 cm.

(a) Show that one interior angle of the polygon is 150°.

$$\frac{(12-2)180}{12} = 150$$

[1]

(b) The polygon is enclosed by a circle, centre O, so that each vertex touches the circumference of the circle.



NOT TO SCALE

(i) Show that the radius, AO, of the circle is 11.6 cm, correct to 1 decimal place.

$$\widehat{AOB} = \frac{360°}{12} = 30°$$

$$6^2 = r^2 + r^2 - 2r \times r\cos 30°$$

$$36 = 2r^2 - 2r^2\frac{\sqrt{3}}{2}$$

$$36 = (2-\sqrt{3})r^2$$

$$r = \sqrt{\frac{36}{2-\sqrt{3}}} \approx 11.5911 \approx 11.6$$

[3]

(ii) Calculate

(a) the circumference of the circle,

$2\pi \times 11.5911$

.................72.8................. cm [2]

(b) the perimeter of the shaded **minor** segment formed by the chord *AB*.

$30^\circ = \frac{\pi}{6}$ radian

Arc length$_{AB}$ $= 11.5911 \times \frac{\pi}{6} \approx 6.0691$

$\Rightarrow$ perimeter$_{\text{shaded}}$ $= 6.0691 + 6$

.................12.1................. cm [2]

(c) The regular 12-sided polygon is the cross-section of a prism of length 2 cm.

Calculate the volume of the prism.

$A_{\Delta AOB} = \frac{1}{2} \times 11.5911 \times 11.5911 \sin 30^\circ \approx 33.588$

$\Rightarrow$ Area of cross-section $= 12 \ A_{\Delta AOB}$

$= 12 \times 33.588 = 403.056$

$\Rightarrow V_{\text{prism}} = 403.056 \times 2 \approx 806$

.................806................. cm^3 [3]

5 The time, t minutes, taken by each of 80 people to travel to work is recorded.
The table shows information about these times.

Mid value	2.5	7.5	15	27.5	47.5
Time (t minutes)	$0 < t \leqslant 5$	$5 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 35$	$35 < t \leqslant 60$
Frequency	3	7	18	28	24
Cumulative Frequency	3	10	28	56	80

(a) **(i)** Write down the class interval containing the median time.

......20...... $< t \leqslant$ ......35...... [1]

(ii) Calculate an estimate of the mean time.

$$\frac{2.5 \times 3 + 7.5 \times 7 + 15 \times 18 + 27.5 \times 28 + 47.5 \times 24}{80}$$

......28...... min [4]

(b) **(i)** One of these 80 people is chosen at random.

Find the probability that this person took longer than 10 minutes to travel to work.
Give your answer as a fraction in its simplest form.

$$\frac{18 + 28 + 24}{80}$$

......$\frac{7}{8}$...... [2]

(ii) Two people are chosen at random from those taking 20 minutes or less to travel to work.

Calculate the probability that one of these people took 5 minutes or less and the other took more than 5 minutes.

$$P(\leqslant 5 \text{ min}) \times P(> 5 \text{ min}) \times 2$$

$$\frac{3}{28} \times \frac{7 + 18}{27} \times 2$$

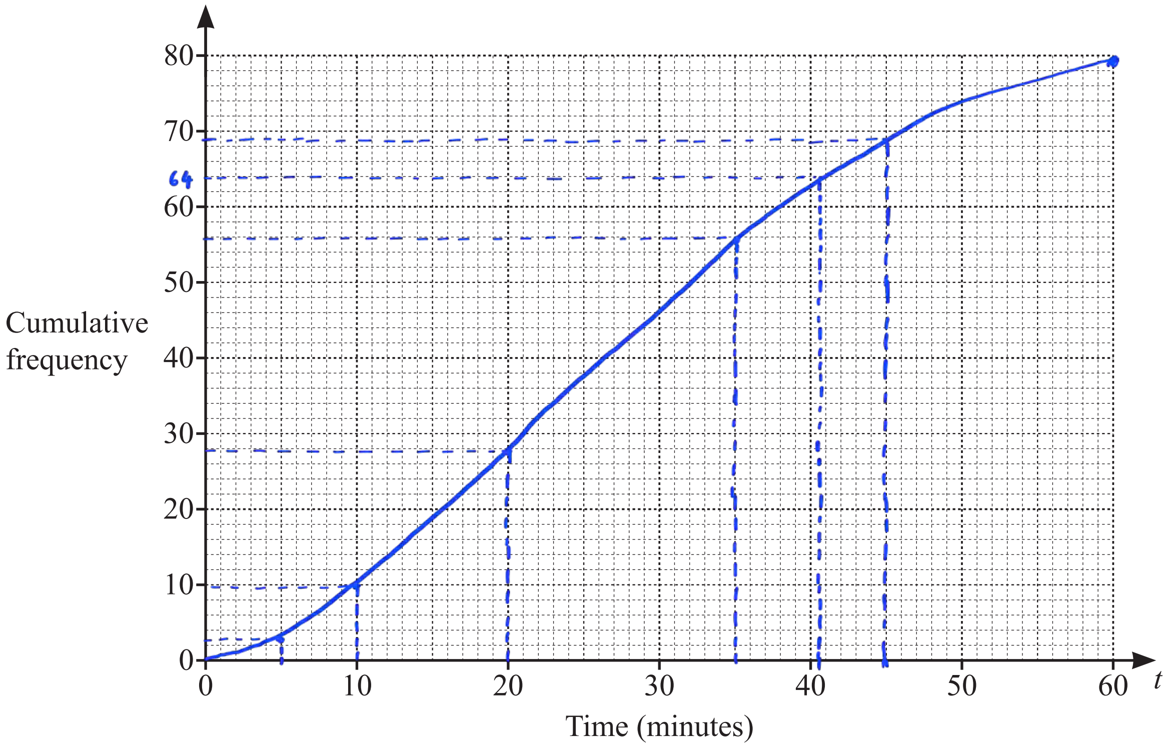
......$\frac{25}{126}$...... [3]

(c) (i) Use the frequency table on page 8 to complete the cumulative frequency table.

Time (t minutes)	$t \leqslant 5$	$t \leqslant 10$	$t \leqslant 20$	$t \leqslant 35$	$t \leqslant 60$
Cumulative frequency	3	10	28	56	80

[1]

(ii) On the grid, draw a cumulative frequency diagram to show this information.



[3]

(iii) Find an estimate for the 80th percentile.

80 × 80% = 64 → 40.5

..........40.5.......... min [2]

(iv) Find an estimate for the percentage of people who took longer than 45 minutes to travel to work.
Show all your working.

80 − 69 = 11

$\frac{11}{80} \times 100$

..........13.75.......... % [3]

6 **(a)** Simplify.

$$a-2b-3a+7b$$

$(a-3a)+(7b-2b)$

$-2a+5b$ [2]

(b) Expand and simplify.

$$4(x-5)-(3-2x)$$

$4x-20-3+2x$

$6x-23$ [2]

(c) Write as a single fraction in its simplest form.

$$\frac{3}{x-5}-\frac{7}{2x}$$

$\dfrac{3 \times 2x - 7(x-5)}{2x(x-5)} = \dfrac{6x-7x+35}{2x^2-10x}$

$\dfrac{-x+35}{2x^2-10x}$ [3]

(d) Solve.

$$\frac{13-4x}{3}=6-x$$

$13-4x=3(6-x)$

$13-4x=18-3x$

$-4x+3x=18-13$

$-x=5$

$x=$ -5 [3]

(e) Make x the subject of the formula.

$$y=\frac{5(p-2x)}{x}$$

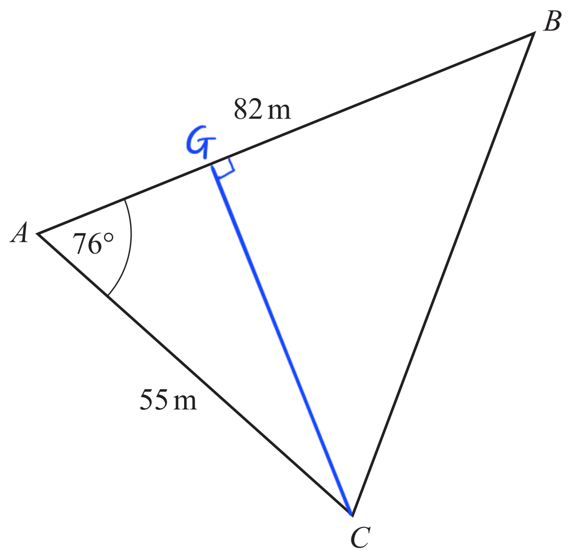
$xy=5p-10x$

$xy+10x=5p$

$x(y+10)=5p$

$x=$ $\dfrac{5p}{y+10}$ [4]

7



The diagram shows a field ABC.

(a) Calculate BC.

$BC^2 = 82^2 + 55^2 - 2 \times 82 \times 55 \times \cos 76° \approx 7566.86$

$BC = \sqrt{7566.86} \approx 87.0$

BC = ...87.0.......... m [3]

(b) Calculate angle ACB.

$$\frac{82}{\sin \widehat{ACB}} = \frac{\sqrt{7566.86}}{\sin 76°}$$

$$\Rightarrow \sin \widehat{ACB} = \frac{82 \sin 76°}{\sqrt{7566.86}} \approx 0.91466$$

$$\widehat{ACB} = 66.16°$$

Angle ACB = ...66.2°.......... [3]

(c) A gate, G, lies on AB at the shortest distance from C.

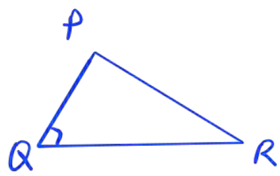
Calculate AG.

ΔAGC : $\cos 76^\circ = \dfrac{AG}{55}$

$\Rightarrow AG = 55 \cos 76^\circ$

$AG =$ ......13.3...... m [3]

(d) A different triangular field PQR has the same area as ABC.
$PQ = 90$ m and $QR = 60$ m.



Work out the two possible values of angle PQR.

$A_{\Delta PQR} = A_{\Delta ABC}$

$\Rightarrow \frac{1}{2}\, 90 \times 60 \times \sin \widehat{PQR} = \frac{1}{2} \times 55 \times 82 \sin 76^\circ$

$\Rightarrow \sin \widehat{PQR} = 0.81038$

$\widehat{PQR} = 54.1^\circ$ or $\widehat{PQR} = 180^\circ - 54.1^\circ = 125.9^\circ$

Angle $PQR =$ ......54.1°...... or ......125.9°...... [5]

8 **(a)** A has coordinates $(-2,7)$, B has coordinates $(1,-5)$ and C has coordinates $(5,4)$.

(i) Find the coordinates of the midpoint of the line AB.

$$\left(\frac{-2+1}{2}, \frac{7+(-5)}{2}\right)$$

(.....-0.5..... , ...1.....) [2]

(ii) Find $\overrightarrow{AC}$.

$$\begin{pmatrix} 5-(-2) \\ 4-7 \end{pmatrix}$$

$\overrightarrow{AC} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ [2]

(iii) Find $|\overrightarrow{AC}|$.

$$\sqrt{7^2+(-3)^2} \simeq 7.62$$

.....7.62..... [2]

(iv) Find the equation of the line AB.
Give your answer in the form $y = mx+c$.

$$m_{AB} = \frac{-5-7}{1-(-2)} = -4$$

$$y - 7 = -4[x-(-2)]$$

$$y - 7 = -4x - 8$$

$$y = -4x - 1$$

$y =$ .....$-4x-1$..... [3]

(v) Find the equation of the line perpendicular to AB that passes through C. ℓ
Give your answer in the form $y = mx + c$.

$m_\ell = -1 : (-4) = 0.25$

$y - 4 = 0.25(x - 5)$

$y - 4 = 0.25x - 1.25$

$y = 0.25x + 2.75$

$y =$ ....$0.25x + 2.75$.... [3]

(b) The graphs of $y + 5x = 8$ (1) and $y = 2x^2 + 6x - 13$ (2) intersect at the points P and Q.

Find the coordinates of P and the coordinates of Q.
Show all your working.

Sub (2) into (1):

$2x^2 + 6x - 13 + 5x = 8$

$2x^2 + 11x - 21 = 0$

$2x^2 - 3x + 14x - 21 = 0$

$x(2x - 3) + 7(2x - 3) = 0$

$(x + 7)(2x - 3) = 0$

$x = -7$ or $x = 1.5$

When $x = -7$, $y = 8 - 5 \times (-7) = 43$

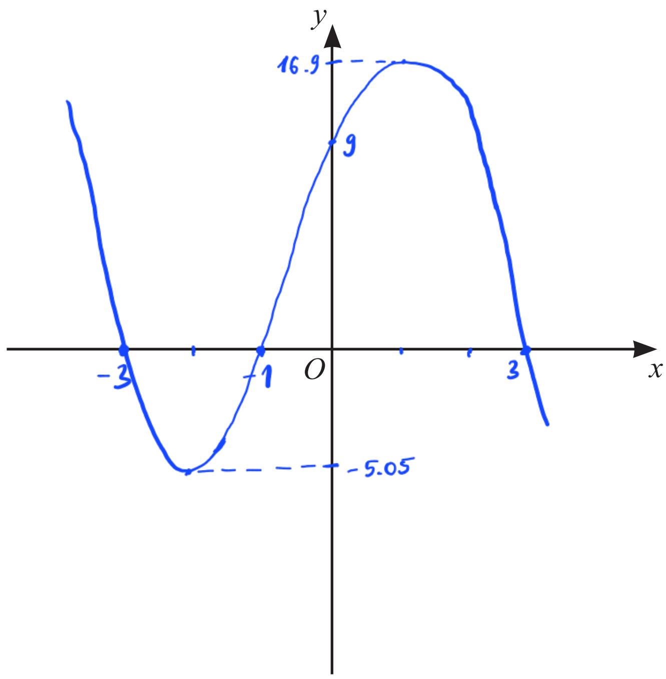
When $x = 1.5$, $y = 8 - 5 \times 1.5 = 0.5$

P (....-7.... , ...43....)

Q (....1.5.... , ...0.5....) [6]

9 **(a)** Sketch the graph of $y = (x+1)(3-x)(3+x)$, indicating the coordinates of the points where the graph crosses the x-axis and the y-axis.

y intercept = (0 + 1)(3 − 0)(3 + 0) = 9



[4]

(b) **(i)** Show that $y = (x+1)(3-x)(3+x)$ can be written as $y = 9 + 9x - x^2 - x^3$.

$y = (3x + 3 - x^2 - x)(3 + x)$

$y = (-x^2 + 2x + 3)(3 + x)$

$y = -3x^2 + 6x + 9 - x^3 + 2x^2 + 3x$

$y = -x^3 - x^2 + 9x + 9$ ✓

[2]

(ii) Calculate the x–values of the turning points of $y = 9 + 9x - x^2 - x^3$.
Show all your working and give your answers correct to 2 decimal places.

$\frac{dy}{dx} = 9 - 2x - 3x^2 = 0$

$3x^2 + 2x - 9 = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-9)}}{2 \times 3}$

$x \approx 1.43$ or $x \approx -2.10$

$x =$ ....1.43........, $x =$ ....-2.10........ [7]

(iii) The equation $9 + 9x - x^2 - x^3 = k$ has one solution only when $k < a$ and when $k > b$, where a and b are integers.

Find the maximum value of a and the minimum value of b.

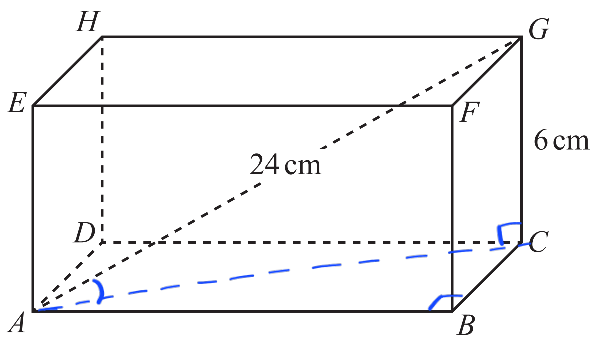
When $x = 1.43$, $y = 9 + 9 \times 1.43 - 1.43^2 - 1.43^3 = 16.90$

When $x = -2.10$, $y = 9 + 9 \times (-2.1) - (-2.1)^2 - (-2.1)^3 = -5.05$

$a =$ ....-6........

$b =$ ....17........ [3]

10



NOT TO SCALE

The diagram shows a cuboid *ABCDEFGH*.
$CG = 6$ cm, $AG = 24$ cm and $AB = 2BC$.

(a) Calculate *AB*.

ΔACG : $AC^2 = 24^2 - 6^2 = 540$

ΔABC : $AC^2 = AB^2 + BC^2$

$540 = (2BC)^2 + BC^2$

$540 = 5BC^2$

$BC^2 = 108$

$BC = 6\sqrt{3}$

$AB = 12\sqrt{3}$

AB = ...20.8.......... cm [4]

(b) Calculate the angle between *AG* and the base *ABCD*.

ΔGAC : $\sin \widehat{GAC} = \frac{6}{24}$

$\Rightarrow \widehat{GAC} = 14.5°$

..........14.5°.......... [3]