0580/42
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SOLVED BY KhanhEdu.com

1 **(a)** Find the lowest common multiple (LCM) of 30 and 75.

$30 = 2 \times 3 \times 5$

30 → 2, 15; 15 → 3, 5

$75 = 3 \times 5^2$

75 → 3, 25; 25 → 5, 5

$2 \times 3 \times 5^2 = 150$ .... [2]

(b) Share \$608 in the ratio 4 : 5 : 7.

$\frac{608}{4+5+7} = 38$

$\times 4 = 152$

$\times 5 = 190$

$\times 7 = 266$

\$ ......152......

\$ ......190......

\$ ......266...... [3]

(c) Work out $\frac{6.39 \times 10^4}{2.45 \times 10^6}$.

Give your answer in standard form.

$0.026082\ldots$

......2.61×10^{-2}...... [2]

(d) Write $0.\dot{2}\dot{7}$ as a fraction.

$x = 0.272727\ldots$

$100x = 27.2727\ldots$

$100x - x = 27 - 0$

$99x = 27$

......$\frac{27}{99}$...... [1]

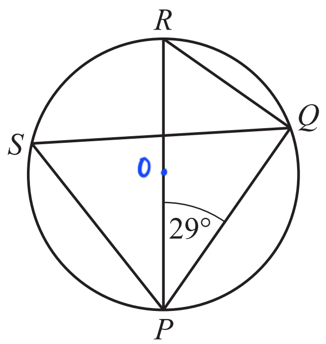
(e) A stone has volume 45 cm^3 and mass 126 g.
Find the density of the stone, giving the units of your answer.

[Density = mass ÷ volume]

$\frac{126\,g}{45\,cm^3} = 2.8\ g/cm^3$

......2.8...... ......g/cm^3 [2]

2 **(a)**



NOT TO SCALE

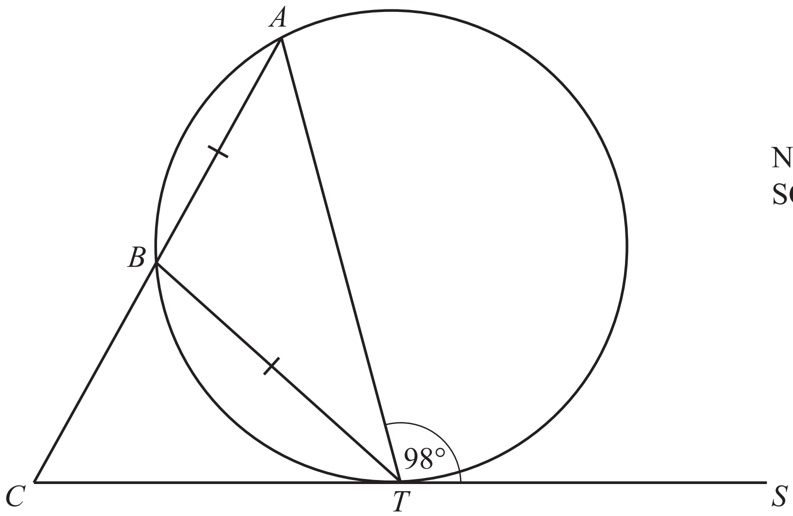
The points P, Q, R and S lie on a circle with diameter PR.

Work out the size of angle PSQ, giving a geometrical reason for each step of your working.

$\widehat{PQR} = 90°$ (angle at semicircle)

$\widehat{PRQ} = 180° - 90° - 29° = 61°$ (sum of 3 angles in a triangle = 180°), $\widehat{PSQ} = \widehat{PRQ} = 61°$ (angles subtend the same arc) [3]

(b)



NOT TO SCALE

The points A, B and T lie on a circle and CTS is a tangent to the circle at T.
ABC is a straight line and $AB = BT$.
Angle $ATS = 98°$.

Work out the size of angle ACT.

$\widehat{ABT} = \widehat{ATS} = 98°$ (alternate segment theorem)

$\widehat{BAT} = \dfrac{180° - 98°}{2} = 41°$

$\widehat{ATC} = 180° - 98° = 82°$

$\widehat{ACT} = 180° - 82° - 41°$

Angle ACT = 57° [4]

3 A line, l, joins point F (3, 2) and point G (− 5, 4).

(a) Calculate the length of line l.

$\sqrt{[3-(-5)]^2 + (2-4)^2} = \sqrt{68} \approx 8.25$

.........8.25......... [3]

(b) Find the equation of the perpendicular bisector of line l in the form $y = mx + c$.

Mid point of FG $= \left(\frac{3+(-5)}{2}, \frac{2+4}{2}\right) = (-1, 3)$

$m_{FG} = \frac{4-2}{-5-3} = -\frac{1}{4}$

=> gradient of perpendicular line $= -1 : \left(-\frac{1}{4}\right) = 4$

=> Equation of perpendicular line : $y - 3 = 4[x - (-1)]$

$y - 3 = 4x + 4$

$y =$ ....$4x + 7$....... [5]

(c) A point H lies on the y-axis such that the distance $GH = 13$ units.

Find the coordinates of the two possible positions of H.

H lies on the y-axis => $x_H = 0$

$\sqrt{[0-(-5)]^2 + (y_H - 4)^2} = 13$

$25 + y_H^2 - 8y_H + 16 = 13^2$

$y_H^2 - 8y_H - 128 = 0$

$y_H^2 - 16y_H + 8y_H - 128 = 0$

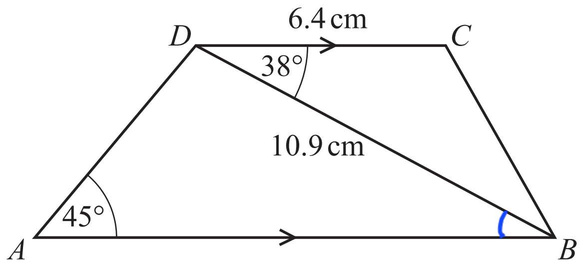
$y_H(y_H - 16) + 8(y_H - 16) = 0$

$(y_H + 8)(y_H - 16) = 0$

$\Rightarrow y_H = -8$ or $y_H = 16$

(.......0......., ...−8.......) and (.......0......., ...16.......) [4]

4



ABCD is a trapezium with *DC* parallel to *AB*.
DC = 6.4 cm, *DB* = 10.9 cm, angle *CDB* = 38° and angle *DAB* = 45°.

(a) Find *CB*.

$CB^2 = 6.4^2 + 10.9^2 - 2 \times 6.4 \times 10.9 \cos 38°$

$CB^2 = 49.8267$

$CB \approx 7.06$

CB = ...7.06........ cm [3]

(b) **(i)** Find angle *ADB*.

$\widehat{ADC} = 180° - 45° = 135°$

$\widehat{ADB} = 135° - 38° = 97°$

Angle *ADB* = ...97°........ [1]

(ii) Find *AB*.

ΔABD : $\dfrac{AB}{\sin \widehat{ADB}} = \dfrac{10.9}{\sin 45°}$

$\Rightarrow AB = \dfrac{10.9 \sin 97°}{\sin 45°} \approx 15.300$

AB = ...15.3........ cm [3]

(c) Calculate the area of the trapezium.

$\widehat{ABD} = \widehat{BDC} = 38°$ (alternating angles)

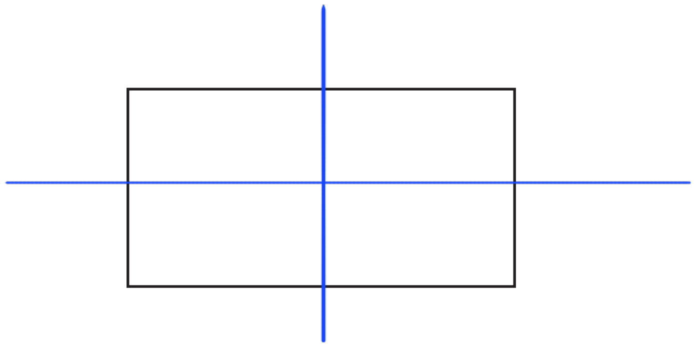
$A_{ABCD} = A_{\Delta ABD} + A_{\Delta BCD}$

$= \frac{1}{2} \times 10.9 \times 15.300 \sin 38° + \frac{1}{2} \times 10.9 \times 6.4 \sin 38°$

≈ 72.8

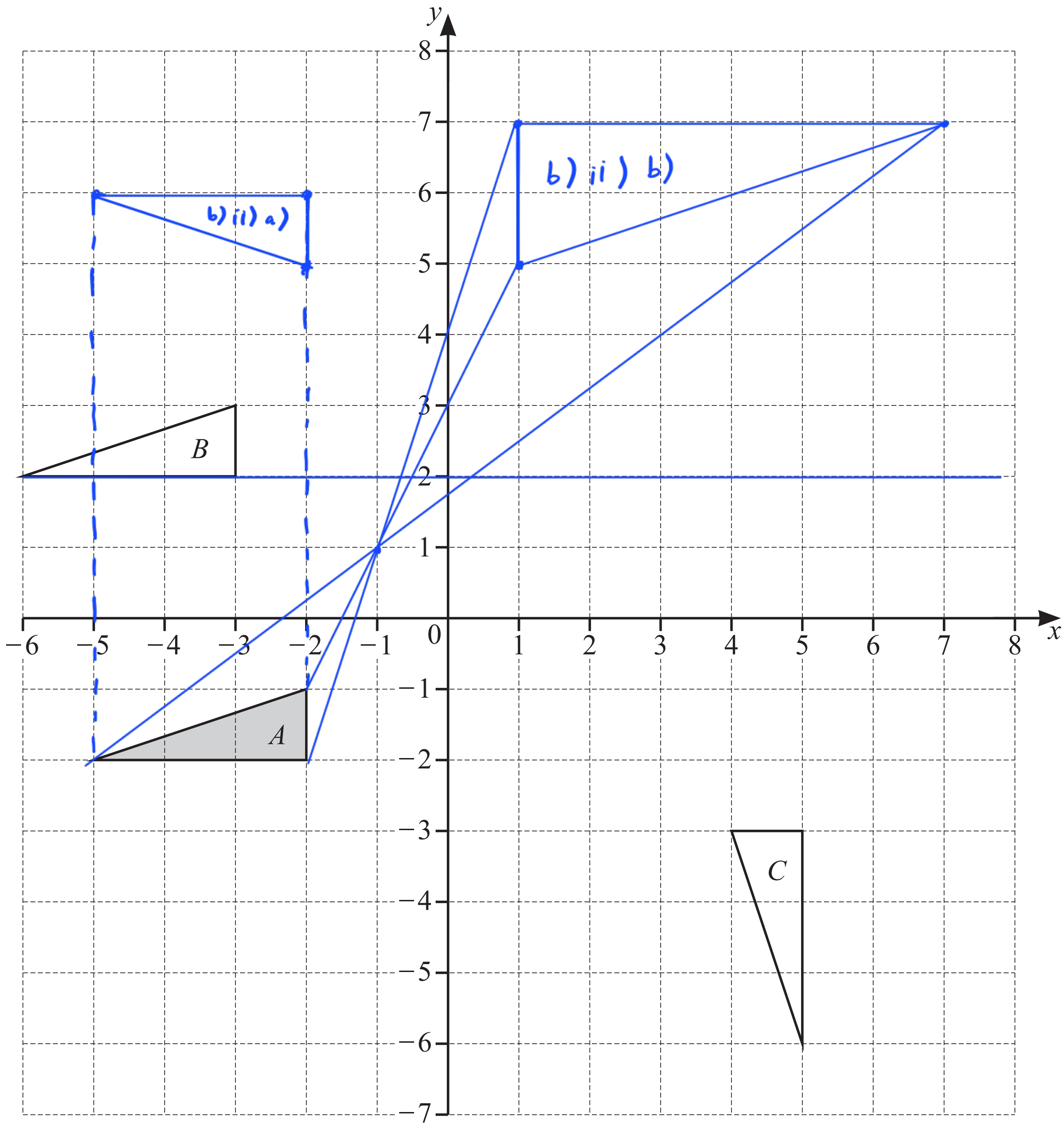
........72.8........ cm^2 [3]

5 **(a)** Draw the lines of symmetry of the rectangle.



[2]

(b)



(i) Describe fully the **single** transformation that maps

(a) triangle A onto triangle B,

Translation , vector $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ ....

.... [2]

(b) triangle A onto triangle C.

Rotation , center $(2, 1)$, anticlockwise $90°$ ....

.... [3]

(ii) **(a)** Draw the image of triangle A after reflection in $y = 2$. [2]

(b) Draw the image of triangle A after enlargement by scale factor -2, centre $(-1, 1)$. [2]

6 **(a)** At a festival, 380 people out of 500 people questioned say that they are camping.
There are 55 300 people at the festival.

Calculate an estimate of the total number of people camping at the festival.

380 — 500

55 300

$$\frac{55360 \times 380}{500}$$

............42 028............ [2]

(b) 12 friends travel to the festival.
5 travel by car, 4 travel by bus and 3 travel by train.
Two people are chosen at random from the 12 friends.

Calculate the probability that they travel by different types of transport.

$$P(C \cap B) \times 2 + P(C \cap T) \times 2 + P(B \cap T)$$

$$\left(\frac{5}{12} \times \frac{4}{11} \times 2\right) + \left(\frac{5}{12} \times \frac{3}{11} \times 2\right) + \left(\frac{4}{12} \times \frac{3}{11} \times 2\right)$$

............$\frac{47}{66}$............ [4]

(c) Arno buys a student ticket for $43.68 .
This is a saving of 16% on the full price of a ticket.

Calculate the full price of a ticket. t

$$t - 16\% t = 43.68$$

$$0.84 t = 43.68$$

$$t = 52$$

$ ....52............ [2]

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(d) At a football match, there are 29 800 people, correct to the nearest 100.

(i) At the end of the football match, the people leave at a rate of 400 people per minute, correct to the nearest 50 people.

Calculate the lower bound for the number of minutes it takes for all the people to leave.

$$time_{min} = \frac{people_{min}}{rate_{max}} = \frac{29800 - \frac{100}{2}}{400 + \frac{50}{2}} = 70$$

..........70.............................. min [3]

(ii) At a cricket match there are 27 500 people, correct to the nearest 100.
Calculate the upper bound for the difference between the number of people at the football match and at the cricket match.

$$differece_{max} = football_{max} - cricket_{min}$$

$$= \left(29800 + \frac{100}{2}\right) - \left(27500 - \frac{100}{2}\right)$$

..........2400.. [2]

7 Information about the mass, m kg, of each of 150 children is recorded in the frequency table.

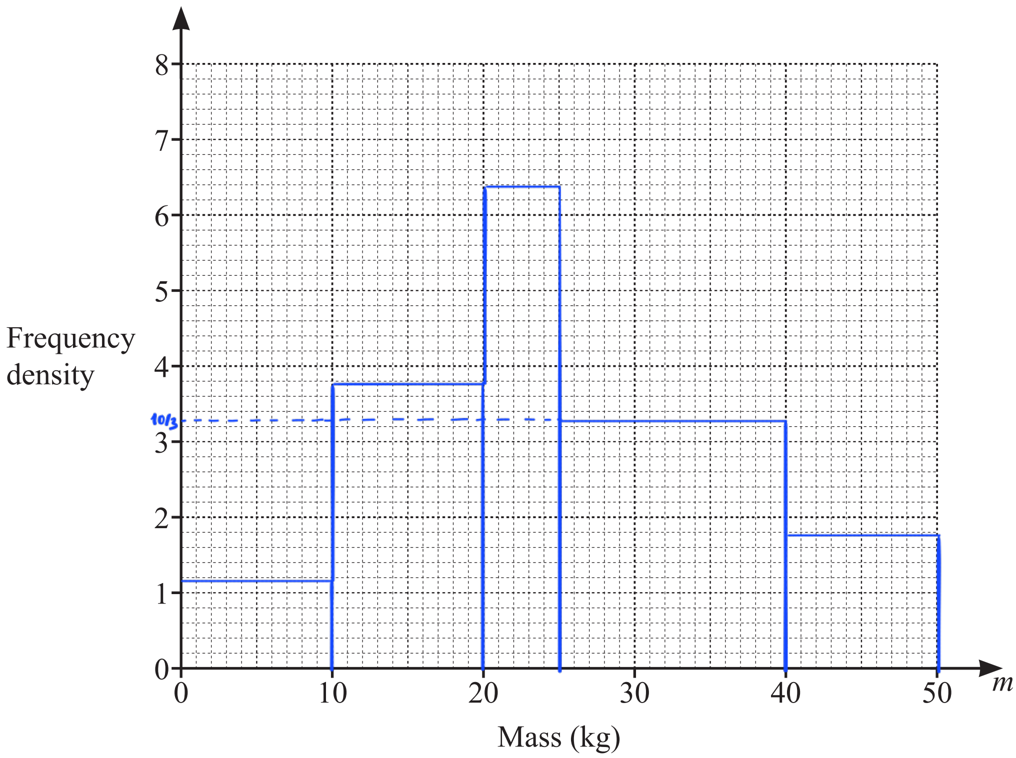
Mid value	5	15	22.5	32.5	45
Mass (m kg)	$0 < m \leqslant 10$	$10 < m \leqslant 20$	$20 < m \leqslant 25$	$25 < m \leqslant 40$	$40 < m \leqslant 50$
Frequency	12	38	32	50	18
Freq density	1.2	3.8	6.4	10/3	1.8

(a) Calculate an estimate of the mean mass.

$$\frac{(5 \times 12) + (15 \times 28) + (22.5 \times 32) + (32.5 \times 50) + (45 \times 18)}{150}$$

$\frac{757}{30}$ kg [4]

(b) Draw a histogram to show the information in the table.



[4]

(c) **(i)** Use the frequency table to complete this cumulative frequency table.

Mass (m kg)	$m \leqslant 10$	$m \leqslant 20$	$m \leqslant 25$	$m \leqslant 40$	$m \leqslant 50$
Cumulative frequency	12	50	82	132	150

[2]

(ii) Calculate the percentage of children with a mass greater than 10 kg.

$$\frac{150 - 12}{150} \times 100$$

92 % [2]

8 **(a)** Solve.

$$10 - 3p = 3 + 11p$$

$10 - 3 = 11p + 3p$

$7 = 14p$

$p = $ ......0.5...... [2]

(b) Make m the subject of the formula.

$$mc^2 - 2k = mg$$

$mc^2 - mg = 2k$

$m(c^2 - g) = 2k$

$m = \frac{2k}{c^2 - g}$

$m = $ ......$\frac{2k}{c^2-g}$...... [3]

(c) Solve.

$$\frac{1}{x-3} + \frac{4}{2x+3} = 1$$

$\frac{(2x+3) + 4(x-3)}{(x-3)(2x+3)} = 1$

$2x + 3 + 4x - 12 = 2x^2 - 6x + 3x - 9$

$6x - 9 = 2x^2 - 3x - 9$

$2x^2 - 9x = 0$

$x(2x - 9) = 0$

$x = 0$ or $x = \frac{9}{2}$

$x = $ ......0...... or $x = $ ......$\frac{9}{2}$...... [5]

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(d) Solve the simultaneous equations.
You must show all your working.

$$x + 2y = 12 \qquad (1)$$
$$5x + y^2 = 39 \qquad (2)$$

$(1) \Rightarrow \quad x = 12 - 2y$

Sub x into (2) : $\quad 5(12 - 2y) + y^2 = 39$

$$60 - 10y + y^2 = 39$$
$$y^2 - 10y + 21 = 0$$
$$y^2 - 3y - 7y + 21 = 0$$
$$y(y-3) - 7(y-3) = 0$$
$$(y-7)(y-3) = 0$$
$$y = 7 \quad \text{or} \quad y = 3$$

When $y = 7$, $x = 12 - 2 \times 7 = -2$

When $y = 3$, $x = 12 - 2 \times 3 = 6$

$x =$ ....-2........ $y =$ ....7........

$x =$ ....6........ $y =$ ....7........ [5]

(e) Expand and simplify.

$$(2x - 3)(x + 6)(x - 4)$$

$$(2x^2 - 3x + 12x - 18)(x - 4)$$
$$(2x^2 + 9x - 18)(x - 4)$$
$$2x^3 + 9x^2 - 18x - 8x^2 - 36x + 72$$

$2x^3 + x^2 - 54x + 72$ [3]

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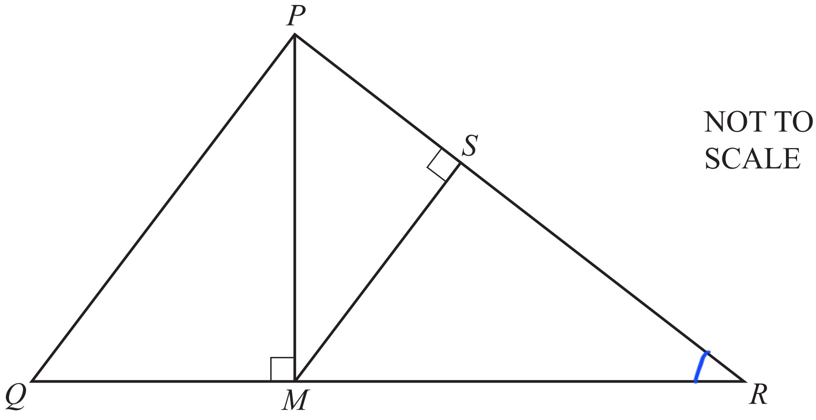
9 **(a)**



In triangle *PQR*, *M* lies on *QR* and *S* lies on *PR*.

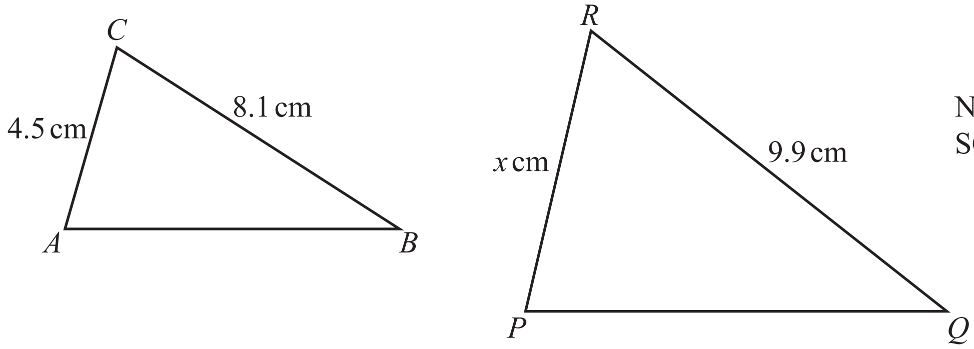
Explain, giving reasons, why triangle *PMR* is similar to triangle *MSR*.

$\widehat{PRQ}$ is a common angle

$\widehat{PMR} = \widehat{MSR}$ ($= 90°$) | corresponding angles

$\Rightarrow \widehat{MPR} = \widehat{SMR}$ | are equal [3]

(b)



Triangle *ABC* is similar to triangle *PQR*.

(i) Find the value of *x*.

$$\frac{x}{4.5} = \frac{9.9}{8.1}$$

$$\Rightarrow x = \frac{4.5 \times 9.9}{8.1}$$

$x =$ 5.5 [2]

(ii) The area of triangle *PQR* is 25 cm^2.

Calculate the area of triangle *ABC*.

$$\frac{A_{\triangle PQR}}{A_{\triangle ABC}} = \left(\frac{QR}{BC}\right)^2 = \left(\frac{9.9}{8.1}\right)^2 = \frac{121}{81}$$

$$A_{\triangle ABC} = 25 : \frac{121}{81} \approx 16.7$$

16.7 cm^2 [2]

10 **(a)** Find all the positive integers which satisfy the inequality.

$$3n - 8 > 5n - 15$$

$3n - 5n > -15 + 8$

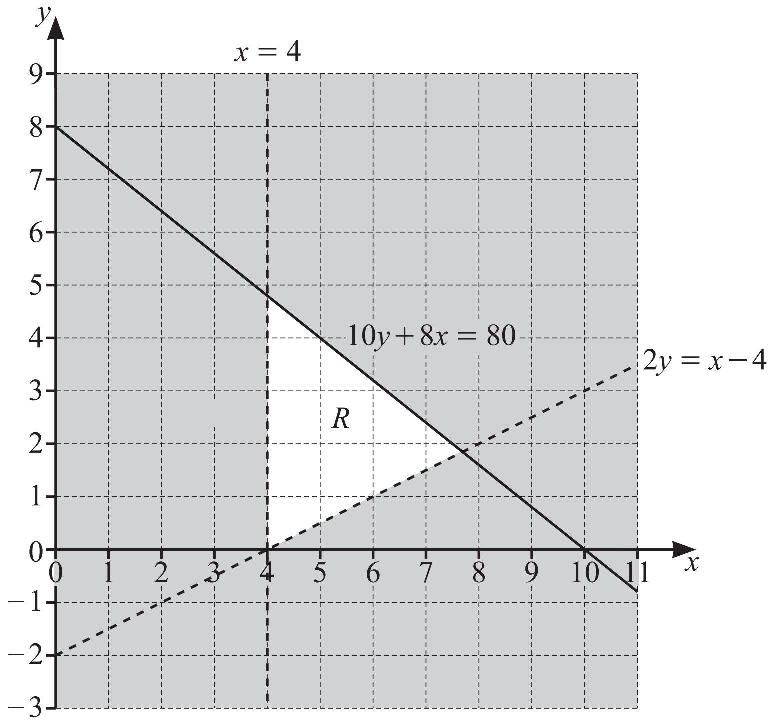
$-2n > -7$

$n < \frac{-7}{-2}$

$n < 3.5$

..........1, 2, 3.......... [2]

(b)



The region marked R is defined by three inequalities.

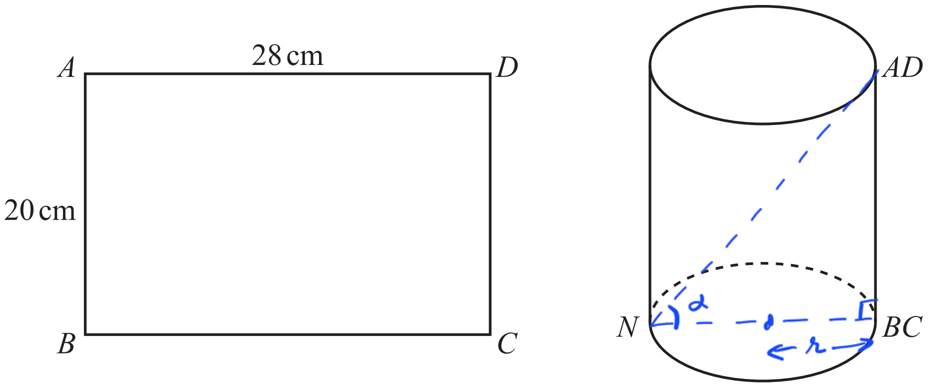
(i) Find these three inequalities.

..........$10y + 8x \leq 80$..........

..........$x > 4$..........

..........$2y > x - 4$.......... [3]

11 (a)



NOT TO SCALE

A rectangular sheet of paper *ABCD* is made into an open cylinder with the edge *AB* meeting the edge *DC*.
AD = 28 cm and *AB* = 20 cm.

(i) Show that the radius of the cylinder is 4.46 cm, correct to 3 significant figures.

AD is perimeter of the base of cylinder

$\Rightarrow 28 = 2\pi r$

$r = \frac{28}{2\pi} \approx 4.4563 \approx 4.46$

[2]

(ii) Calculate the volume of the cylinder.

$V = \pi \times 4.4563^2 \times 20 \approx 1250$

........1250........ cm^3 [2]

(iii) *N* is a point on the base of the cylinder, such that *BN* is a diameter.

Calculate the angle between $\overset{\alpha}{AN}$ and the base of the cylinder.

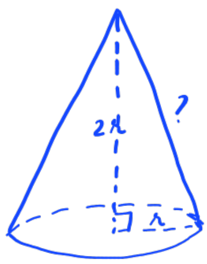
$NB = 2 \times 4.4563 = 8.9126$

$\tan\alpha = \frac{20}{8.9126}$

$\alpha \approx 66.0^\circ$

........66.0°........ [3]

(b) The volume of a solid cone is $310\,\text{cm}^3$.
The height of the cone is twice the radius of its base.
Calculate the slant height of the cone.



radius = r => height = $2r$

$V_{cone} = 310$

$\Rightarrow \frac{1}{3}\pi r^2 (2r) = 310$

$\frac{2}{3}\pi r^3 = 310$

$r^3 = \frac{465}{\pi}$

$r = \sqrt[3]{\frac{465}{\pi}} \approx 5.28974$

=> slant height $= \sqrt{5.28974^2 + (2 \times 5.28974)^2}$

≈ 11.8

........11.8........ cm [5]

12 A curve has equation $y = x^3 - kx^2 + 1$.
When $x = 2$, the gradient of the curve is 6.

(a) Show that $k = 1.5$.

$\frac{dy}{dx} = 3x^2 - 2kx$

When $x = 2$: $6 = 3 \times 2^2 - 2k \times 2$

$\Rightarrow 4k = 6$

$k = 1.5$

[5]

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(b) Find the coordinates of the two stationary points of $y = x^3 - 1.5x^2 + 1$.
You must show all your working.

$\frac{dy}{dx} = 3x^2 - 2 \times 1.5x = 0$

$\Rightarrow 3x(x - 1) = 0$

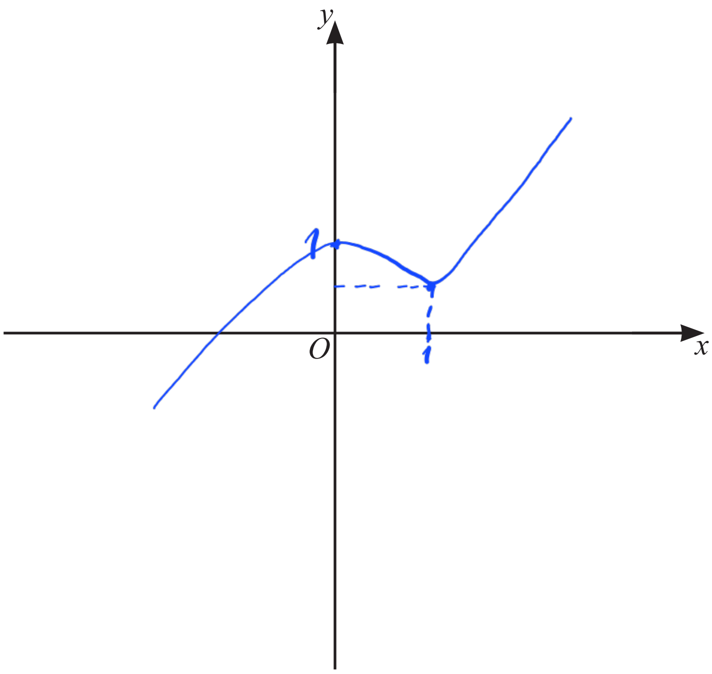
$x = 0$ or $x = 1$

When $x = 0$, $y = 0^3 - 1.5 \times 0^2 + 1 = 1$

When $x = 1$, $y = 1^3 - 1.5 \times 1^2 + 1 = 0.5$

(......0...... , ...1......) and (......1...... , ...0.5......) [4]

(c) Sketch the curve $y = x^3 - 1.5x^2 + 1$.



[2]