1 **(a)** Calculate the volume of

(i) a solid cylinder with radius 6 cm and height 14 cm,

$\pi 6^2 \times 14 = 504\pi$

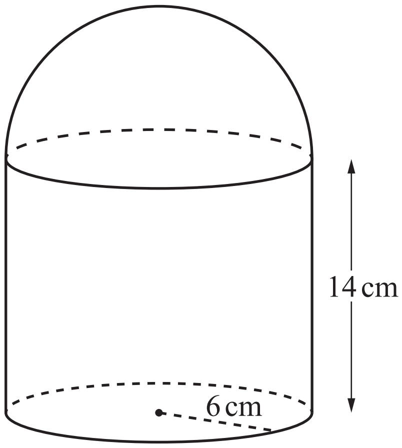
.........1580......... cm^3 [2]

(ii) a solid hemisphere with radius 6 cm.

$\frac{1}{2} \times \frac{4}{3} \pi 6^3 = 144\pi$

.........452......... cm^3 [2]

(b)



NOT TO SCALE

The cylinder and hemisphere in **part (a)** are joined to form the solid in the diagram.
The solid is made of steel and 1 cm^3 of steel has a mass of 7.85 g.

(i) Show that 1 cm^3 of steel has a mass of 0.007 85 kg.

$7.85\ g = \frac{7.85}{1000}\ kg = 0.00785\ kg$

[1]

(ii) Calculate the total mass of the solid.

$V_{solid} = 504\pi + 144\pi = 648\pi$

$\Rightarrow mass_{solid} = 648\pi \times 0.00785$

.........16.0......... kg [2]

(c) 2000 cm^3 of iron is melted down and some of it is used to make 50 spheres with radius 2 cm.

(i) Calculate the percentage of iron that is left over.

$V_{50 \text{ spheres}} = 50 \times \frac{4}{3} \pi 2^3 = \frac{1600\pi}{3}$

$\text{percentage}_{\text{left over}} = \frac{2000 - \frac{1600\pi}{3}}{2000} \times 100$

.........16.2......... % [3]

(ii) The iron left over is then made into a cube.

Calculate the length of an edge of the cube.

$\text{length} = \sqrt[3]{2000 - \frac{1600\pi}{3}} \approx 6.87$

.........6.87......... cm [1]

(d) A solid cone has radius $3R$ cm and slant height $9R$ cm.
A solid cylinder has radius x cm and height $7x$ cm.
The **total** surface area of the cone is equal to the **total** surface area of the cylinder.

Given that $R = kx$, find the value of k.

$A_{\text{cone}} = A_{\text{cylinder}}$

$A_{\text{curved}} + A_{\text{base}} = A_{2 \text{ bases}} + A_{\text{curved}}$

$\pi \times 3R \times 9R + \pi \times (3R)^2 = 2 \times \pi x^2 + 2\pi x\, 7x$

$27R^2\pi + 9R^2\pi = 2\pi x^2 + 14\pi x^2$

$36R^2\pi = 16\pi x^2$

$36R^2 = 16x^2$

$R^2 = \frac{4}{9}x^2$

$R = \frac{2}{3}x$

$k =$ ......$\frac{2}{3}$...... [4]

2 **(a)** Write

(i) 2994.99 correct to the nearest 10,

.......2 990....... [1]

(ii) 0.983 correct to 1 decimal place,

.......1.0....... [1]

(iii) 2090 correct to 2 significant figures.

.......2100....... [1]

(b) Write down a prime number between 90 and 100.

.......97....... [1]

(c) Write 2^{-6} as a fraction.

$\frac{1}{2^6}$

.... $\frac{1}{64}$ .... [1]

(d) Write 0.007 01 in standard form.

.......7.01×10^{-3}....... [1]

(e) Simplify $1.5 \times 10^{x} + 1.5 \times 10^{x-1}$ giving your answer in standard form.

$15 \times 10^{x-1} + 1.5 \times 10^{x-1}$

$(15 + 1.5)\ 10^{x-1}$

$16.5 \times 10^{x-1}$

.......1.65×10^{x}....... [2]

(f) Write $0.3\dot{7}$ as a fraction.
You must show all your working.

$x = 0.3777\ldots$

$10x = 3.777\ldots$

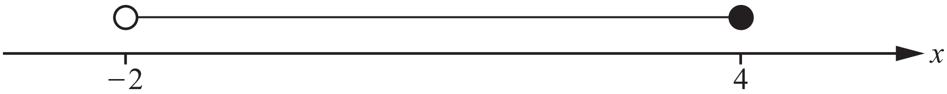
$100x = 37.777\ldots$

$100x - 10x = 37 - 3$

$90x = 34$

.... $\frac{34}{90}$ .... [2]

3 **(a)**



Write down the inequality shown by the number line.

$-2 < x \leq 4$ .... [1]

(b) $-3 \leqslant 2x+3 < 9$

(i) Solve the inequality.

$-3 \leq 2x + 3$ $\quad\quad$ $2x + 3 < 9$

$\Rightarrow -6 \leq 2x$ $\quad\quad$ $2x < 6$

$-3 \leq x$ $\quad\quad$ $x < 3$

$-3 \leq x < 3$ .... [3]

(ii) Write down all the integer values of x that satisfy the inequality.

$-3, -2, -1, 0, 1, 2$ .... [2]

(c) Solve the equations.

(i) $3(3-x)-\dfrac{2(x+2)}{5}=1$

$$9 - 3x - \frac{2x+4}{5} = 1$$

$$\frac{(9-3x)5 - (2x+4)}{5} = 1$$

$$45 - 15x - 2x - 4 = 5$$

$$-17x = -36$$

$x =$ $\dfrac{36}{17}$ .... [4]

(ii) $\dfrac{5}{x+3}=\dfrac{3}{x+5}$

$$5(x+5) = 3(x+3)$$

$$5x + 25 = 3x + 9$$

$$5x - 3x = 9 - 25$$

$$2x = -16$$

$$x = -8$$

$x =$ -8 .... [3]

4 **(a)** **(i)** Zak invests \$500 at a rate of 2% per year simple interest.

Calculate the value of Zak's investment at the end of 5 years.

interest $= 500 \times \frac{2}{100} \times 5 = 50$

Total value $= 500 + 50$

\$ ....550.................................... [3]

(ii) Yasmin invests \$500 at a rate of 1.8% per year compound interest.

Calculate the value of Yasmin's investment at the end of 5 years.

$500\left(1 + \frac{1.8}{100}\right)^5 \approx 546.649$

\$ ....547.................................... [2]

(iii) Zak and Yasmin continue with these investments.

How many **more complete** years is it before the value of Yasmin's investment is greater than the value of Zak's investment?

$500\left(1 + \frac{1.8}{100}\right)^t > 500 + 500 \times \frac{2}{100} \times t$

$500 \times 1.018^t > 500 + 10t$

use trial & error method:

when $t = 12$, Yasmin < Zak

when $t = 13$, Yasmin > Zak

$\Rightarrow t = 13$

$\Rightarrow 13 - 5 = 8$ more years

....8.................................... [3]

(b) Xavier buys a car for \$2500.
The value of the car decreases exponentially at a rate of 10% each year.

Calculate the value of Xavier's car at the end of 5 years.
Give your answer correct to the nearest dollar.

$$2500\left(1-\frac{10}{100}\right)^5 = 1476.225$$

\$ ....1476.................................... [3]

(c) The number of a certain type of bacteria increases exponentially at a rate of r% each day.
After 22 days, the number of this bacteria has doubled.

Find the value of r.

Put b is the number of bacteria at the beginning

$$b\left(1+\frac{r}{100}\right)^{22} = 2b$$

$$\Rightarrow \left(1+\frac{r}{100}\right)^{22} = 2$$

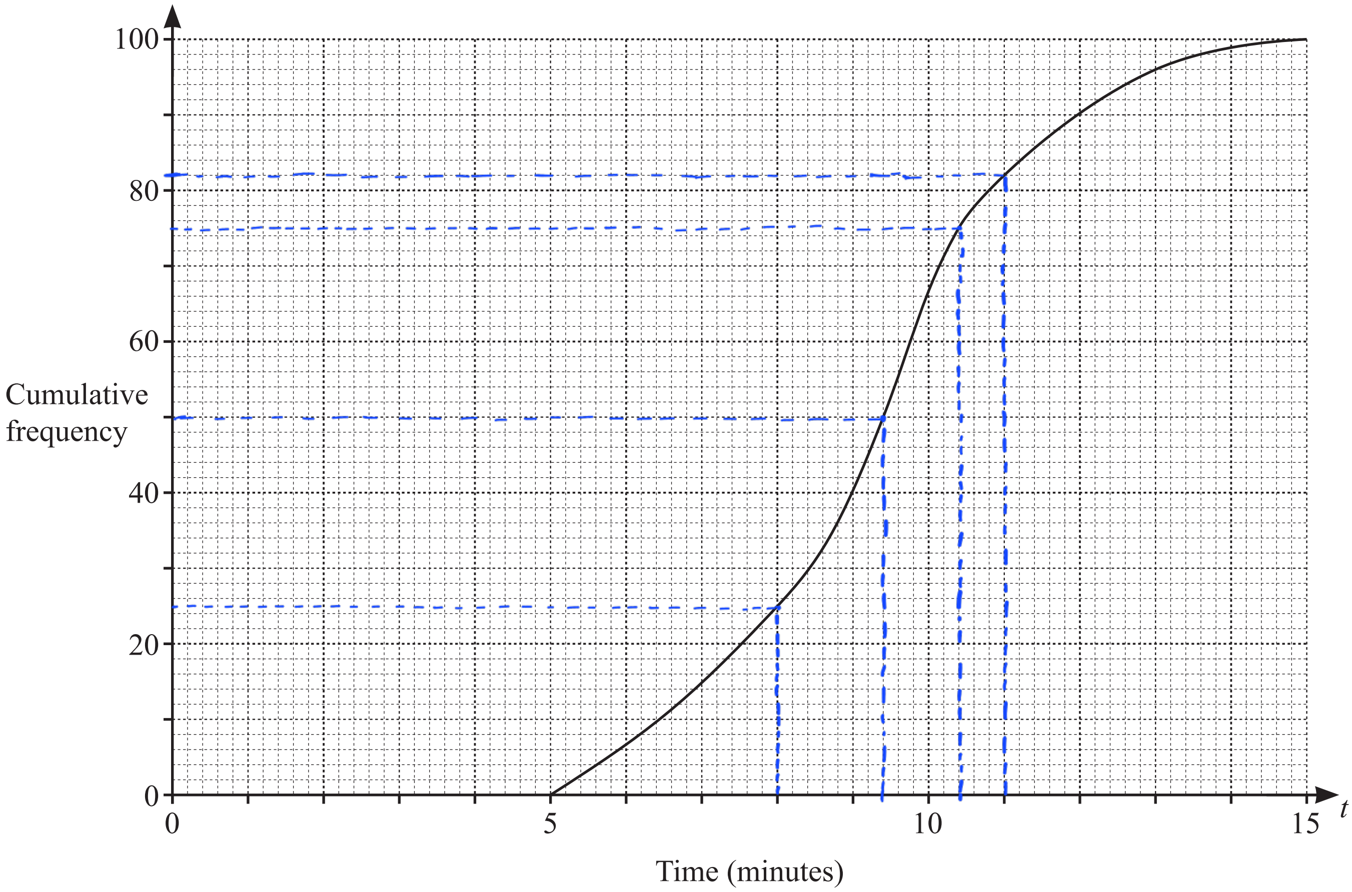
$$\frac{r}{100} = \sqrt[22]{2} - 1$$

$$r = 100\left(\sqrt[22]{2} - 1\right)$$

$$r \approx 3.201$$

$r =$ ....3.20.................................... [3]

5 **(a)** 100 students each record the time, t minutes, taken to eat a pizza.
The cumulative frequency diagram shows the results.



Find an estimate of

(i) the median,

................9.4................ min [1]

(ii) the interquartile range,

10.4 − 8

................2.4................ min [2]

(iii) the number of students taking more than 11 minutes to eat a pizza.

100 − 82

................18................ [2]

(b) 150 students each record how far they can throw a tennis ball.
The table shows the results.

Mid value	10	25	32.5	40	52.5
Distance (d metres)	$0 < d \leqslant 20$	$20 < d \leqslant 30$	$30 < d \leqslant 35$	$35 < d \leqslant 45$	$45 < d \leqslant 60$
Frequency	4	38	40	53	15

(i) Calculate an estimate of the mean.

$$\frac{(10 \times 4) + (25 \times 38) + (32.5 \times 40) + (40 \times 53) + (52.5 \times 15)}{150}$$

........34.65........ m [4]

(ii) A histogram is drawn to show this information.
The height of the bar representing $30 < d \leqslant 35$ is 12 cm.

Calculate the height of each of the other bars.

Distance (d metres)	Frequency	Height of bar (cm)	Freq. density
$0 < d \leqslant 20$	4	0.3	0.2
$20 < d \leqslant 30$	38	5.7	3.8
$30 < d \leqslant 35$	40	12	8
$35 < d \leqslant 45$	53	7.95	5.3
$45 < d \leqslant 60$	15	1.5	1

[3]

(iii) Two students are chosen at random.

Find the probability that they both threw the ball more than 45 m.

$$\frac{15}{150} \times \frac{14}{149}$$

.... $\frac{7}{745}$ .... [2]

6 **(a)** $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Find

(i) $3\mathbf{q}$,

$\begin{pmatrix} -3 \\ 3 \end{pmatrix}$ [1]

(ii) $\mathbf{p} - \mathbf{q}$,

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ [1]

(iii) $|\mathbf{p}|$.

$\sqrt{2^2 + 3^2} = \sqrt{13}$

........3.61........ [2]

(b) B is the point (2, 7) and $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$.

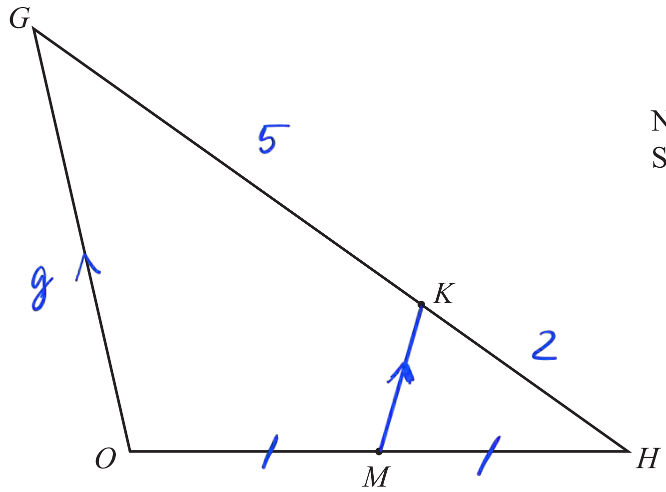
Find the coordinates of A.

$2 - x_A = -4$

$7 - y_A = 6$

(........6........ , ...1........) [2]

(c)



NOT TO SCALE

In triangle OGH, M is the midpoint of OH and K divides GH in the ratio 5 : 2.
$\overrightarrow{OG} = \mathbf{g}$ and $\overrightarrow{OH} = \mathbf{h}$.

Find $\overrightarrow{MK}$ in terms of $\mathbf{g}$ and $\mathbf{h}$.
Give your answer in its simplest form.

$\overrightarrow{GH} = \overrightarrow{GO} + \overrightarrow{OH} = -g + h$

$\Rightarrow \overrightarrow{KH} = \frac{2}{7}\overrightarrow{GH} = \frac{2}{7}(-g+h)$

$\overrightarrow{MK} = \overrightarrow{MH} + \overrightarrow{HK}$

$= \frac{1}{2}h - \frac{2}{7}(-g+h)$

$= \frac{1}{2}h + \frac{2}{7}g - \frac{2}{7}h$

$\overrightarrow{MK} = \frac{2}{7}g + \frac{3}{14}h$ [4]

7 $\mathrm{f}(x) = 10 - x$ $\quad \mathrm{g}(x) = \frac{2}{x},\ x \neq 0$ $\quad \mathrm{h}(x) = 2^x$ $\quad \mathrm{j}(x) = 5 - 2x$

(a) (i) Find $\mathrm{g}\left(\frac{1}{2}\right)$.

$\frac{2}{\frac{1}{2}} = 4$

.... 4 .... [1]

(ii) Find $\mathrm{hg}\left(\frac{1}{2}\right)$.

$h(4) = 2^4 = 16$

.... 16 .... [1]

(b) Find x when $\mathrm{f}(x) = 7$.

$10 - x = 7$

$x = 10 - 7 = 3$

$x =$ .... 3 .... [1]

(c) Find x when $\mathrm{g}(x) = \mathrm{h}(3)$.

$\frac{2}{x} = 2^3 = 8$

$x = \frac{2}{8}$

$x =$ .... $\frac{2}{8}$ .... [2]

(d) Find $\mathrm{j}^{-1}(x)$.

$\times(-2) \longrightarrow +5$

$:(-2) \longleftarrow -5$

$\mathrm{j}^{-1}(x) =$ .... $\frac{x-5}{-2}$ .... [2]

(e) Write $\mathrm{f}(x) + \mathrm{g}(x) + 1$ as a single fraction in its simplest form.

$10 - x + \frac{2}{x} + 1$

$\frac{(10-x)x + 2 + x}{x}$

.... $\frac{-x^2 + 11x + 2}{x}$ .... [3]

(f) $(\mathrm{f}(x))^2 - \mathrm{ff}(x) = ax^2 + bx + c$

Find the values of a, b and c.

$(10-x)^2 - [10 - (10-x)]$

$= 100 - 20x + x^2 - x$

$= x^2 - 21x + 100$

$a =$ .... 1 ....

$b =$ .... -21 ....

$c =$ .... 100 .... [4]

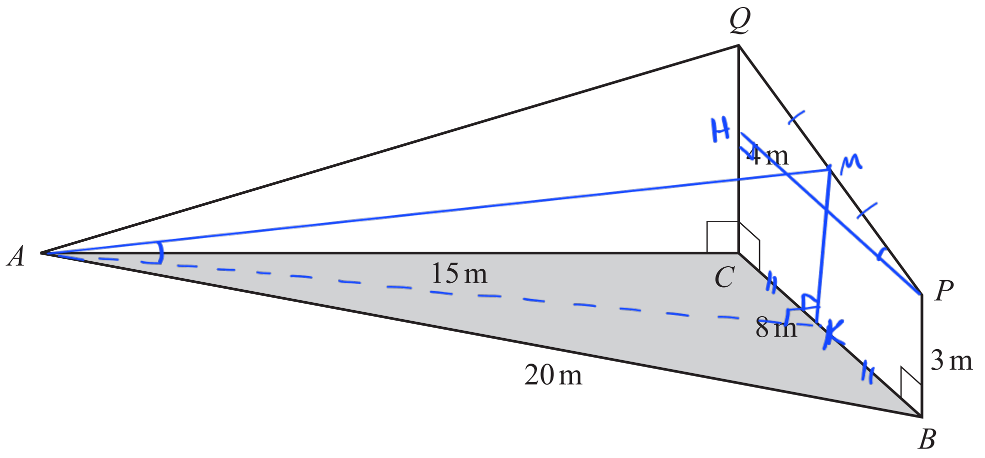
(g) Find x when $\mathrm{h}^{-1}(x) = 10$.

$y = 2^x$

Swap : $x = 2^y = 2^{h^{-1}(x)} = 2^{10}$

$x =$ .... 1024 .... [2]

8



The diagram shows triangle ABC on horizontal ground.
$AC = 15\text{ m}$, $BC = 8\text{ m}$ and $AB = 20\text{ m}$.

BP and CQ are vertical poles of different heights.
$BP = 3\text{ m}$ and $CQ = 4\text{ m}$.
AQ and PQ are straight wires.

(a) Show that angle $ACB = 117.5°$, correct to 1 decimal place.

$\triangle ABC$: $20^2 = 15^2 + 8^2 - 2 \times 15 \times 8 \cos \widehat{ACB}$

$111 = -240 \cos \widehat{ACB}$

$\cos \widehat{ACB} = -\frac{37}{80}$

$\widehat{ACB} \approx 117.549°$

$\approx 117.5°$

[4]

(b) Calculate the area of triangle ABC.

$A_{\Delta ABC} = \frac{1}{2} \times 15 \times 8 \sin 117.549°$

..........53.2.......................... m^2 [2]

(c) Calculate the length of AQ.

ΔACQ : $AQ = \sqrt{4^2 + 15^2} = \sqrt{241}$

.........15.5......... m [2]

(d) Calculate the angle of elevation of Q from P.

$PH \perp QC \Rightarrow PH = 8, \quad HQ = 4 - 3 = 1$

$\tan \widehat{QPH} = \frac{1}{3}$

$\Rightarrow \widehat{QPH} \approx 7.125°$

.........7.1°......... [3]

(e) Another straight wire connects A to the midpoint of PQ.

Calculate the angle between this wire and the horizontal ground.

$MK \perp BC \Rightarrow K$ is mid point of $BC \Rightarrow CK = 4$

$MK = \frac{3+4}{2} = 3.5$

ΔACK : $AK^2 = 15^2 + 4^2 - 2 \times 15 \times 4 \cos \widehat{ACB}$

$AK^2 = 15^2 + 4^2 - 2 \times 15 \times 4 \times \left(-\frac{37}{80}\right)$

$AK^2 = 296.5$

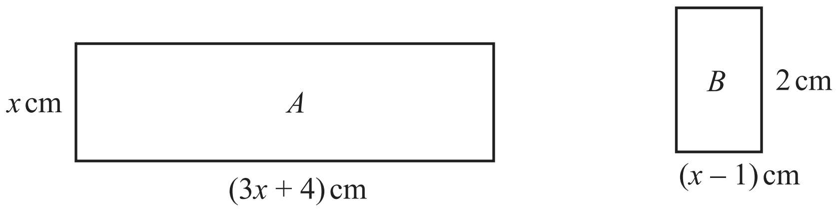
$AK = \sqrt{296.5}$

ΔAMK : $\tan \widehat{MAK} = \frac{MK}{AK} = \frac{3.5}{\sqrt{296.5}}$

$\widehat{MAK} \approx 11.5°$

.........11.5°......... [5]

9 **(a)**



The total of the areas of rectangles A and B is $20\,\text{cm}^2$.

(i) Show that $3x^2 + 6x - 22 = 0$.

$x(3x+4) + 2(x-1) = 20$

$3x^2 + 4x + 2x - 2 = 20$

$3x^2 + 6x - 22 = 0$ ✓

[2]

(ii) Solve the equation $3x^2 + 6x - 22 = 0$, giving your answers correct to 4 significant figures.
You must show all your working.

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 3(-22)}}{2 \times 3}$$

$x =$ ......-3.887...... or $x =$ ......1.887...... [4]

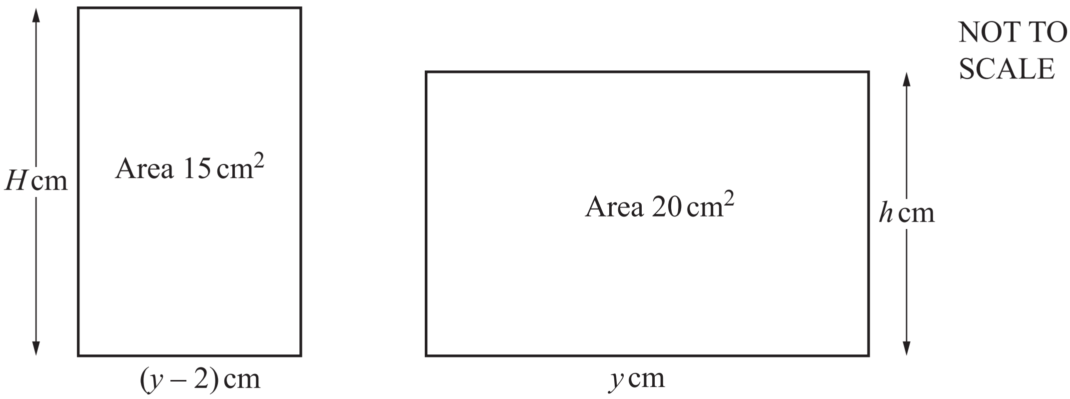
(iii) Find the perimeter of rectangle B.

Because $x > 1$ so $x = 1.887$

Perimeter$_B$ $= 2(2 + 1.887 - 1)$

......5.77...... cm [1]

(b)



The diagram shows two rectangles where $H - h = 1$.

By forming a quadratic equation and factorising, find the value of y.

$$\frac{15}{y-2} - \frac{20}{y} = 1$$

$$\frac{15y - 20(y-2)}{(y-2)y} = 1$$

$$15y - 20y + 40 = y^2 - 2y$$

$$y^2 + 3y - 40 = 0$$

$$y^2 + 8y - 5y - 40 = 0$$

$$y(y+8) - 5(y+8) = 0$$

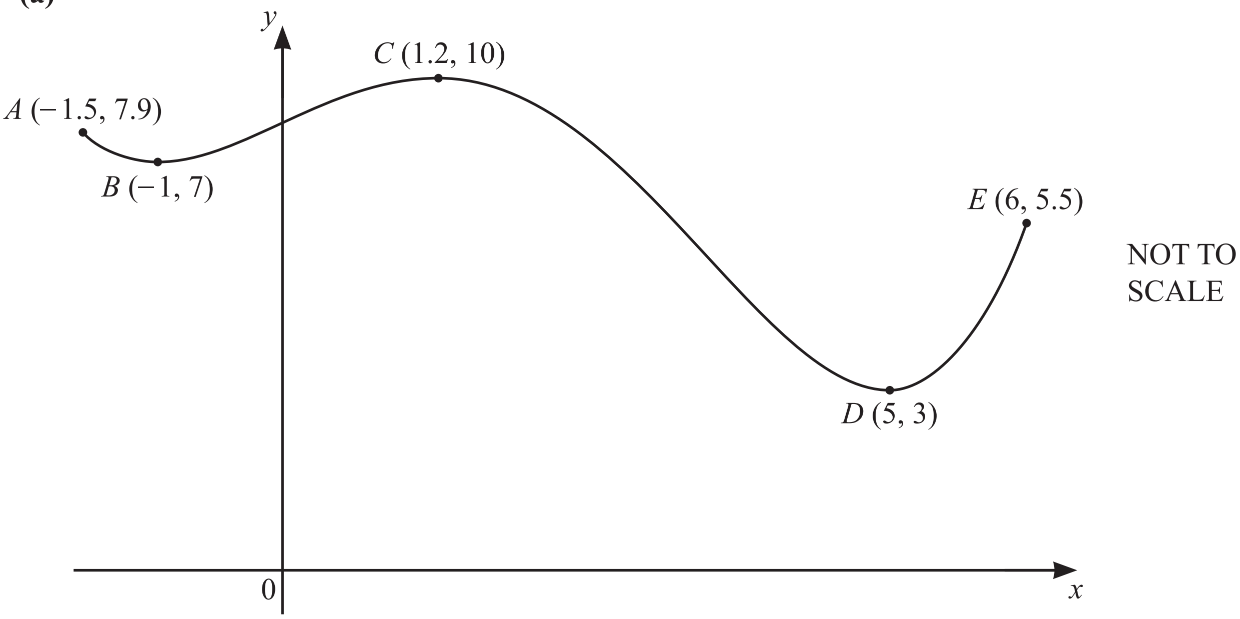
$$(y-5)(y+8) = 0$$

$$y = 5 \quad \text{or} \quad y = -8$$

Because $y > 0$ so $y = 5$

$y =$ ....5.. [7]

10 **(a)**



The diagram shows a sketch of the graph of $y = \mathrm{f}(x)$ for $-1.5 \leqslant x \leqslant 6$.
The coordinates of five points on the graph of $y = \mathrm{f}(x)$ are shown on the diagram.

(i) $\mathrm{f}(x) = k$ has two solutions in the interval $-1.5 \leqslant x \leqslant 6$.

Write down a possible integer value of k.

$k =$ ......4...... [1]

(ii) $\mathrm{f}(x) = j$ has no solutions in the interval $-1.5 \leqslant x \leqslant 6$ when $j < a$ or $j > b$.

Find the maximum value of a and the minimum value of b.

$a =$ ......3......

$b =$ ......10...... [2]

(b) Find the coordinates of the two stationary points on the graph of $y = x^6 - 6x^5$.
You must show all your working.

$$\frac{dy}{dx} = 6x^5 - 30x^4 = 0$$

$$6x^4(x-5) = 0$$

$$x = 0 \text{ or } x = 5$$

when $x = 0$, $y = 0^6 - 6 \times 0^5 = 0$

when $x = 5$, $y = 5^6 - 6 \times 5^5 = -3125$

(......0...... , ...0......)

(......5...... , ...−3125......) [5]

