1 (a) Here are the ingredients needed to make a pasta bake to serve 12 people.

250 g butter
600 g pasta
460 g mushrooms
280 g cheese
800 ml milk

(i) Find the mass of the cheese as a percentage of the mass of the mushrooms.

$\frac{280}{460} \times 100 \approx 60.9$

........60.9........ % [1]

(ii) Find the mass of butter needed to make a pasta bake to serve 18 people.

12 people → 250 g

18 people → $\frac{18 \times 250}{12} = 375$

........375........ g [2]

(iii) Monica has 2.2 litres of milk and 1.5 kg of each other ingredient.

2200 ml 1500 g

Calculate the greatest number of people she can serve with pasta bake.

milk : $\frac{2200 \times 12}{800} = 33$ people

pasta : $\frac{1500 \times 12}{600} = 30$ people

For butter, mushrooms, cheese, the number of people served is greater than 30

........30........ [3]

(b) In 2019, a packet of pasta cost \$2.40 .
This was an increase of 25% of the cost of a packet in 2018.

(i) Work out the cost in 2018. P_{18}

$P_{18} + P_{18} \times 25\% = 2.4$

$\Rightarrow 1.25 P_{18} = 2.4$

$P_{18} = 1.92$

\$ ....1.92.... [2]

(ii) In 2020, the cost of a packet increased by 15% from the cost in 2019.

Work out the total percentage increase in the cost of a packet from 2018 to 2020.

$P_{20} = P_{19} + P_{19} \times 15\%$

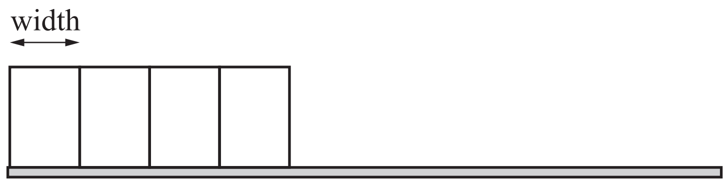
$= 1.15 P_{19}$

$= 1.15 \times 2.4 = 2.76$

percentage increase $= \dfrac{2.76 - 1.92}{1.92} \times 100$

....43.75.... % [3]

(c)



NOT TO SCALE

Pasta is sold in packets with width 11.5 cm, correct to the nearest 0.5 cm.
A shop places these packets in a single line on a shelf of length 2 m, correct to the nearest 0.1 m.

200cm 10cm

Find the maximum number of these packets that will fit along this shelf.
You must show all your working.

Max number of packet $= \dfrac{\text{Longest shelf}}{\text{shortest packet}}$

$= \dfrac{200 + \frac{10}{2}}{11.5 - \frac{0.5}{2}} \approx 18.2$

Because the number of packet is a natural number so the answer is 18

....18.... [3]

2 **(a)** Simplify fully.

(i) $p^3 \times p^{11}$

p^{14} [1]

(ii) $\dfrac{18m^6}{3m^2}$

$6m^4$ [2]

(iii) $\left(\dfrac{27x^9y^{27}}{64}\right)^{-\frac{1}{3}}$

$$\frac{1}{\left(\frac{27x^9y^{27}}{64}\right)^{\frac{1}{3}}} = \frac{1}{\frac{27^{1/3}(x^9)^{1/3}(y^{27})^{1/3}}{64^{1/3}}} = \frac{4}{3x^3y^9}$$

$\dfrac{4}{3x^3y^9}$ [3]

(b) A sequence has nth term $3n^2$.

Write down the first 3 terms of this sequence.

3 , 12 , 27 [2]

(c) Find the nth term for each of these sequences.

(i) 13, 16, 19, 22, 25, …

+3 +3 +3 +3

$3n + 10$ [2]

(ii) 3, 17, 55, 129, 251, …

14 38 74 122

24 36 48

12 12

$\frac{12}{6} = 2$ => There is a term $2n^3$

$2n^3 + 1$ [2]

(d) Solve.

$$\frac{3x-22}{4} = 23$$

$3x - 22 = 92$

$3x = 114$

$x = 38$

$x = $ ......38...................... [3]

(e) Use the quadratic formula to solve $3x^2 + 8x - 20 = 0$.
Show all your working and give your answers correct to 2 decimal places.

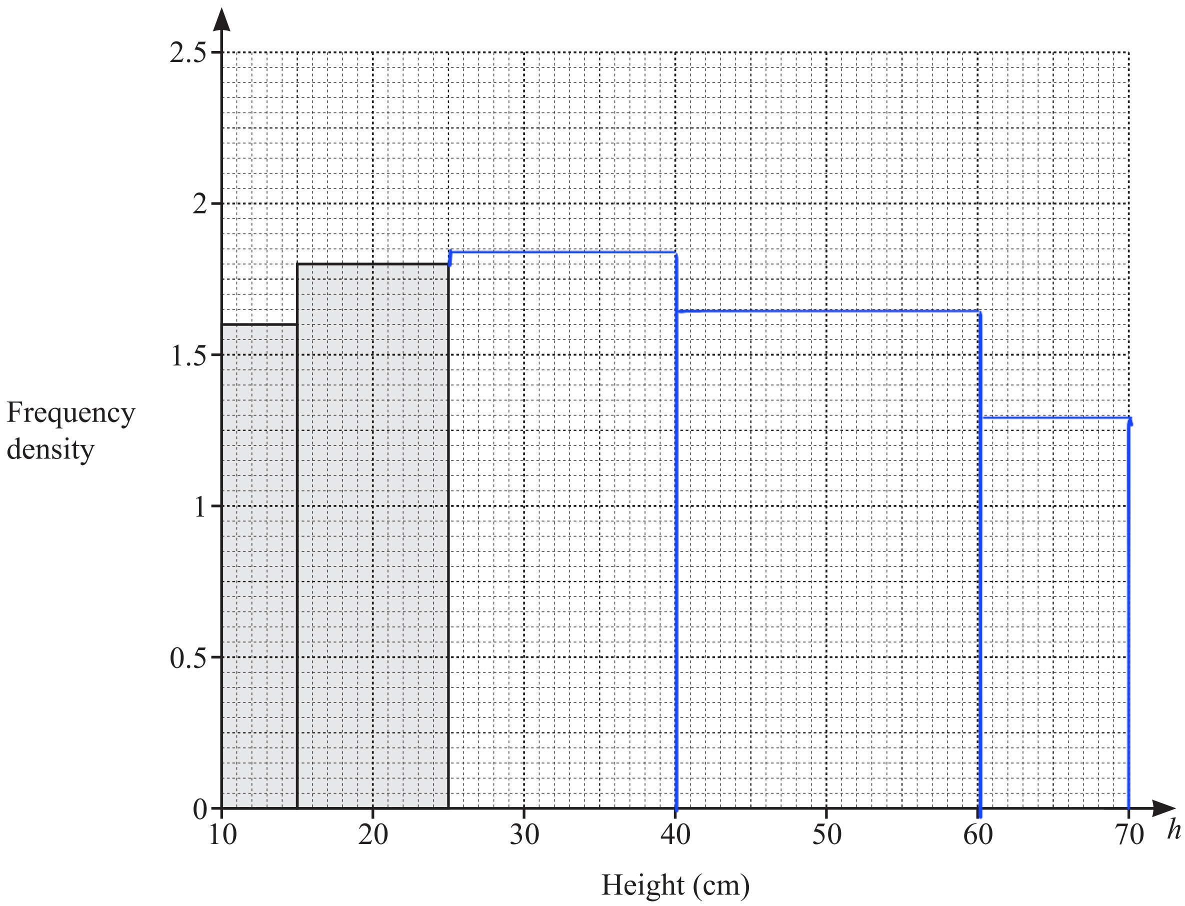
$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 3(-20)}}{2 \times 3}$$

$x = $ ......−4.24...... , $x = $ ......1.57...... [4]

3 The height, h cm, of each of 100 plants is recorded.
The table shows information about the heights of these plants.

Class width	5	10	15	20	10
Height (h cm)	$10 < h \leqslant 15$	$15 < h \leqslant 25$	$25 < h \leqslant 40$	$40 < h \leqslant 60$	$60 < h \leqslant 70$
Frequency	8	18	28	33	13
Freq density			28/15	1.65	1.3

(a) Complete the histogram to show this information.
The first two blocks have been drawn for you.



[3]

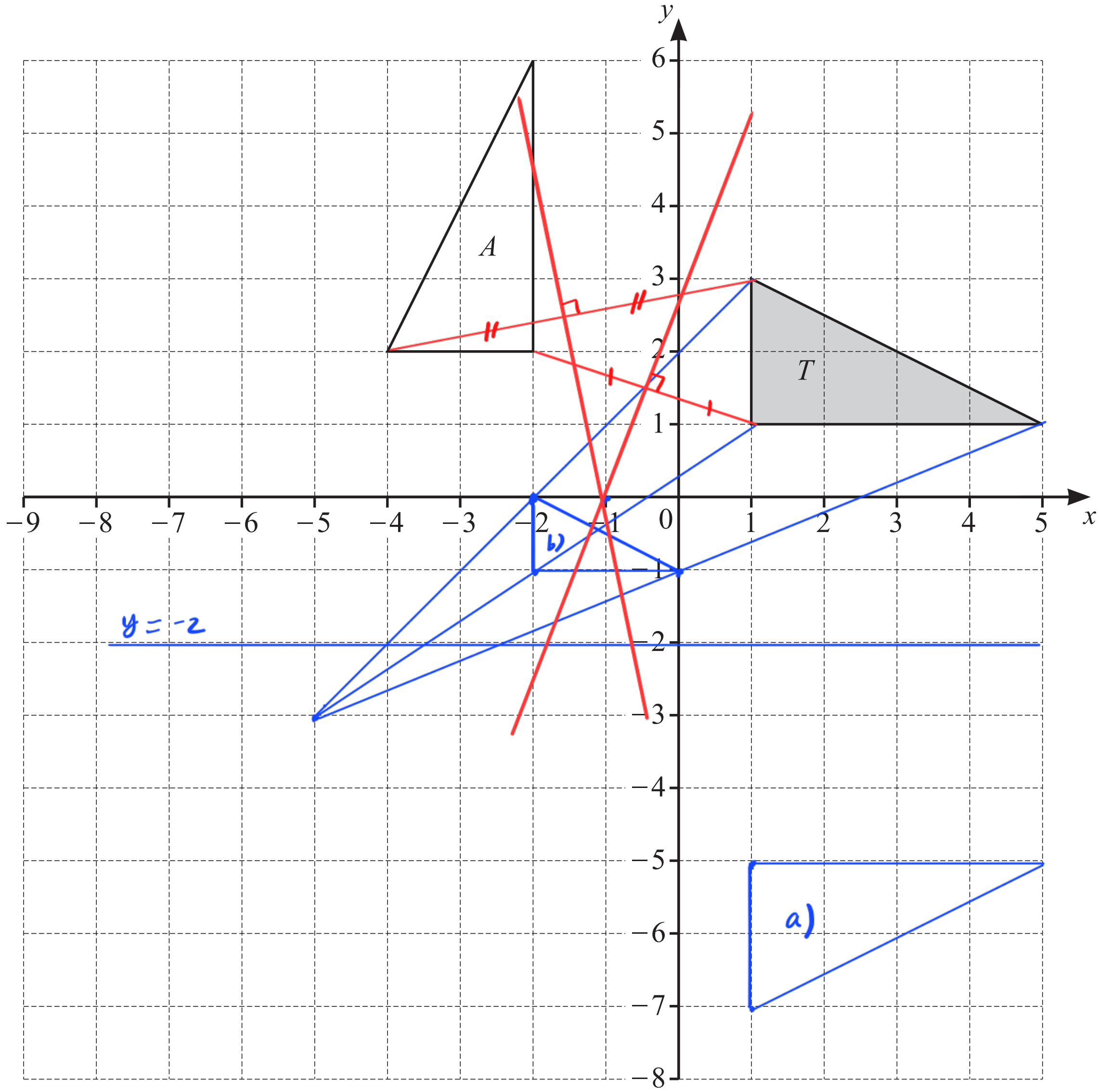
(b) Calculate an estimate of the mean height.

Mid value	12.5	20	32.5	50	65
Frequency	8	18	28	33	13

$$\frac{(12.5 \times 8) + (20 \times 18) + (32.5 \times 28) + (50 \times 33) + (65 \times 13)}{100}$$

..........38.65.................... cm [4]

4



(a) Draw the reflection of triangle T in the line $y = -2$. [2]

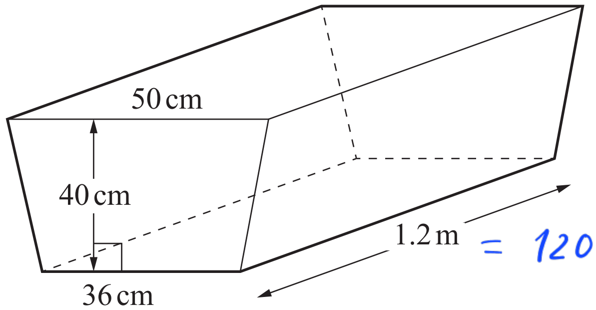
(b) Draw the enlargement of triangle T with scale factor $\frac{1}{2}$ and centre of enlargement $(-5, -3)$. [2]

(c) Describe fully the **single** transformation that maps triangle T onto triangle A.

Rotation, center (-1, 0), anticlockwise 90°

.... [3]

5



$= 120\text{ cm}$

NOT TO SCALE

The diagram shows a water trough in the shape of a prism.
The prism has a cross-section in the shape of an isosceles trapezium.
The trough is completely filled with water.

(a) Show that the volume of water in the trough is 206.4 litres.

$$V = \frac{1}{2}(36 + 50)40 \times 120 = 206400\text{ cm}^3$$
$$= 206.4\ \ell$$

[3]

(b) The water from the trough is emptied at a rate of 600 ml per second.

Calculate the time taken, in minutes and seconds, for the trough to be emptied.

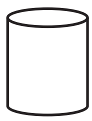
$206.4\ \ell = 206400\text{ m}\ell$

$\Rightarrow \text{Time} = \dfrac{206400}{600} = 344\text{ s}$

$344 : 60 = 5\text{ r }44$

..........5...... minutes ....44..... seconds [3]

(c) All the water from the trough is emptied into a vertical cylindrical tank.
The depth of the water in the tank is 84 cm.



(i) Calculate the radius of the tank.

$$V_{\text{water}} = \pi r^2 \times 84 = 206400$$
$$\Rightarrow r^2 = \frac{206400}{84\pi}$$
$$r = \sqrt{\frac{206400}{84\pi}} \approx 27.9666$$

..........28.0........................ cm [3]

(ii) The tank is 60% full.

Calculate the height of the tank.

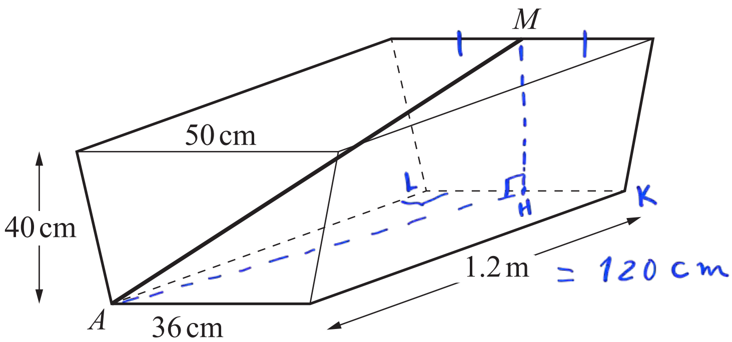
$V_{water} = 60\% \ V_{tank}$

$\Rightarrow V_{tank} = \dfrac{206400}{60\%} = 344\,000$

$\Rightarrow height_{tank} = \dfrac{344\,000}{\pi \times 27.966^2} \approx 140$

.............140............. cm [2]

(d)



NOT TO SCALE

A steel rod *AM* is placed inside the empty water trough as shown in the diagram.
A is a vertex at the base of the isosceles trapezium and *M* is the midpoint of the top edge on the opposite face.

Calculate the length of the steel rod, *AM*.

$MH \perp LK \Rightarrow \begin{cases} MH = 40\text{ cm} \\ LH = \frac{1}{2} LK = \frac{1}{2} \times 36 = 18 \end{cases}$

$\Delta ALH: \ AH^2 = 120^2 + 18^2 = 14724$

$\Delta AMH: \ AM = \sqrt{14724 + 40^2} \approx 127.765$

AM = ...128............. cm [4]

6 **(a)** $P = 5k^2 - 7$

(i) Find the value of P when $k = 3$.

$P = 5 \times 3^2 - 7$

$P =$ ......38...... [2]

(ii) Rearrange the formula to make k the subject.

$P + 7 = 5k^2$

$\frac{P+7}{5} = k^2$

$k =$ ......$\pm\sqrt{\frac{P+7}{5}}$...... [3]

(b) **(i)** Solve.

$x - 3 \leqslant 5x + 7$

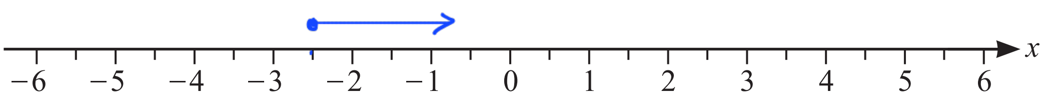
$x - 5x \leqslant 7 + 3$

$-4x \leqslant 10$

$x \geqslant \frac{10}{-4}$

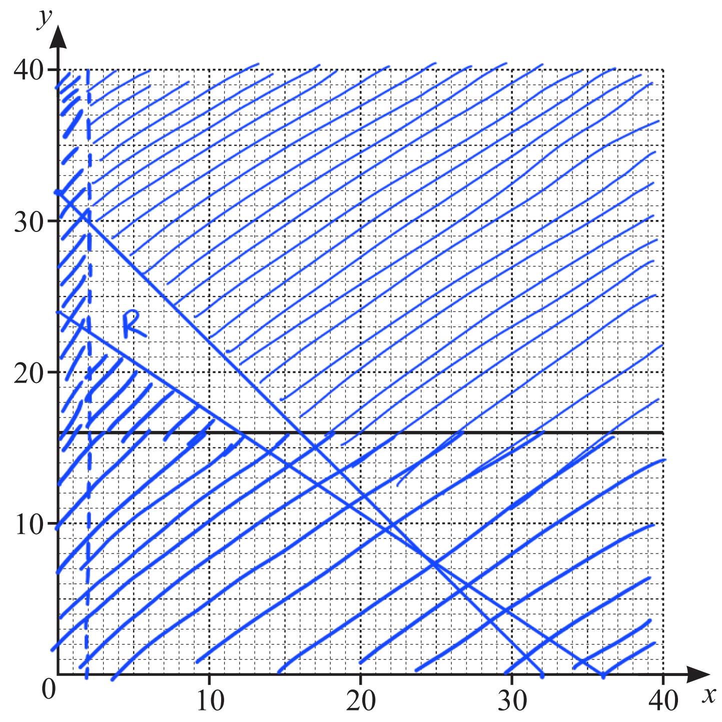
......$x \geqslant -2.5$...... [2]

(ii) Show your answer to **part (b)(i)** on the number line.



[1]

(c) The line $y = 16$ is drawn on the grid.



The region R satisfies the following inequalities.

$y \geqslant 16$ $\quad$ $x > 2$ $\quad$ $2x + 3y \geqslant 72$ $\quad$ $y \leqslant 32 - x$

(i) By drawing three more lines and shading the region **not required**, find and label region R. [6]

7 Regan is playing a game with these six number cards.

−3	−2	2	3	5	7

(a) She takes two cards at random, without replacement, and **multiplies** the two numbers to give a score.

Find the probability that

(i) the score is 35

$P(1st_5 \text{ and } 2nd_7) \times 2$

$\left(\frac{1}{6} \times \frac{1}{5}\right) \times 2$

.... $\frac{1}{15}$ .... [3]

(ii) the score is a positive number.

possible pairs of cards $(-3,-2)$, $(2,3)$, $(2,5)$, $(2,7)$, $(3,5)$, $(3,7)$, $(5,7)$

Each pair has probability of $\frac{1}{15}$

$P(\text{positive}) = 7 \times \frac{1}{15}$

.... $\frac{7}{15}$ .... [3]

(b) Regan now takes three cards at random from the six cards, without replacement, and **adds** the three numbers to give a total.

Find the probability that her total is 5.

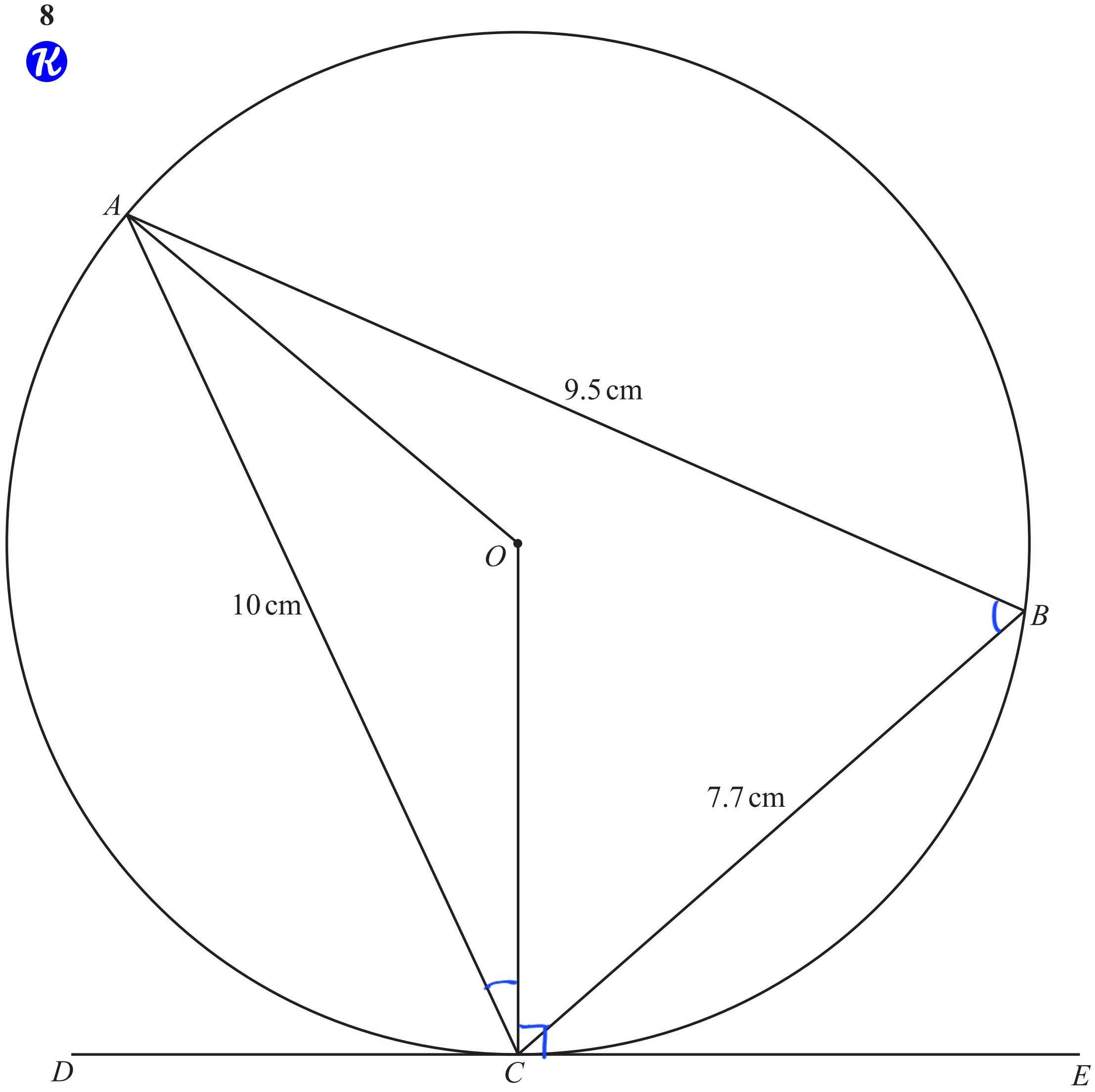
The combination of cards $(-2, 2, 5)$ or $(-3, 3, 5)$

$$P = \left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)6 + \left(\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}\right)6$$

$$P = \frac{1}{10}$$

.... [4]

8



NOT TO SCALE

A, B and C are points on the circle, centre O.
DE is a tangent to the circle at C.
$AC = 10\text{ cm}$, $AB = 9.5\text{ cm}$ and $BC = 7.7\text{ cm}$.

(a) Show that angle $ABC = 70.2°$, correct to 1 decimal place.

$\triangle ABC$: $10^2 = 9.5^2 + 7.7^2 - 2 \times 9.5 \times 7.7 \cos \widehat{ABC}$

$\frac{-2477}{50} = \frac{-1463}{10} \cos \widehat{ABC}$

$\cos \widehat{ABC} = \frac{2477}{7315}$

$\widehat{ABC} \simeq 70.207°$

[4]

(b) Find

(i) angle AOC

$\widehat{AOC} = 2\widehat{ABC} = 2 \times 70.207^\circ = 140.414^\circ$

Angle AOC = ...140.4°...... [1]

(ii) angle ACO

$\widehat{ACO} = \dfrac{180^\circ - 140.414^\circ}{2} \approx 19.8^\circ$

Angle ACO = ...19.8°...... [1]

(iii) angle ACD.

$\widehat{ACD} = \widehat{ABC} = 70.207^\circ$

Angle ACD = ...70.2°...... [1]

(c) Calculate the radius, OC, of the circle.

ΔAOC : $10^2 = OA^2 + OC^2 - 2\,OA.OC.\cos\widehat{AOC}$

$100 = 2 \times OC^2 - 2 \times OC^2 \cos 140.414^\circ$

$100 = 3.5413\,OC^2$

$OC^2 = 28.238$

OC = ...5.31...... cm [3]

(d) Calculate the area of triangle ABC as a percentage of the area of the circle.

$A_{\Delta ABC} = \frac{1}{2} \times 9.5 \times 7.7 \sin 70.207^\circ$

$= 34.414$

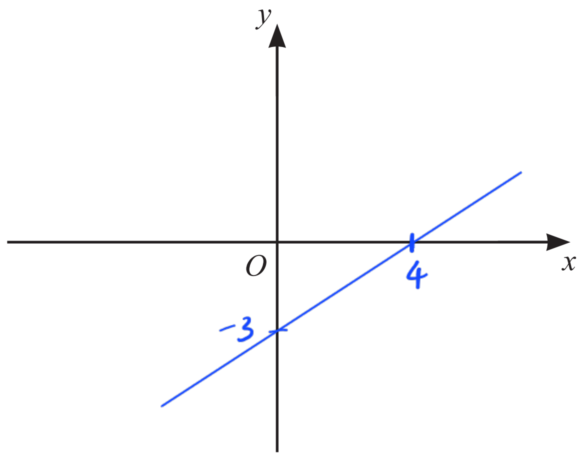
$A_{circle} = \pi\, OC^2 = 28.238\,\pi$

$\dfrac{34.414}{28.238\,\pi} \times 100 \approx 38.8$

......38.8...... % [4]

9 **(a)** Sketch the following graphs.
On each sketch, indicate any intercepts with the axes.

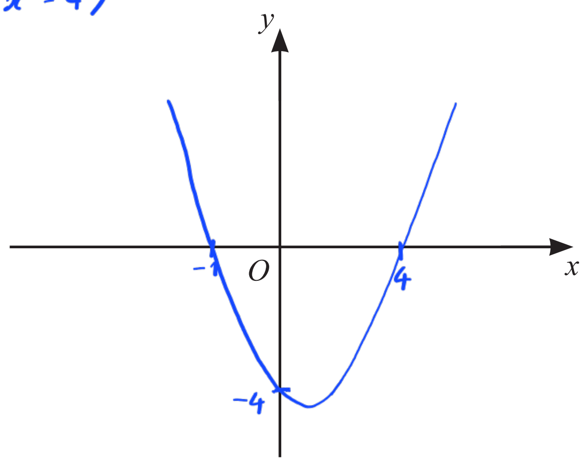
(i) $3x - 4y = 12$



[2]

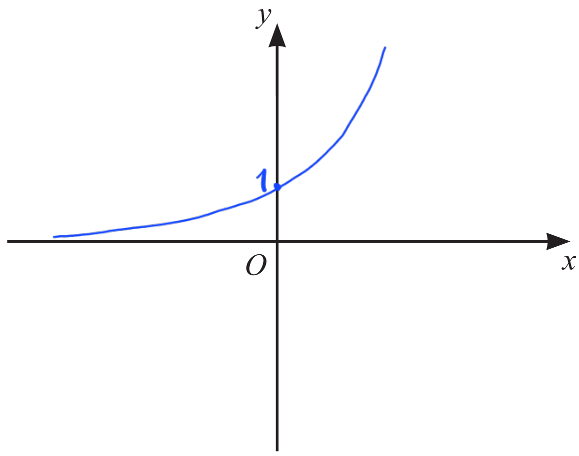
(ii) $y = x^2 - 3x - 4$

$y = (x+1)(x-4)$



[4]

(iii) $y = 6^x$



[2]

(b) **(i)** Find the derivative, $\frac{dy}{dx}$, of $y = 5 + 8x - \frac{4}{3}x^3$.

$8 - \frac{4}{3} \times 3x^2$

.........$-4x^2 + 8$......... [2]

(ii) Find the gradient of $y = 5 + 8x - \frac{4}{3}x^3$ at $x = -1$.

$-4(-1)^2 + 8$

.........4......... [2]

(iii) A tangent is drawn to the graph of $y = 5 + 8x - \frac{4}{3}x^3$.
The gradient of the tangent is -28.

Find the coordinates of the two possible points where this tangent meets the graph.

gradient $= \frac{dy}{dx} = -28$

$\Rightarrow -4x^2 + 8 = -28$

$4x^2 = 36$

$x^2 = 9$

$x = \pm 3$

When $x = 3$, $y = 5 + 8 \times 3 - \frac{4}{3} \times 3^3 = -7$

When $x = -3$, $y = 5 + 8(-3) - \frac{4}{3}(-3)^3 = 17$

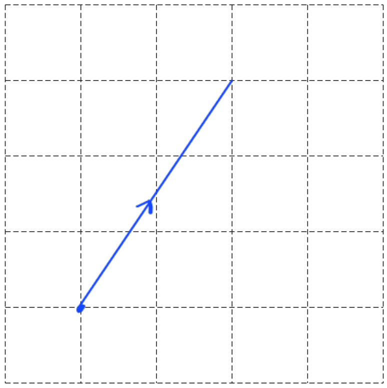
(.........3......... , ...-7.........)

(.........-3......... , ...17.........) [5]

10 **(a)** $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

(i) On the grid, draw and label vector 2**a**.

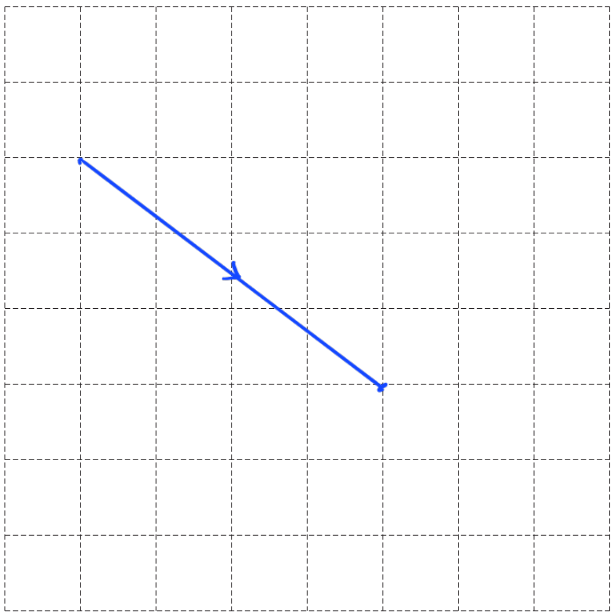
$2a = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$



[1]

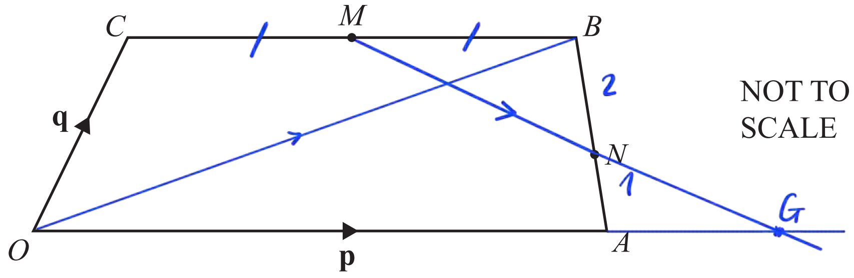
(ii) On the grid, draw and label vector (**a** − **b**).

$a - b = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$



[2]

(b)



$OABC$ is a trapezium with OA parallel to CB.
M is the midpoint of CB and N is the point on AB such that $AN : NB = 1 : 2$.
O is the origin, $\overrightarrow{OA} = \mathbf{p}$, $\overrightarrow{OC} = \mathbf{q}$ and $\overrightarrow{CB} = \frac{3}{4}\mathbf{p}$.

(i) Find, in terms of **p** and/or **q**, in its simplest form

(a) $\overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB} = q + \frac{3}{4}p$

$\overrightarrow{OB} =$ ...... $q + \frac{3}{4}p$ ...... [1]

(b) $\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -p + q + \frac{3}{4}p$

$\overrightarrow{AB} =$ ...... $q - \frac{1}{4}p$ ...... [2]

(c) $\overrightarrow{MN}$.

$\overrightarrow{MB} = \frac{1}{2}\overrightarrow{CB} = \frac{1}{2} \times \frac{3}{4}p = \frac{3}{8}p$

$\overrightarrow{NB} = \frac{2}{3}\overrightarrow{AB} = \frac{2}{3}(q - \frac{1}{4}p)$

$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN} = \frac{3}{8}p - \frac{2}{3}(q - \frac{1}{4}p)$

$\overrightarrow{MN} =$ ...... $\frac{13}{24}p - \frac{2}{3}q$ ...... [3]

(ii) OA and MN are extended to meet at G.

Find the position vector of G in terms of **p**.

$\Delta MNB \sim \Delta GNA$

$\frac{MB}{GA} = \frac{NB}{NA} = 2 \Rightarrow GA = \frac{MB}{2}$

$\overrightarrow{AG} = \frac{1}{2}\overrightarrow{MB} = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16}p$

$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG} = p + \frac{3}{16}p$

...... $\frac{19}{16}p$ ...... [2]