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1 (a) (i) At a football club, season tickets are sold for seated areas and for standing areas.
The cost of season tickets are in the ratio seated : standing = 5 : 3.
The cost of a season ticket for the standing area is \$45.

Find the cost of a season ticket for the seated area.

$$\frac{45 \times 5}{3}$$

\$ ......75...... [2]

(ii) In 2021, the value of the team's players was \$2.65 million.
In 2022 this value has decreased by 12%.

Find the value in 2022.

$$2.65 - 2.65 \times 12\%$$

\$ ......2.332...... million [2]

(iii) The number of people at a football match is 1455.
This is 6.25% of the total number of people allowed in the stadium.

Find the total number of people allowed in the stadium.

$$1455 = 6.25\% \, P$$

$$P = \frac{1455}{6.25\%}$$

......23280...... [2]

(iv) The average attendance increased exponentially by 4% each year for the three years from 2016 to 2019.
In 2019 the average attendance was 1631.

Find the average attendance for 2016.

$$1631 = a_{16}\left(1 + \frac{4}{100}\right)^3$$

$$a_{16} = \frac{1631}{1.04^3} \approx 1450$$

......1450...... [3]

(b) Another club sells season tickets for individuals and for families.
In 2018, the number of season tickets sold is in the ratio family : individual = 2 : 7.

(i) The number of family season tickets sold is x.

Write an expression, in terms of x, for the number of individual season tickets sold.

$\frac{x \times 7}{2}$

$\frac{7x}{2}$ [1]

(ii) In 2019, the number of family season tickets sold increases by 12 and the number of individual season tickets sold decreases by 26.

Complete the table by writing expressions, in terms of x, for the number of tickets sold each year.

Year	Family tickets	Individual tickets
2018	x	$\frac{7x}{2}$
2019	$x + 12$	$\frac{7x}{2} - 26$

[2]

(iii) In 2019, the number of individual season tickets sold is 3 times the number of family season tickets sold.

Write an equation in x and solve it to find the number of family tickets sold in 2018.

$\frac{7x}{2} - 26 = 3(x + 12)$

$3.5x - 26 = 3x + 36$

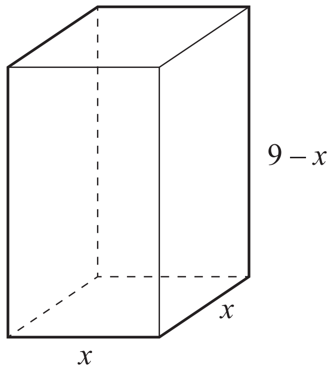
$3.5x - 3x = 36 + 26$

$0.5x = 62$

$x = 124$

$x =$ 124 [4]

2 All the lengths in this question are measured in centimetres.



NOT TO SCALE

The diagram shows a solid cuboid with a square base.

(a) The volume, $V\,\text{cm}^3$, of the cuboid is $V = x^2(9-x)$.
The table shows some values of V for $0 \leqslant x \leqslant 9$.

x	0	1	2	3	4	5	6	7	8	9
V	0	8	28	54	80	100	108	98	64	0

(i) Complete the table.

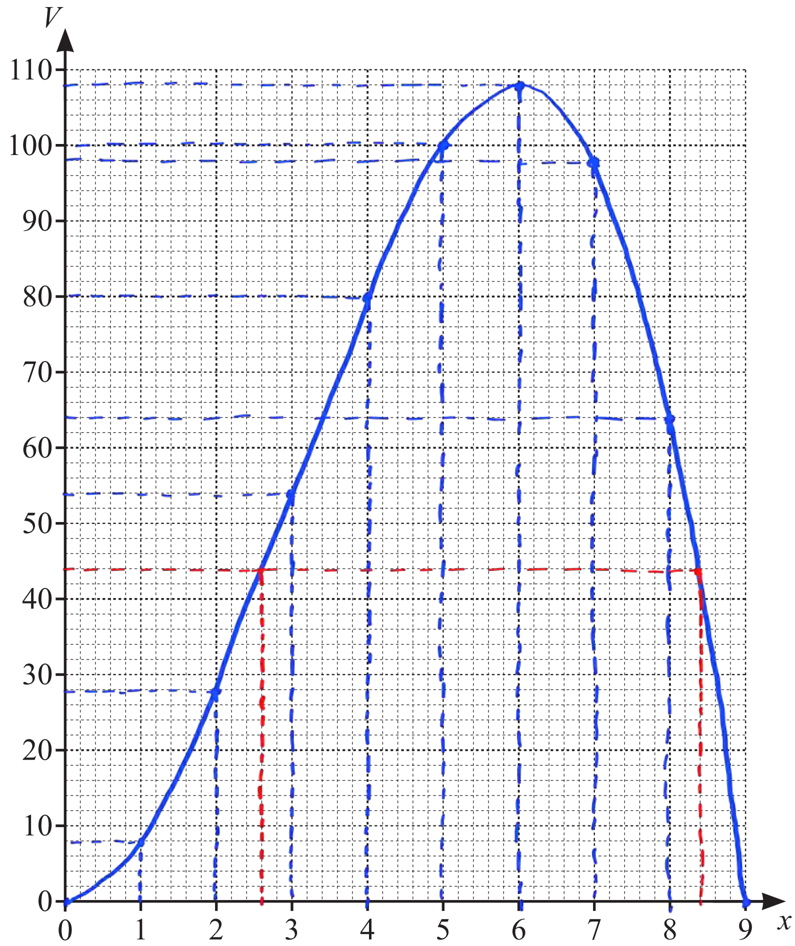
$2^2(9-2)$

[1]

(ii) On the grid on the opposite page, draw the graph of $V = x^2(9-x)$ for $0 \leqslant x \leqslant 9$. [4]

(iii) Find the values of x when the volume of the cuboid is $44\,\text{cm}^3$.

$x =$ ....2.6...... or $x =$ ....8.4...... [2]



(b) **(i)** Show that the total surface area of the cuboid is $(36x - 2x^2)\,\text{cm}^2$.

$2x^2 + 4x(9-x)$

$2x^2 + 36x - 4x^2$

$36x - 2x^2$

[2]

(ii) Find the surface area when the volume of the cuboid is a maximum.

From the graph we can see V_{max} is when $x = 6$

$\Rightarrow$ surface area $= 36 \times 6 - 2 \times 6^2$

.........144......... cm^2 [3]

3 Kai and Ann carry out a survey on the distances travelled, in kilometres, by 200 cars.

Kai completes this frequency table for the data collected.

Mid value	90	125	175	250	350
Distance (d km)	$80 < d \leqslant 100$	$100 < d \leqslant 150$	$150 < d \leqslant 200$	$200 < d \leqslant 300$	$300 < d \leqslant 400$
Frequency	7	33	76	52	32

(a) **(i)** Calculate an estimate of the mean.

$$\frac{(90 \times 7) + (125 \times 33) + (175 \times 76) + (250 \times 52) + (350 \times 32)}{200}$$

......211.275...... km [4]

(ii) Ann uses this frequency table for the same data.
There is a different interval for the final group.

Distance (d km)	$80 < d \leqslant 100$	$100 < d \leqslant 150$	$150 < d \leqslant 200$	$200 < d \leqslant 300$	$300 < d \leqslant 360$ (330)
Frequency	7	33	76	52	32

Without calculating an estimate of the mean for this data, find the difference between Ann's and Kai's estimate of the mean.
You must show all your working.

The only difference is the last column

difference $= 350 \times 32 - 330 \times 32 = 640$

$\text{difference}_{\text{mean}} = \frac{640}{200} = 3.2$

......3.2...... km [2]

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(iii) A histogram is drawn showing the information in **Kai's** frequency table.
The height of the block for the interval $200 < d \leqslant 300$ is 2.6 cm.

Calculate the height of the block for each of the following intervals.

Class width	20	50	50	100	100
Freq density	0.35	0.66	1.52	0.52	0.32
Height (cm)				2.6	

$80 < d \leqslant 100$ ......1.75...... cm

$150 < d \leqslant 200$ ......7.6...... cm

$300 < d \leqslant 400$ ......1.6...... cm [3]

(b) One car is picked at random.

Find the probability that the car has travelled more than 300 km.

$\frac{32}{200}$ [1]

(c) Two of the 200 cars are picked at random.

Find the probability that

(i) both cars have travelled 150 km or less,

$$\frac{7+33}{200} \times \frac{7+33-1}{199}$$

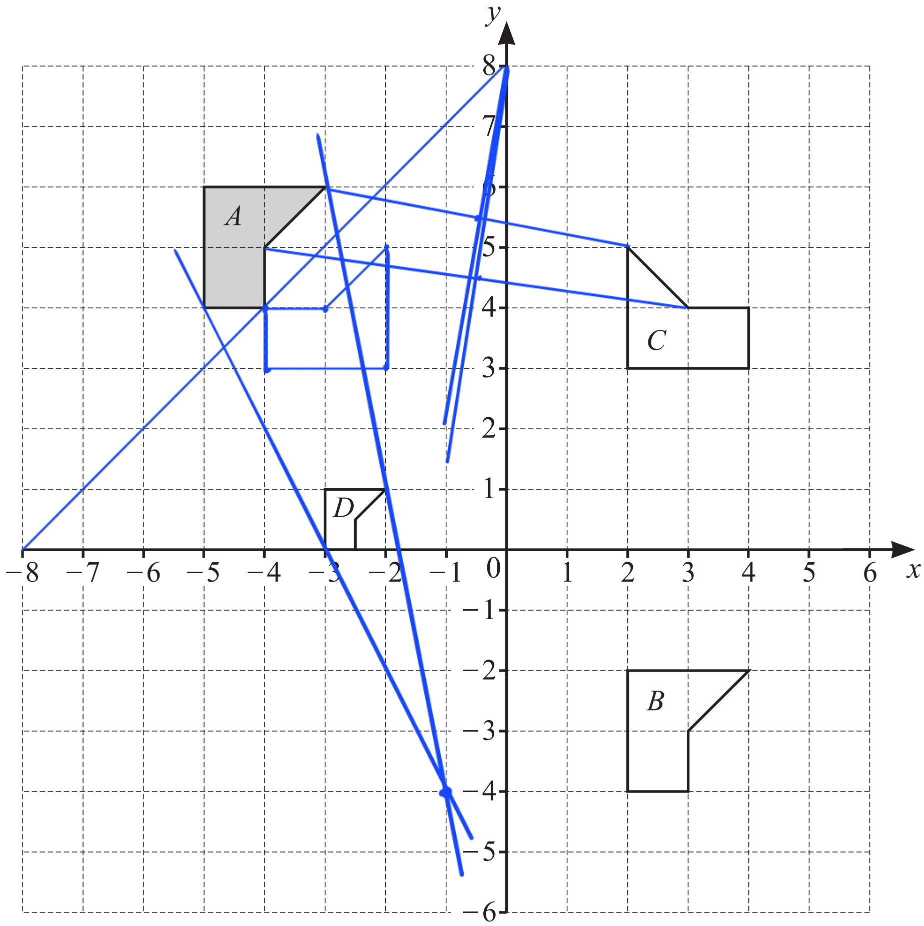
$\frac{39}{995}$ [2]

(ii) one car has travelled more than 200 km and the other car has travelled 100 km or less.

$$\frac{52+32}{200} \times \frac{7}{199} \times 2$$

$\frac{147}{4975}$ [3]

4



(a) Describe fully the **single** transformation that maps

(i) shape A onto shape B,

Translation by vector $\begin{pmatrix} 7 \\ -8 \end{pmatrix}$

.... [2]

(ii) shape A onto shape C,

Rotation, center $(0, 8)$, anticlockwise $90°$

.... [3]

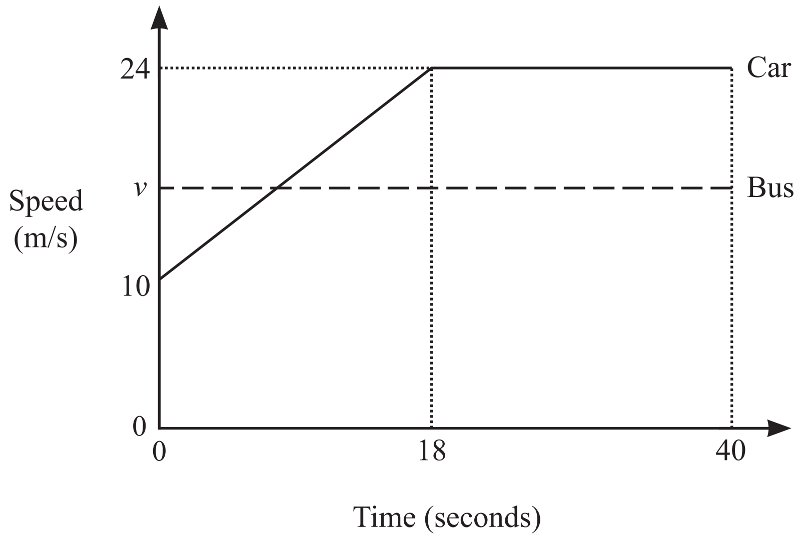
(iii) shape A onto shape D.

Enlargement, center $(-1, -4)$, scale factor $= \frac{1}{2}$

.... [3]

(b) On the grid, draw the image of shape A after a reflection in the line $y = x + 8$. [2]

5 **(a)** The diagram shows the speed–time graph for part of a journey for two vehicles, a car and a bus.



NOT TO SCALE

(i) Calculate the acceleration of the car during the first 18 seconds.

$\frac{24-10}{18}$

$\frac{14}{18}$ m/s^2 [1]

(ii) In the first 40 seconds the car travelled 134 m more than the bus.

Calculate the constant speed, v, of the bus.

$\frac{10+24}{2} \times 18 + (40-18)24 = 134 + 40v$

$834 = 134 + 40v$

$40v = 700$

$v =$ 17.5 m/s [4]

(b) A train takes 10 minutes 30 seconds to travel 16 240 m.

16.24 km

Calculate the average speed of the train.
Give your answer in kilometres per hour.

$10 \text{ min } 30 \text{ s} = \left(\frac{10}{60} + \frac{30}{3600}\right) \text{hours} = 0.175 \text{ h}$

$\text{Speed} = \frac{16.24}{0.175} = 92.8$

92.8 km/h [3]

6 **(a)** Solve.

$$4x + 15 = 9$$

$4x = 9 - 15 = -6$

$x = \frac{-6}{4}$

$x =$ ......$\frac{-6}{4}$...... [2]

(b) Factorise.

$$a^2 - 9$$

$a^2 - 3^2$

......$(a-3)(a+3)$...... [1]

(c) Write as a single fraction in its simplest form.

$$\frac{4a}{5} \div \frac{3ad}{10c}$$

$\frac{4a}{5} \times \frac{10c}{3ad} = \frac{40ac}{15ad}$

......$\frac{8c}{3d}$...... [3]

(d) $5^n + 5^n + 5^n + 5^n + 5^n = 5^m$

Find an expression for m in terms of n.

$5 \times 5^n = 5^m$

$5^{n+1} = 5^m$

$m =$ ......$n+1$...... [2]

(e) Solve by factorisation.

$$4x^2 + 8x - 5 = 0$$

$4x^2 - 2x + 10x - 5 = 0$

$2x(2x-1) + 5(2x-1) = 0$

$(2x+5)(2x-1) = 0$

$2x + 5 = 0$ or $2x - 1 = 0$

$x =$ ......$\frac{-5}{2}$...... or $x =$ ......$\frac{1}{2}$...... [3]

(f) **(i)** y is directly proportional to $(x+3)^3$.
When $x = 2$, $y = 13.5$.

Find x when $y = 108$.

$y = k(x+3)^3$

$13.5 = k(2+3)^3 \Rightarrow k = 0.108$

$\Rightarrow y = 0.108(x+3)^3$

$108 = 0.108(x+3)^3$

$x =$ ......7...... [3]

(ii) g is inversely proportional to the square of d.
When d is halved, the value of g is multiplied by a factor n.

Find n.

$g = \frac{k}{d^2}$, $d_{new} = \frac{d}{2}$

$g_{new} = \frac{k}{d_{new}^2} = \frac{k}{\frac{d^2}{4}} = 4\frac{k}{d^2} = 4g$

$n =$ ......4...... [2]

(g) Expand and simplify.

$$(2x+3)(x-1)(x+3)$$

$(2x^2 + 3x - 2x - 3)(x+3)$

$(2x^2 + x - 3)(x+3)$

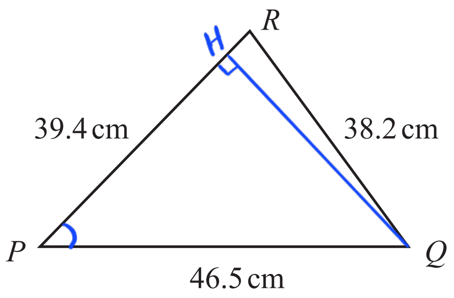
$2x^3 + x^2 - 3x + 6x^2 + 3x - 9$

......$2x^3 + 7x^2 - 9$...... [3]

(h) Find the derivative, $\frac{dy}{dx}$, of $y = 3x^2 + 4x - 1$.

......$6x + 4$...... [2]

7 **(a)**



NOT TO SCALE

(i) Calculate angle *QPR*.

$38.2^2 = 39.4^2 + 46.5^2 - 2 \times 39.4 \times 46.5 \times \cos \widehat{QPR}$

$-2255.37 = -3664.2 \cos \widehat{QPR}$

$\cos \widehat{QPR} = 0.61551$

$\widehat{QPR} \approx 52.011°$

Angle *QPR* = ...52.0°...................................... [4]

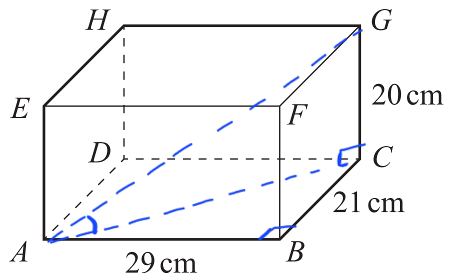
(ii) Find the shortest distance from *Q* to *PR*.

$\sin \widehat{QPH} = \dfrac{QH}{46.5}$

$\Rightarrow QH = 46.5 \sin 52.011° \approx 36.6$

..............36.6...................... cm [3]

(b) The diagram shows a cuboid.



NOT TO SCALE

(i) Calculate the length *AG*.

$AC^2 = 29^2 + 21^2 = 1282$

$AG = \sqrt{1282 + 20^2} = 29\sqrt{2} \approx 41.012$

AG = ...41.0................................ cm [3]

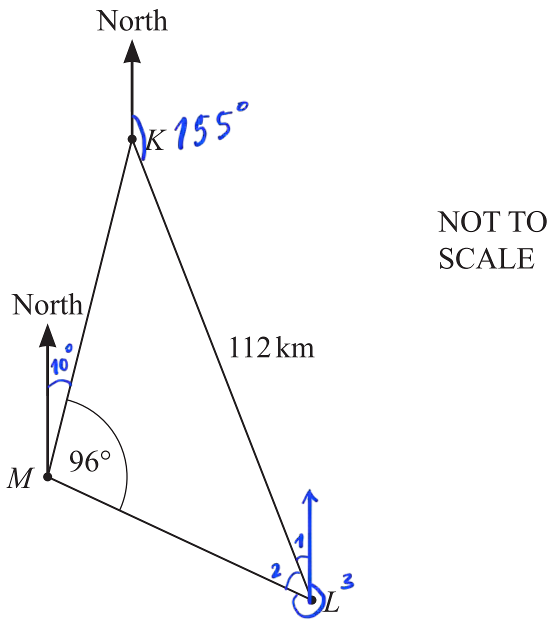
(ii) Calculate the angle between AG and the base $ABCD$.

$\tan \widehat{GAC} = \dfrac{20}{\sqrt{1282}}$

$\Rightarrow \widehat{GAC} \approx 29.187°$

$29.2°$ .... [3]

(c)



The diagram shows the positions of a lighthouse, L, and two ships, K and M.
The bearing of L from K is 155° and $KL = 112$ km.
The bearing of K from M is 010° and angle $KML = 96°$.

Find the bearing and distance of ship M from the lighthouse, L.

$\widehat{L_1} = 180° - 155° = 25°$

$\widehat{L_1} + \widehat{L_2} = 180° - (10° + 96°) = 74°$

$\widehat{L_2} = 74° - 25° = 49°$

Bearing$_{L \to M}$ $= \widehat{L_3} = 360° - 25° - 49° = 286°$

$\widehat{MKL} = 180° - 96° - 49° = 35°$

$\triangle MKL: \quad \dfrac{112}{\sin 96°} = \dfrac{ML}{\sin 35°}$

$\Rightarrow ML = \dfrac{112 \sin 35°}{\sin 96°} \approx 64.6$

Bearing .... $286°$ ....

Distance .... 64.6 .... km [5]

8 AB is a line with midpoint M.
A is the point (2, 3) and M is the point (12, 7).

(a) Find the coordinates of B.

$\frac{2 + x_B}{2} = 12$, $\frac{3 + y_B}{2} = 7$

(........22........ , ...11........) [2]

(b) Show that the equation of the perpendicular bisector of AB is $2y + 5x = 74$.

ℓ

$m_{AB} = \frac{11 - 3}{22 - 2} = \frac{2}{5}$

$m_\ell = -1 : \frac{2}{5} = \frac{-5}{2}$

M lies on line ℓ

$\Rightarrow$ Equation of ℓ: $y - 7 = \frac{-5}{2}(x - 12)$

$y - 7 = -2.5x + 30$

$y + 2.5x = 37$

multiply by 2: $2y + 5x = 74$

[4]

(c) The perpendicular bisector of AB passes through the point N.
The point N has coordinates (2, n).

Find the value of n.

$2n + 5 \times 2 = 74$

$2n = 64$

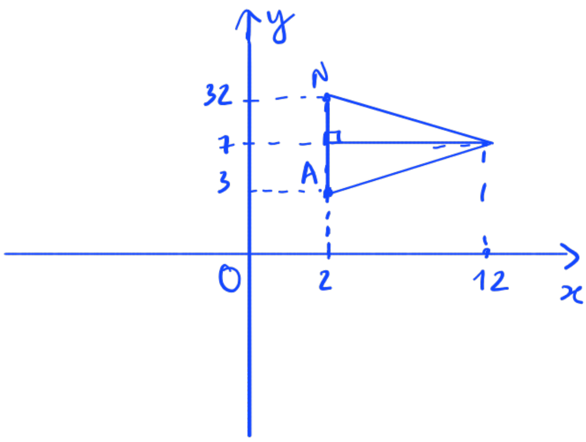
$n =$ ....32........ [1]

(d) Points A, M and N form a triangle.

Find the area of the triangle.

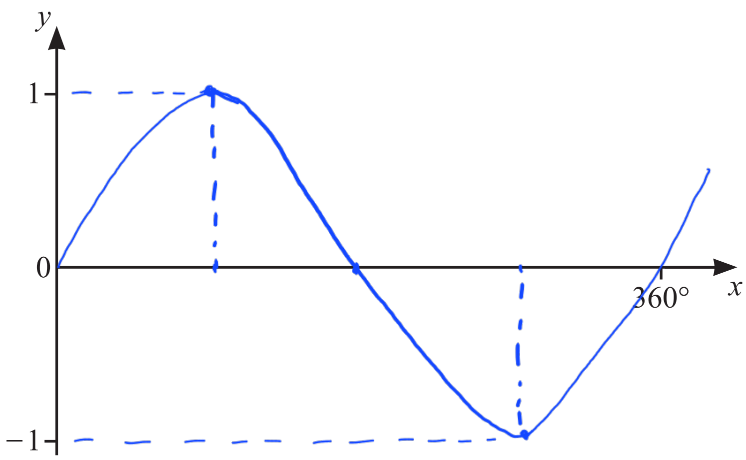
$\frac{1}{2}(32 - 3)(12 - 2)$

$= 145$



........145........ [2]

9



(a) On the diagram, sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$. [2]

(b) Solve the equation $5\sin x + 4 = 0$ for $0° \leqslant x \leqslant 360°$.

$5 \sin x = -4$

$\sin x = \frac{-4}{5}$

$x = -53.1°$ or $x = 180° - (-53.1°) = 233.1°$

or $x = -53.1° + 360° = 306.9°$

$x =$ ....233.1°.... or $x =$ ....306.9°.... [3]

10 **(a)** The lengths of the sides of a triangle are 11.4 cm, 14.8 cm and 15.7 cm, all correct to 1 decimal place.

0.1

Calculate the upper bound of the perimeter of the triangle.

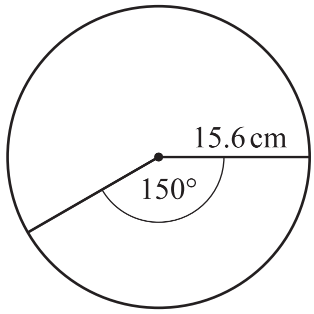
$\text{perimeter}_{max} = \text{side 1}_{max} + \text{side 2}_{max} + \text{side 3}_{max}$

$= \left(11.4 + \frac{0.1}{2}\right) + \left(14.8 + \frac{0.1}{2}\right) + \left(15.7 + \frac{0.1}{2}\right)$

$= 42.05$

....42.05.... cm [2]

(b) The diagram shows a circle, radius 15.6 cm.
The angle of the minor sector is 150°.



NOT TO SCALE

Calculate the area of the minor sector.

$$150^\circ = 150 \times \frac{\pi}{180} \text{ radian} = \frac{5\pi}{6} \text{ rad}$$

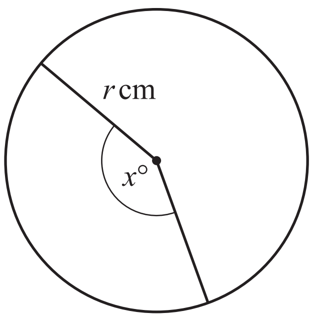
$$A_{\text{minor sector}} = \frac{1}{2} \times 15.6^2 \ \frac{5\pi}{6}$$

$$= 101.4\pi$$

$$\approx 319$$

.........319......... cm^2 [2]

(c)



NOT TO SCALE

The diagram shows a circle, radius r cm and minor sector angle $x°$.
The **perimeter** of the major sector is three times the **perimeter** of the minor sector.

Show that $x = \dfrac{90(\pi - 2)}{\pi}$.

$$x^\circ = x\frac{\pi}{180} \text{ radian} \Rightarrow \text{Perimeter}_{\text{minor sector}} = rx\frac{\pi}{180} + 2r$$

$$\text{Perimeter}_{\text{major sector}} = \left(2\pi r - rx\frac{\pi}{180}\right) + 2r$$

$$\Rightarrow 2\pi r - rx\frac{\pi}{180} + 2r = 3\left(rx\frac{\pi}{180} + 2r\right)$$

$$2\pi r + 2r - 6r = rx\frac{\pi}{60} + rx\frac{\pi}{180}$$

$$2\pi r - 4r = \frac{\pi}{45}rx$$

$$x = \frac{2\pi r - 4r}{\frac{\pi}{45}r} = \frac{2\pi - 4}{\frac{\pi}{45}} = \frac{90(\pi - 2)}{\pi}$$

[4]

11 (a) $\left|\begin{pmatrix} 9m \\ 40m \end{pmatrix}\right| = \dfrac{205}{2}$

Find the two possible values of m.

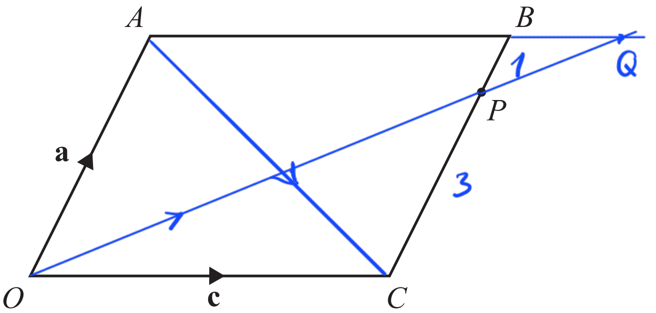
$$(9m)^2 + (40m)^2 = \left(\frac{205}{2}\right)^2$$

$$1681\,m^2 = 10\,506.25$$

$$m^2 = 6.25$$

m = ...2.5........ or ....-2.5........ [3]

(b)



NOT TO SCALE

$OABC$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.
P is the point on CB such that $CP : PB = 3 : 1$.

(i) Find, in terms of **a** and/or **c**, in their simplest form,

(a) $\overrightarrow{AC}$,

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$
$$= -a + c$$

$\overrightarrow{AC}$ = ...$-a + c$........ [1]

(b) $\overrightarrow{CP}$,

$$\overrightarrow{CP} = \frac{3}{4}\overrightarrow{CB} = \frac{3}{4}a$$

$\overrightarrow{CP}$ = ...$\frac{3}{4}a$........ [1]

(c) $\overrightarrow{OP}$.

$$\overrightarrow{OP} = \overrightarrow{OC} + \overrightarrow{CP}$$
$$= c + \frac{3}{4}a$$

$\overrightarrow{OP}$ = ...$c + \frac{3}{4}a$........ [1]

(ii) OP and AB are extended to meet at Q.

Find the position vector of Q.

$$\Delta BPQ \sim \Delta CPO \Rightarrow \frac{PQ}{PO} = \frac{BP}{CP} = \frac{1}{3}$$

$$\Rightarrow \frac{PQ}{1} = \frac{PO}{3} = \frac{PQ + PO}{1 + 3} = \frac{OQ}{4}$$

$$\Rightarrow OQ = \frac{4}{3}PO$$

$$\Rightarrow \overrightarrow{OQ} = \frac{4}{3}\overrightarrow{OP} = \frac{4}{3}\left(c + \frac{3}{4}a\right)$$

........$\frac{4}{3}c + a$........ [2]